A Pull Planning Framework

We think in generalities, we live in detail.

– Alfred North Whitehead

Purpose of Production Control

Objective: Meet customer expectations with on-time delivery of correct quantities of desired specification without excessive lead times or large inventory levels.

Two Basic Approaches:

Push Systems: Material Requirements Planning
- General.
- Provides a planning hierarchy.
- Underlying model often inappropriate.

Pull Systems: Kanban
- Reduces congestion.
- Improves production environment.
- Suitable only for repetitive manufacturing.
Advantages of Pull

Advantages:

• **Observability**: we can see WIP but not capacity.

• **Efficiency**: pull systems require less average WIP to attain same throughput as equivalent push system.

• **Robustness**: pull systems are less sensitive to errors in WIP level than push systems are to errors in release rate.

• **Quality**: pull systems require and promote improved quality.

Magic of Pull: WIP Cap

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A Dilemma

**Question**: If pull is so great, why do people still buy ERP systems?

**Answer**: Manufacturing involves *planning* as well as *execution*.

<table>
<thead>
<tr>
<th></th>
<th>Planning</th>
<th>Execution</th>
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<tbody>
<tr>
<td>Push</td>
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<tr>
<td>Pull</td>
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Hierarchical Pull Planning Framework

Goals:
- To attain the benefits of a pull environment.
- To gain the generality of hierarchical production planning systems.

The Environment:
- CONWIP production lines.
- Daily/Weekly production quota.

The Hierarchy:
- Based on CONWIP for predictability and generality.
- Consistency between levels.
- Accommodate different implementations of modules for different environments.
- Use feedback.
CONWIP as the Foundation

Pull:
- jobs into the line whenever parts are used.
- jobs with the same routing.
- jobs for different part numbers.

Push:
- jobs between stations on line.
- jobs into buffer storage between lines.

A CONWIP Line:
- represents a level in a bill of material.
- is between stock points.
- maintains a constant amount of work in process.
Benefits of CONWIP

CONWIP vs. Push:
- Easier and more robust control.
- Less congestion.
- Greater predictability.

CONWIP vs. Kanban:
- Can accommodate a changing product mix.
- Can be used with setups.
- Suitable for short runs of small lots.
- More predictable.

Conveyor Model of CONWIP

Predicting Completion Times:
- Practical production rate: $r_P$ parts per hour
- Minimum practical lead time: $T_P$ hours
- $X_i$ is number of parts in job $i$ on the backlog.
- Then the expected completion time of the $n^{th}$ job, $c_n$, will be:
  $$c_n = \sum_{i=1}^{n-1} \frac{X_i}{r_P} + T_P$$

Quoting Due Dates: need to add a “fudge factor” (which should consider cycle time variability) to ensure a reasonable service level.
Aggregating Planning by Time Horizon

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<tr>
<th>Time Horizon</th>
<th>Length</th>
<th>Representative Decisions</th>
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<td>Decisions, Process Control, Quality Compliance Decisions,</td>
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<td>Emergency Equipment Repairs</td>
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</table>

Other Levels of Aggregation

**Processes:** Treat several workstations as one. Leave out unimportant (never bottleneck) workstations.

**Products:** Group different part numbers into product families, which have
- have roughly the same routing
- have roughly the same price
- share setups

**Personnel:** Categorize people according to
- management vs. labor
- shift
- workstation
- craft
- permanent vs. temporary
**Forecasting**

**Basic Problem:** predict demand for planning purposes.

**Laws of Forecasting:**
1. *Forecasts are always wrong!*
2. *Forecasts always change!*
3. *The further into the future, the less reliable the forecast will be!*

**Forecasting Tools:**
- **Qualitative:**
  - Delphi
  - Analogies
  - Many others
- **Quantitative:**
  - Causal models (e.g., regression models)
  - Time series models

**Capacity/Facility Planning**

**Basic Problem:** how much and what kind of physical equipment is needed to support production goals?

**Issues:**
- **Basic Capacity Calculations:** stand-alone capacities and congestion effects (e.g., blocking)
- **Capacity Strategy:** lead or follow demand
- **Make-or-Buy:** vendoring, long-term identity
- **Flexibility:** with regard to product, volume, mix
- **Speed:** scalability, learning curves
Workforce Planning

**Basic Problem:** how much and what kind of labor is needed to support production goals?

**Issues:**

- **Basic Staffing Calculations:** standard labor hours adjusted for worker availability.
- **Working Environment:** stability, morale, learning.
- **Flexibility/Agility:** ability of workforce to support plant's ability to respond to short and long term shifts.
- **Quality:** procedures are only as good as the people who carry them out.

Aggregate Planning

**Basic Problem:** generate a long-term production plan that establishes a rough product mix, anticipates bottlenecks, and is consistent with capacity and workforce plans.

**Issues:**

- **Aggregation:** product families and time periods must be set appropriately for the environment.
- **Coordination:** AP is the link between the high level functions of forecasting/capacity planning and intermediate level functions of quota setting and scheduling.
- **Anticipating Execution:** AP is virtually always done deterministically, while production is carried out in a stochastic environment.
- **Linear Programming:** is a powerful tool well-suited to AP and other optimization problems.
Quota Setting

**Basic Problem:** set target production quota for pull system

**Issues:** Larger quotas yield

**Benefits:**
- Increased throughput.
- Increased utilization.
- Lower unit labor hour.
- Lower allocation of overhead.

**Costs:**
- More overtime.
- Higher WIP levels.
- More expediting.
- Increased difficulties in quality control.

Planned Catch-Up Times

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Economic Production Quota Notation

\[ p = \text{unit profit} \]
\[ C_{or} = \text{fixed overtime cost} \]
\[ Y = \text{regular time production (random variable)} \]
\[ \mu = \text{mean regular time production (} E[Y] \text{)} \]
\[ \sigma = \text{std dev of regular time production (} \sqrt{\text{Var}(Y)} \text{)} \]
\[ M = \text{maximum overtime production} \]
\[ Q = \text{regular time production quota (decision variable)} \]

Simple “Sell-All-You-Can-Make” Model

**Objective Function:** Average weekly profit

\[ \max_{Q} Z = pQ - C_{or} \Pr\{Y \leq Q\} \]

**Reasonability Test:** We want the probability of not being able to catch up on overtime to be small (i.e., \( \alpha \)):

\[ \Pr\{Q^* - Y > M\} \leq \alpha \]

If this is not true, another (lost sales) model should be used.
Simple “Sell-All-You-Can-Make” Model (cont.)

Normal Approximation: Express $Q = \mu - k\sigma$, so the objective and reasonability test can be written:

$$\max_k Z = p(\mu - k\sigma) - C_{ot}(1 - \Phi(k))$$

$$\Phi(k + M / \sigma) \geq 1 - \alpha$$

Solution: The objective function is maximized by:

$$k^* = \sqrt{2\ln\left(\frac{C_{ot}}{\sqrt{2\pi\sigma p}}\right)}$$

$$Q^* = \mu - k^*\sigma$$ buffer capacity

Intuition from Model

- Optimal production quota depends on both mean and variance of regular time production ($Q^*$ increases with $\mu$ and decreases with $\sigma$).
- Increasing capacity increases profit, since
  $$\frac{\partial Z^*}{\partial \mu} = p$$
- Decreasing variance increases profit, since
  $$\frac{\partial Z^*}{\partial \sigma} = -pk^*$$
- Model is valid (i.e., has a solution $0 < k^* < \infty$) only if
  $$p \leq \frac{C_{ot}}{\sqrt{2\pi\sigma}}$$
  since otherwise the term in the $\sqrt{\cdot}$ becomes negative. If this occurs, then OT cost does not exceed revenue lost to make-up period and a different model is required.
Other Quota Setting Models

Model 2: Lost Sales
- Run continuously.
- Choose periodic production quota $Q$.
- Demand above $Q$ is lost (or vendored) at a cost.
- Solution looks like that to the Newsboy problem

Model 3: Fixed plus Variable Cost of Overtime
- Same as Model 1, except that cost of overtime has a fixed component, $C_{ot}$, and a component proportional to the amount of the shortage.
- Solution looks like that to Model 1 except term under $\sqrt{\cdot}$ is more complex.

Other Quota Setting Models (cont.)

Model 4: Backlogging
- Fixed plus variable cost of overtime.
- Decision maker can choose to carry shortage to next period at a cost.
- Dependence between periods requires more sophisticated solution techniques (e.g., dynamic programming).
- Solution consists of $Q^*$, optimal quota, plus $S^*$, an “overtime trigger” such that we use overtime only if the shortage is at least $S$. 
Quota Setting Implementation

- Iteration between quota setting and aggregate planning may be necessary for consistency.

- Motivation (setting the “bar”) vs. Prediction (quoting due dates).

- MPS smoothing – necessary to keep steady quota.

- Gross capacity control through shift addition/deletion, rather than production slow-down.

Setting WIP Levels

Basic Problem: establish WIP levels (card counts) in pull system.

Issues:
- Mean regular time production increases with WIP level.
- Variance of regular time production also affected by WIP level.
- WIP levels should be set to facilitate desired throughput.
- Adjustment may be necessary as system evolves (feedback).
- Easy method:
  1. Specify feasible cycle time, CT, and identify practical production rate, \( r_P \).
  2. Set WIP from

\[
WIP = r_P \times CT
\]
Demand Management

**Basic Problem:** establish an interface between the customer and the plant floor, that supports both competitive customer service and workable production schedules.

**Issues:**
- **Customer Lead Times:** shorter is more competitive.
- **Customer Service:** on-time delivery.
- **Batching:** grouping like product families can reduce lost capacity due to setups.
- **Interface with Scheduling:** customer due dates are an enormously important control in the overall scheduling process.

Sequencing and Scheduling

**Basic Problem:** develop a plan to guide the release of work into the system and coordination with needed resources (e.g., machines, staffing, materials).

**Methods:**
- **Sequencing:**
  - Gives order of releases but not times.
  - Adequate for simple CONWIP lines where FISFO is maintained.
  - The “CONWIP backlog.”
- **Scheduling:**
  - Gives detailed release times.
  - Attractive where complex routings make simple sequence impractical.
  - MRP-C.
Sequencing CONWIP Lines

Objectives:
- Maximize profit.
- No late jobs.
- All firm jobs selected.

Job Sequencing System:
- Sequences bottleneck line.
- Uses Quota to explicitly consider capacity.
- Tries to group like families of jobs to reduce setups.
- Identifies the “offensive” jobs in an infeasible schedule.
- Suggests when more work could start in a lightly loaded schedule.
- Provides sequence for other lines.

Real-Time Simulation

Basic Problem: anticipate problems in schedule execution and provide vehicle for exploring solutions.

Approaches:
- **Deterministic Simulation:**
  - Given release schedule and dispatching rules, predict output times.
  - Commercial packages (e.g., FACTOR).
- **Conveyor Model:**
  - Allow hot jobs to pass in buffers, not in the lines.
  - Use simplified simulation based on conveyor model to predict output times.
Shop Floor Control

**Basic Problem:** control flow of work through plant and coordinate with other activities (e.g., quality control, preventive maintenance, etc.)

**Issues:**

- **Customization:** SFC is often the most highly customized activity in a plant.
- **Information Collection:** SFC represents the interface with the actual production processes and is therefore a good place to collect data.
- **Simplicity:** departures from simple mechanisms must be carefully justified.

Tracking and Feedback

**Basic Problems:**

- Signal quota shortfall.
- Update capacity data.
- Quote delivery dates.

**Functions:**

**Statistical Throughput Control:**

- Monitored at critical tools.
- Like SPC, only measuring throughput.
- Problems are apparent with time to act.
- Workers aware of situation.

**Feedback:**

- Collect capacity data.
- Measure continual improvement.
Conclusions

Pull Environment Provides:
- Less WIP and thereby earlier detection of quality problems.
- Shorter lead times allowing increased customer response and less reliance on forecasts.
- Less buffer stock and therefore less exposure to schedule and engineering changes.

CONWIP Provides: a pull environment that
- Has greater throughput for equivalent WIP than kanban.
- Can accommodate a changing product mix.
- Can be used with setups.
- Is suitable for short runs of small lots.
- Is predictable.

Conclusions (cont.)

Planning Hierarchy Provides:
- Consistent framework for planning.
- Links between levels.
- Feedback.
Forecasting

The future is made of the same stuff as the present.

– Simone Weil

Forecasting “Laws”

1) Forecasts are always wrong!
2) Forecasts always change!
3) The further into the future, the less reliable the forecast!

Trumpet of Doom

Start of season

40%

20%

+10%

-10%

16 weeks

26 weeks
Quantitative Forecasting

Goals:
• Predict future from past
• Smooth out “noise”
• Standardize forecasting procedure

Methodologies:
• Causal Forecasting:
  – regression analysis
  – other approaches
• Time Series Forecasting:
  – moving average
  – exponential smoothing
  – regression analysis
  – seasonal models
  – many others

Time Series Forecasting

Historical Data   Forecast

\[ A(i), i = 1, \ldots, t \] \rightarrow \text{Time series model} \rightarrow f(t+t), i = 1, 2, \ldots

Time Series Approach

Notation:

\[ A(i) = \text{observation in period } i \]

\[ t = \text{current period} \]

\[ f(t + \tau) = \text{forecast for period } t + \tau \]

\[ F(t) = \text{smoothed estimate as of period } t \]

\[ T(t) = \text{smoothed trend as of period } t \]

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Time Series Approach (cont.)

Procedure:

1. Select model that computes \( f(t + \tau) \) from \( A(i) \), \( i = 1, \ldots, t \)

2. Forecast existing data and evaluate quality of fit by using:

\[
\text{MAD} = \frac{\sum |f(t) - A(t)|}{n}
\]

\[
\text{MSD} = \frac{\sum (f(t) - A(t))^2}{n}
\]

\[
\text{BIAS} = \frac{\sum (f(t) - A(t))}{n}
\]

3. Stop if fit is acceptable. Otherwise, adjust model constants and go to (2) or reject model and go to (1).
Moving Average

Assumptions:
• No trend
• Equal weight to last \( m \) observations

Model:

\[
F(t) = \frac{\sum_{i=1}^{m} A(i)}{t}
\]

\[
f(t+\tau) = F(t), \quad \tau = 1, 2, ...
\]

Moving Average (cont.)

Example: Moving Average with \( m = 3 \) and \( m = 5 \).

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<th>Month</th>
<th>Demand A(t)</th>
<th>Forecast (m=3)</th>
<th>Forecast (m=5)</th>
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Note: bigger \( m \) makes forecast more stable, but less responsive.
Exponential Smoothing

Assumptions:
- No trend
- Exponentially declining weight given to past observations

Model:
\[ F(t) = \alpha A(t) + (1-\alpha) F(t-1) \]
\[ f(t+\tau) = F(t), \quad \tau = 1, 2, \ldots \]

Example: Exponential Smoothing with \( \alpha = 0.2 \) and \( \alpha = 0.6 \).

<table>
<thead>
<tr>
<th>Month</th>
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<th>Forecast (( \alpha = 0.6 ))</th>
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Note: we are still lagging behind actual values.
Exponential Smoothing with a Trend

Assumptions:
• Linear trend
• Exponentially declining weights to past observations/trends

Model:
\[ F(t) = \alpha A(t) + (1-\alpha)(F(t-1)+T(t-1)) \]
\[ T(t) = \beta (F(t) - F(t-1)) + (1 - \beta) T(t-1) \]
\[ f(t+\tau) = F(t) + \tau T(t) \]

Note: these calculations are easy, but there is some "art" in choosing \( F(0) \) and \( T(0) \) to start the time series.

Exponential Smoothing with a Trend (cont.)

Example: Exponential Smoothing with Trend, \( \alpha = 0.2, \beta = 0.5 \).

Note: we start with trend equal to difference between first two demands.

<table>
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<tr>
<th>Month</th>
<th>Demand</th>
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<th>Smoothed Estimate ( F(t) )</th>
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Exponential Smoothing with a Trend (cont.)

Example: Exponential Smoothing with Trend, $\alpha = 0.2$, $\beta = 0.5$.  

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Note: we start with trend equal to zero.

Effects of Altering Smoothing Constants

Exponential Smoothing with Trend: various values of $\alpha$ and $\beta$

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<tr>
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<th>BIAS</th>
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Note: these assume we start with trend equal diff between first two demands.
Effects of Altering Smoothing Constants

Exponential Smoothing with Trend: various values of $\alpha$ and $\beta$

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<th>MSD</th>
<th>BIAS</th>
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Note: these assume we start with trend equal to zero.

Observations: assuming we start with zero trend

- $\alpha = 0.3$, $\beta = 0.5$ work well for MAD and MSD
- $\alpha = 0.6$, $\beta = 0.6$ work better for BIAS
- Our original choice of $\alpha = 0.2$, $\beta = 0.5$ had MAD = 3.73, MSD = 22.32, BIAS = -2.02, which is pretty good, although $\alpha = 0.3$, $\beta = 0.6$, with MAD = 3.65, MSD = 21.78, BIAS = -1.52 is better.
Winters Method for Seasonal Series

**Seasonal series:** a series that has a pattern that repeats every \( N \) periods for some value of \( N \) (which is at least 3).

**Seasonal factors:** a set of multipliers \( c_t \), representing the average amount that the demand in the \( t \)th period of the season is above or below the overall average.

**Winter’s Method:**
- The series:
  \[
  F(t) = \alpha(A(t) / c(t - N)) + (1 - \alpha)(F(t - 1) + T(t - 1))
  \]
- The trend:
  \[
  T(t) = \beta(F(t) - F(t - 1) + (1 - \beta)T(t - 1)
  \]
- The seasonal factors:
  \[
  c(t) = \gamma(A(t) / F(t)) + (1 - \gamma)\lambda(c(t - N)
  \]
- The forecast:
  \[
  f(t + \tau) = (F(t) + \tau T(t))c(t)
  \]

| Year | Quarter | t | A(t) | F(t) | T(t) | c(t) | f(t) | f(t)-A(t) | |f(t)-A(t)| (f(t)-A(t))^2 |
|------|---------|---|------|------|------|------|------|----------|----------------|------------------|
| 1997 | 1       | 1 | 4    | ---  | ---  | 0.480| ---  | 0.480    | 0.480          | 2.304            |
|      | 2       | 2 | 2    | ---  | ---  | 0.240| ---  | 0.240    | 0.240          | 0.576            |
|      | 3       | 3 | 5    | ---  | ---  | 0.600| ---  | 0.600    | 0.600          | 3.600            |
|      | 4       | 4 | 8    | ---  | ---  | 0.960| ---  | 0.960    | 0.960          | 9.216            |
|      | 5       | 5 | 11   | ---  | ---  | 1.320| ---  | 1.320    | 1.320          | 1.744            |
|      | 6       | 6 | 13   | ---  | ---  | 1.560| ---  | 1.560    | 1.560          | 2.384            |
|      | 7       | 7 | 18   | ---  | ---  | 2.160| ---  | 2.160    | 2.160          | 4.624            |
|      | 8       | 8 | 15   | ---  | ---  | 1.800| ---  | 1.800    | 1.800          | 3.240            |
|      | 9       | 9 | 9    | ---  | ---  | 1.080| ---  | 1.080    | 1.080          | 1.156            |
|      | 10      | 10| 6    | ---  | ---  | 0.720| ---  | 0.720    | 0.720          | 0.512            |
|      | 11      | 11| 5    | ---  | ---  | 0.600| ---  | 0.600    | 0.600          | 0.360            |
|      | 12      | 12| 4    | 8.33 | 0.00 | 0.480| ---  | 0.480    | 0.480          | 2.304            |
| 1998 | 1       | 13| 5    | 8.54 | 0.02 | 0.491| 4.00 | -1.00    | 1.00           | 1.00             |
|      | 2       | 14| 4    | 9.37 | 0.10 | 0.259| 2.96 | -1.95    | 1.945          | 3.78             |
|      | 3       | 15| 7    | 8.89 | 0.12 | 0.612| 5.66 | -1.32    | 1.31513        | 1.73             |
|      | 4       | 16| 7    | 9.57 | 0.10 | 0.937| 9.43 | 2.43     | 2.42506        | 5.88             |
|      | 5       | 17| 15   | 9.83 | 0.12 | 1.341| 12.76| -2.24    | 2.24392        | 5.04             |
|      | 6       | 18| 17   | 10.04| 0.13 | 1.573| 15.52| -1.48    | 1.47921        | 2.19             |
|      | 7       | 19| 24   | 10.26| 0.13 | 2.178| 21.97| -2.03    | 2.03484        | 4.14             |
|      | 8       | 20| 18   | 10.36| 0.13 | 1.794| 18.72| 0.72     | 0.71585        | 0.51             |
|      | 9       | 21| 12   | 10.55| 0.14 | 1.086| 11.33| -0.67    | 0.67254        | 0.45             |
|      | 10      | 22| 7    | 10.59| 0.13 | 0.714| 7.69 | 0.69     | 0.69489        | 0.48             |
|      | 11      | 23| 8    | 10.98| 0.15 | 0.613| 6.43 | -1.57    | 1.56928       | 2.46             |
|      | 12      | 24| 6    | 11.27| 0.17 | 0.485| 5.34 | -0.66    | 0.65635        | 0.43             |

**alpha** 0.100  **beta** 0.100  **gamma** 0.100

- **bias** 1.40  **MAD** 2.34  **MSD** 5.2
**Winters Method - Sample Calculations**

Initially we set:
- smoothed estimate = first season average
- smoothed trend = zero \( T(N) = T(12) = 0 \)
- seasonality factor = ratio of actual to average demand

\[
F(12) = \frac{\sum_{i=1}^{12} A(i)}{12} = \frac{4 + 2 + \ldots + 4}{12} = 8.33
\]

\[
c(l) = \frac{A(l)}{F(12)} = \frac{4}{8.33} = 0.480
\]

\[
F(13) = \alpha(A(13)/c(13-12) + (1-\alpha)/(F(12) + T(12)) = 0.1(5/0.480) + (1-0.1)(8.33 + 0) = 8.54
\]

\[
T(13) = \beta(F(13) - F(12)) + (1-\beta)T(12) = 0.1(8.54 - 8.33) + (1-0.1)(0) = 0.02
\]

\[
c(13) = \gamma(A(13)/F(13)) + (1-\gamma)c(l) = 0.1(5/8.54) + (1-0.1)(0.48) = 0.491
\]

From period 13 on we can use initial values and standard formulas...

---

**Winters Method Example**

![Graph showing demand over months with data points and trend lines](http://factory-physics.com)
Conclusions

**Sensitivity:** Lower values of $m$ or higher values of $\alpha$ will make moving average and exponential smoothing models (without trend) more sensitive to data changes (and hence less stable).

**Trends:** Models without a trend will underestimate observations in time series with an increasing trend and overestimate observations in time series with a decreasing trend.

**Smoothing Constants:** Choosing smoothing constants is an art; the best we can do is choose constants that fit past data reasonably well.

**Seasonality:** Methods exist for fitting time series with seasonal behavior (e.g., Winters method), but require more past data to fit than the simpler models.

**Judgement:** No time series model can anticipate structural changes not signaled by past observations; these require judicious overriding of the model by the user.