Unit 18: Accelerated Test Models

Review: Log Location-Scale Model Representation

\[ \log T = \mu + \sigma Z, \quad P(Z \leq t) = \Phi(t) \]

\[ \iff \]

\[ P(T \leq t) = P(\log T \leq \log t) = P(\mu + \sigma Z \leq \log t) = P\left(Z \leq \frac{\log t - \mu}{\sigma}\right) = \Phi\left(\frac{\log t - \mu}{\sigma}\right) \]
Review: Log Location-Scale Families and the SAFT Model

Baseline Distribution at $x_0$:

$$P\left(T(x_0) \leq t\right) = \Phi\left(\frac{\log t - \mu(x_0)}{\sigma}\right)$$

SAFT Model:

$$T(x) = \frac{T(x_0)}{AF(x)}$$
Review: Log Location-Scale Families and the SAFT Model

$$P(T(x) \leq t) = P\left(\frac{T(x_0)}{AF(x)} \leq t\right)$$

$$= P(T(x_0) \leq t AF(x))$$

$$= \Phi\left(\frac{\log t AF(x) - \mu(x_0)}{\sigma}\right)$$

$$= \Phi\left(\frac{\log t - (\mu(x_0) - \log AF(x))}{\sigma}\right)$$

$$= \Phi\left(\frac{\log t - \mu(x)}{\sigma}\right)$$

$$\Leftrightarrow$$

$$\mu(x) = \mu(x_0) - \log AF(x)$$
Unit 18 Objectives

• Describe motivation and application of accelerated reliability testing
• Explain the connection between degradation, physical failure, and acceleration of reliability tests
• Examine the basis for temperature and humidity acceleration
• Examine the basis for voltage and pressure stress acceleration
• Show how to compute time-acceleration factors
• Review other accelerated test models and assumptions
Accelerated Test Increasingly Important

• Today’s manufacturers need to develop newer, higher technology products in record time while improving productivity, reliability, and quality
• Important Issues:
  – Rapid product development
  – Rapidly changing technologies
  – More complicated products with more components
  – Higher customer expectations for better reliability
Need for Accelerated Tests

• Need timely information on high reliability products
  – Modern products designed to last for years or decades
  – Accelerated Tests (ATs) used for timely assessment of reliability of product components and materials
  – Test at high levels of use rate, temperature, voltage, pressure, humidity, etc.
  – Estimate life at use conditions

• Note: Estimation/prediction from ATs involves extrapolation
Application of Accelerated Tests

- Applications of Accelerated Tests include:
  - Evaluating the effect of stress on life
  - Assessing component reliability
  - Demonstrating component reliability
  - Detecting failure modes
  - Comparing two or more competing products
  - Establishing safe warranty times
Methods of Acceleration

• Three fundamentally different methods of accelerating a reliability test:
  – Increase the use-rate of the product (e.g., test a toaster 400 times/day). Higher use rate reduces test time
  – Use elevated temperature or humidity to increase rate of failure-causing chemical/physical process
  – Increase stress (e.g., voltage or pressure) to make degrading units fail more quickly.

• Use a physical/chemical (preference) or empirical model relating degradation or lifetime at use conditions.
Change in Resistance Over Time of Carbon-Film Resistors
(Shiorni and Yanagisawa, 1979)
Accelerated Degradation Tests (ADTs)

• **Response:** Amount of degradation at points in time

• **Model components**
  – Model for degradation over time
  – A definition of failure as a function of degradation variable
  – Relationship(s) between degradation model parameters (e.g., chemical process reaction rates) and acceleration variables (e.g., temperature or humidity).
Breakdown Times in Minutes of a Mylar-Polyurethane Insulating Structure (from Kalkanis and Rosso, 1989)
Accelerated Life Tests (ALTs)

• Response
  – Failure time (or interval) for units that fail
  – Censoring time for units that do not fail

• Model Components:
  – Constant-stress time-to-failure distribution
  – Relationship(s) between one (or more) of the constant-stress model parameters and the accelerating variables
Use-Rate Acceleration

• **Basic Idea:** Increase use-rate to accelerate failure-causing wear or degradation

• **Examples:**
  – Running automobile engines or appliances continuously
  – Rapid cycling of relays and switches
  – Cycles to failure in fatigue testing

• **Simple assumption:**
  – Useful if life adequately modeled by cycles of operation.
    Reasonable if cycling simulates actual use and if test units return to steady state after each cycle.

• **More complicated situation:**
  – Wear rate or degradation rate depends on cycling frequency or product deteriorates in stand-by as well as during actual use
Elevated Temperature Acceleration of Chemical Reaction Rates

- The **Arrhenius model Reaction Rate**, $R_{\text{temp}}$, is

  $$R_{\text{temp}} = \gamma_0 \exp \left( \frac{-E_a}{k_B(\text{temp} \, ^\circ \text{C} + 273.15)} \right) = \gamma_0 \exp \left( -\frac{E_a \times 11605}{\text{temp} \, \text{K}} \right)$$

  where $\text{temp} \, \text{K} = \text{temp} \, ^\circ \text{C} + 273.15$ is temperature in degrees Kelvin and $k_B = 1/11605$ is Boltzmann’s constant in units of electron volts per K. The reaction activation energy, $E_a$, and $\gamma_0$ are characteristics of the product or material being tested.

- The reaction rate **Acceleration Factor** is

  $$A_{\text{F}}(\text{temp}, \text{temp}_U, E_a) = \frac{R_{\text{temp}}}{R_{\text{temp}_U}}$$

  $$= \exp \left[ E_a \left( \frac{11605}{\text{temp}_U \, \text{K}} - \frac{11605}{\text{temp} \, \text{K}} \right) \right]$$

- When $\text{temp} > \text{temp}_U$, $A_{\text{F}}(\text{temp}, \text{temp}_U, E_a) > 1$. 
Acceleration Factors for the SAFT Arrhenius Model

- Table 18.2 gives the **Temperature Differential Factors** (TDF)

\[
TDF = \left( \frac{11605}{\text{temp}_{\text{Low}} \, K} - \frac{11605}{\text{temp}_{\text{High}} \, K} \right).
\]

- Figure 18.3 gives

\[
AF(\text{temp}_{\text{High}}, \text{temp}_{\text{Low}}, E_a) = \exp (E_a \times TDF)
\]

- We use \( AF(\text{temp}) = AF(\text{temp}, \text{temp}_U, E_a) \) when \( \text{temp}_U \) and \( E_a \) are understood to be, respectively, product use temperature and reaction-specific activation energy.
Time-Acceleration Factor as a Function of Temperature Factor (Figure 18.3)
Nonlinear Degradation Reaction-Rate Acceleration

- Consider the simple chemical degradation path model
  \[ D(t; \text{temp}) = D_\infty \times \{1 - \exp [-R_U \times AF(\text{temp}) \times t]\} \]
  where \( R_U \) is the rate reaction at use temperature (\( \text{temp}_U \))
  and for \( \text{temp} > \text{temp}_U \), \( AF(\text{temp}) > 1 \).

- For \( D_\infty > 0 \), failure occurs when \( D(T; \text{temp}) > D_f \). Equating \( D(T; \text{temp}) \) to \( D_f \) and solving for failure time, \( T(\text{temp}) \), gives
  \[ T(\text{temp}) = \frac{T(\text{temp}_U)}{AF(\text{temp})} = \left[-\frac{1}{R_U} \log \left(1 - \frac{D_f}{D_\infty}\right)\right] \]
  where \( T(\text{temp}_U) \) is failure time at use conditions.

- This is an SAFT model.
SAFT Model from Nonlinear Degradation Path

\[ D(t; \text{temp}) = D_\infty \times \{1 - \exp[-R_U \times A.F(\text{temp}) \times t]\} \]

\[ D_\infty < 0 \]
The Arrhenius-Lognormal Regression Model

The Arrhenius-lognormal regression model is

\[ \Pr[T \leq t; \text{temp}] = \Phi_{\text{nor}} \left[ \frac{\log(t) - \mu}{\sigma} \right] \]

where

- \( \mu = \beta_0 + \beta_1 x \),
- \( x = 11605/(\text{temp \, K}) = 11605/(\text{temp \, °C} + 273.15) \)
- and \( \beta_1 = E_a \) is the activation energy
- \( \sigma \) is constant
- This implies that

\[ t_p(\text{temp}_U) = t_p(\text{temp}) \times \mathcal{A}\mathcal{F}(\text{temp}) \]
Example Arrhenius-Lognormal Life Model

\[ \log[t_p(\text{temp})] = \beta_0 + \beta_1 x + \Phi_{\text{nor}}^{-1}(p)\sigma \]

\[ \log T = \beta_0 + \beta_1 x + \sigma Z, \quad Z \sim N(0,1) \]
SAFT Model from Linear Degradation Paths

![Graph showing power drop in dB over hours for different temperatures (80, 150, 195, 237 degrees C).]
Linear Degradation Reaction-Rate Acceleration

Note:

\[
\exp(-x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots
\]

\Rightarrow

\[
x \simeq 1 - \exp(-x) \text{ if } x \text{ is small}
\]

If \( \mathcal{R}_U \times \mathcal{A} \mathcal{F}(\text{temp}) \times t \) is small so that \( \mathcal{D}(t) \) is small relative to \( \mathcal{D}_\infty \), then

\[
\mathcal{D}(t;\text{temp}) = \mathcal{D}_\infty \times \{1 - \exp[-\mathcal{R}_U \times \mathcal{A} \mathcal{F}(\text{temp}) \times t]\}
\]

\[
\approx \mathcal{D}_\infty \times \mathcal{R}_U \times \mathcal{A} \mathcal{F}(\text{temp}) \times t = \mathcal{R}_U^+ \times \mathcal{A} \mathcal{F}(\text{temp}) \times t
\]

is approximately linear in \( t \).
Linear Degradation Reaction-Rate Acceleration

- Also some degradation processes are linear in time:
  \[ D(t; \text{temp}) = R_U \times AF(\text{temp}) \times t. \]

- Failure occurs when \( D(T; \text{temp}) > D_f \). Equating \( D(T; \text{temp}) \) to \( D_f \) and solving for failure time, \( T(\text{temp}) \),
  \[ T(\text{temp}) = \frac{T(\text{temp}_U)}{AF(\text{temp})} \]
  where \( T(\text{temp}_U) = D_f/R_U \) is failure time at use conditions.

- This is an SAFT model and, for example, \( T(\text{temp}_U) \sim \text{WEIB}(\mu, \sigma) \) implies \( T(\text{temp}) \sim \text{WEIB} [\mu - \log(AF(\text{temp})), \sigma] \).
Non-SAFT Degradation Reaction-Rate Acceleration

Consider the more complicated chemical degradation path

\[ D(t; \text{temp}) = D_{1\infty} \times \{1 - \exp [-R_{1U} \times AF_1(\text{temp}) \times t]\} \\
+ D_{2\infty} \times \{1 - \exp [-R_{2U} \times AF_2(\text{temp}) \times t]\} \]

\(R_{1U}, R_{2U}\) are the rates of the reactions contributing to failure.

This is **not** an SAFT model. Temperature affects the two degradation processes differently, inducing a nonlinearity into the acceleration function relating times at two different temperatures.
Voltage Acceleration and Voltage Stress
Inverse Power Relationship

- Depending on the failure mode, voltage can be raised to:
  - Increase the strength of electric fields. This can accelerate some failure-causing reactions.
  - Increase the stress level (e.g., voltage stress relative to declining voltage strength).

- An empirical model for life at volt relative to use conditions volt$\_U$ is

\[ T(\text{volt}) = \frac{T(\text{volt}_U)}{A_F(\text{volt})} = \left( \frac{\text{volt}}{\text{volt}_U} \right)^{\beta_1} T(\text{volt}_U) \]

where $A_F(\text{volt}) = A_F(\text{volt}, \text{volt}_U, \beta_1)$,

\[ A_F(\text{volt}) = A_F(\text{volt}, \text{volt}_U, \beta_1) = \frac{T(\text{volt}_U)}{T(\text{volt})} = \left( \frac{\text{volt}}{\text{volt}_U} \right)^{-\beta_1} \]

and $\beta_1$ is a material characteristic. $T(\text{volt}), T(\text{volt}_U)$ are the failure times at increased voltage and use conditions.
Inverse Power Relationship-Weibull Model

The inverse power relationship-Weibull model is

\[ \Pr[T \leq t; \text{volt}] = \Phi_{\text{sev}} \left[ \frac{\log(t) - \mu}{\sigma} \right] \]

where

- \( \mu = \beta_0 + \beta_1 x \), and

- \( x = \log(\text{Voltage Stress}) \).

- \( \sigma \) assumed to be constant.
Example Weibull Inverse Power Relationship Between Life and Voltage Stress

\[ \log[t_p(\text{volt})] = \beta_0 + \beta_1 x + \Phi_{SEV(p)}^{-1}(p)\sigma \]

\[ \log T = \beta_0 + \beta_1 x + \sigma Z, \quad Z \sim \text{Gumbel}(0,1) \]
Other Commonly Used Life-Stress Relationships

Other commonly used SAFT models have the simple form:

$$T(x) = \frac{T(x_U)}{AF(x)}$$

where $AF(x) = AF(x, x_U, \beta_1) = \exp [\beta_1(x - x_U)]$. $\beta_1$ is a material characteristic.

Examples include:

- **Cycling rate**: $x = \log(\text{frequency})$.
- **Current density**: $x = \log(\text{current})$.
- **Size**: $x = \log(\text{thickness})$.
- **Humidity 1**: $x = \log(\text{RH})$, RH is % relative humidity/100.
- **Humidity 2**: $x = \log[\text{RH}/(1 - \text{RH})]$.

Some of these models are empirical. For a location-scale time-to-failure distribution $\mu = \beta_0 + \beta_1x$. 
Eyring Temperature Relationship

- Arrhenius relationship obtained from empirical observation.

- Eyring developed physical theory describing the effect that temperature has on a reaction rate:

\[
\mathcal{R}(\text{temp}) = \gamma_0 \times A(\text{temp}) \times \exp\left(\frac{-E_a}{k_B \times \text{temp K}}\right)
\]

- \(A(\text{temp})\) is a function of temperature depending on the specifics of the reaction dynamics; \(\gamma_0\) and \(E_a\) are constants.

- Applications in the literature have used \(A(\text{temp}) = (\text{temp K})^m\) with a fixed value of \(m\) ranging between \(m = 0\) (Boccaletti et al. 1989), \(m = .5\) (Klinger 1991a), to \(m = 1\) (Nelson 1990a and Mann Schafer and Singpurwalla 1974). Difficult to identify \(m\) from limited data.

- Eyring showed how to include other accelerating variables.
The Eyring Regression Model (e.g., for Weibull or Lognormal Distribution)

The Eyring temperature-acceleration regression model is

\[
Pr(T \leq t; \text{temp}) = \Phi \left[ \frac{\log(t) - \mu}{\sigma} \right]
\]

where

- \( \mu = -m \log(\text{temp}^\circ C + 273.15) + \beta_0 + \beta_1 x. \)
- \( x = 11605/(\text{temp}^\circ C + 273.15). \)
- \( \beta_1 = E_a \) is the activation energy.
- \( m \) is usually given; \( \sigma \) is constant, but usually unknown.
- With \( m > 0 \), Arrhenius provides a useful first order approximation to the Eyring model, with conservative extrapolation to lower temperatures.
Humidity Acceleration Models

- Useful for accelerating failure mechanisms involving corrosion and certain other kinds of chemical degradation.
- Often used in conjunction with elevated temperature.
- Most humidity models have been developed empirically.
- Empirical and limited theoretical results for corrosion on thin films (Gillen and Mead 1980, Peck 1986, and Klinger 1991b) suggest the use of RH instead of $P_v$ (vapor pressure) as the independent (or experimental) variable in humidity relationships when temperature is also to be varied.

- RH is the preferred variable because the change in life, as a function of RH, does not depend on temperature. That is,

$$\frac{\partial^2 \text{Life}}{\partial \text{RH} \partial \text{temp}} = 0$$

or no statistical interaction.
Humidity Regression Relationships

- Consider the Weibull/lognormal lifetime regression model

\[
\Pr(T \leq t; \text{humidity}) = \Phi_{\text{sev}} \left[ \frac{\log(t) - \mu}{\sigma} \right]
\]

where \( \mu = \beta_0 + \beta_1 x_1 \) and \( \sigma \) is constant. Letting \( 0 < \text{RH} < 1 \) denote \textbf{relative humidity}, possible humidity relationships are:

- \( x_1 = \text{RH} \) [Intel, empirical].
- \( x_1 = \log(\text{RH}) \) [Peck, empirical].
- \( x_1 = \log[\text{RH}/(1 - \text{RH})] \) [Klinger, corrosion on thin films].

- For temperature and humidity acceleration, possible relationships include

\[
\begin{align*}
\mu &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 \\
\text{or} \quad \mu &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 \\
\text{or} \quad \mu &= \beta_0 + \beta_2 x_2 + \beta_3 x_1 x_2
\end{align*}
\]

where \( x_2 = 11605/(\text{temp} \, ^\circ\text{C} + 273.15) \).
Temperature/Humidity Acceleration Factors with RH and Temp K and No Interaction

- **Peck’s relationship**

\[
AF(t_{\text{temp}}, RH) = \frac{R(t_{\text{temp}}, RH)}{R(t_{\text{temp}U}, RH_U)}
\]

\[
= \left( \frac{RH_U}{RH} \right)^{\beta_1} \exp \left[ E_\alpha \left( \frac{11605}{temp_U K} - \frac{11605}{temp K} \right) \right].
\]

- **Klinger’s relationship**

\[
AF(t_{\text{temp}}, RH) = \frac{R(t_{\text{temp}}, RH)}{R(t_{\text{temp}U}, RH_U)}
\]

\[
= \left( \frac{RH_U}{1 - RH_U} \right) \left( \frac{1 - RH}{RH} \right)^{\beta_1} \times \exp \left[ E_\alpha \left( \frac{11605}{temp_U K} - \frac{11605}{temp K} \right) \right].
\]
Thermal Cycling

- Fatigue is an important failure mechanism for many products and materials.

- Mechanical expansion and contraction from thermal cycling can lead to fatigue cracking and failure.

- Applications include:
  
  - Power-on/power-off cycling of electronic equipment and effect on component encapsulement and solder joints.
  - Take-off power-thrust in jet engines and its effect on crack initiation and growth in fan disks.
  - Power-up/power-down of nuclear power plants and effect on the growth of cracks in heat generator tubes.
  - Thermal inkjet printhead delamination could be caused by temperature cycling during normal use.
Coffin-Manson Relationship

- The **Coffin-Manson** relationship says that the *typical* number of cycles to failure is

\[ N = \frac{\delta}{(\Delta \text{temp})^{\beta_1}} \]

where \( \Delta \text{temp} \) is the temperature range and \( \delta \) and \( \beta_1 \) are properties of the material and test setup. This power-rule relationship explains the effect that temperature range has on thermal-fatigue life. For some metals, \( \beta_1 \approx 2 \).

- Letting \( T \) be the random number of cycles to failure (e.g., \( T = N \times \epsilon \) where \( \epsilon \) is a random variable), the acceleration factor when \( \Delta \text{temp} \), relative to the number of cycles when \( \Delta \text{temp}_U \), is

\[ A_F(\Delta \text{temp}) = \frac{T(\Delta \text{temp}_U)}{T(\Delta \text{temp})} = \left( \frac{\Delta \text{temp}}{\Delta \text{temp}_U} \right)^{\beta_1} \]

- There may be a \( \Delta \text{temp} \) threshold below which little or no fatigue damage is done during thermal cycling.
Generalized Coffin-Manson Relationship

- Empirical evidence has shown that effect of temperature cycling can depend importantly on $\text{temp}_{\max}K$, the maximum temperature in the cycling (e.g., if $\text{temp}_{\max}K$ is more than .2 or .3 times a metal’s melting point).

- The effect of temperature cycling can also depend on the cycling rate (e.g., due to heat buildup).

- An **empirical** extension of the Coffin-Manson relationship is

$$N = \frac{\delta}{(\Delta \text{temp})^{\beta_1}} \times \frac{1}{(\text{freq})^{\beta_2}} \times \exp \left( \frac{E_a \times 11605}{\text{temp}_{\max}K} \right)$$

where freq is the cycling frequency, and $E_a$ is an activation energy.

- Caution must be used when using such a model outside the range of available data and past experience.
Other Topics in Chapter 18

• Other accelerated degradation models and relationships to accelerated time models
• Discussion of stress-cycling models
• Other models for two or more experimental factors.