Basic Factory Dynamics

*Physics should be explained as simply as possible, but no simpler.*

– Albert Einstein

HAL Case

**Large Panel Line:** produces unpopulated printed circuit boards

**Line runs 24 hr/day**

**Recent Performance:**

- throughput = 1,100 panels per day (45.8 panels/hr)
- WIP = 37,000 panels
- CT = 34 days (816 hr)
- customer service = 75% on-time delivery

*How is HAL doing?*

*What data do we need to decide?*
HAL - Large Panel Line Processes

**Lamination:** press copper and prepreg into core blanks
**Machining:** trim cores to size
**Circuitize:** etch circuitry into copper
**Optical Test and Repair:** scan panels optically for defects
**Drilling:** holes to provide connections between layers
**Copper Plate:** deposits copper in holes to establish connections
**Procoat:** apply plastic coating to protect boards
**Sizing:** cut panels into boards
**End of Line Test:** final electrical test

HAL Case - Science?

**External Benchmarking**
- but other plants may not be comparable

**Internal Benchmarking**
- capacity data: what is utilization?
- but this ignores WIP effects

*Need relationships between WIP, TH, CT, service!*
Definitions

**Workstations:** a collection of one or more identical machines.

**Parts:** a component, sub-assembly, or an assembly that moves through the workstations.

**End Items:** parts sold directly to customers; relationship to constituent parts defined in *bill of material*.

**Consumables:** bits, chemicals, gasses, etc., used in process but do not become part of the product that is sold.

**Routing:** sequence of workstations needed to make a part.

**Order:** request from customer.

**Job:** transfer quantity on the line.

---

Definitions (cont.)

**Throughput (TH):** for a line, throughput is the average quantity of *good* (non-defective) parts produced per unit time.

**Work in Process (WIP):** inventory between the start and endpoints of a product routing.

**Raw Material Inventory (RMI):** material stocked at beginning of routing.

**Crib and Finished Goods Inventory (FGI):** crib inventory is material held in a stockpoint at the end of a routing; FGI is material held in inventory prior to shipping to the customer.

**Cycle Time (CT):** time between release of the job at the beginning of the routing until it reaches an inventory point at the end of the routing.
Factory Physics

Definition: A manufacturing system is a network of processes through which parts flow and whose purpose is to generate profit now and in the future.

Structure: Plant is made up of routings (lines), which in turn are made up of processes.

Focus: Factory Physics is concerned with the network and flows at the routing (line) level.

Parameters

Descriptors of a Line: 

1) Bottleneck Rate \((r_b)\): Rate (parts/unit time or jobs/unit time) of the process center having the highest long-term utilization.

2) Raw Process Time \((T_0)\): Sum of the long-term average process times of each station in the line.

3) Congestion Coefficient \((\alpha)\): A unitless measure of congestion.
   
   - Zero variability case, \(\alpha = 0\).
   - “Practical worst case,” \(\alpha = 1\).
   - “Worst possible case,” \(\alpha = W_0\).

   Note: we won’t use \(\alpha\) quantitatively, but point it out to recognize that lines with same \(r_b\) and \(T_0\) can behave very differently.
Parameters (cont.)

Relationship:

**Critical WIP** ($W_0$): WIP level in which a line having no congestion would achieve maximum throughput (i.e., $r_b$) with minimum cycle time (i.e., $T_0$).

\[ W_0 = r_b T_0 \]

The Penny Fab

**Characteristics:**
- Four identical tools in series.
- Each takes 2 hours per piece (penny).
- No variability.
- CONWIP job releases.

**Parameters:**

- $r_b = 0.5$ pennies/hour
- $T_0 = 8$ hours
- $W_0 = 0.5 \times 8 = 4$ pennies
- $\alpha = 0$ (no variability, best case conditions)
## The Penny Fab

<table>
<thead>
<tr>
<th>WIP</th>
<th>TH</th>
<th>CT</th>
<th>TH x CT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## TH vs. WIP: Best Case

Throughput (Jobs/hr) vs. WIP (Jobs) chart with axes indicating throughput ranging from 0 to 0.5 and WIP ranging from 0 to 14.
Best Case Law: The minimum cycle time ($CT_{\text{best}}$) for a given WIP level, $w$, is given by

$$CT_{\text{best}} = \begin{cases} T_0, & \text{if } w \leq W_0 \\ w / r_b, & \text{otherwise.} \end{cases}$$

The maximum throughput ($TH_{\text{best}}$) for a given WIP level, $w$ is given by,

$$TH_{\text{best}} = \begin{cases} w / T_0, & \text{if } w \leq W_0 \\ r_b, & \text{otherwise.} \end{cases}$$
Best Case Performance (cont.)

**Example:** For Penny Fab, \( r_b = 0.5 \) and \( T_0 = 8 \), so \( W_0 = 0.5 \times 8 = 4 \).

\[
CT_{\text{best}} = \begin{cases} 
8, & \text{if } w \leq 4 \\
2w, & \text{otherwise.}
\end{cases}
\]

\[
TH_{\text{best}} = \begin{cases} 
w / 8, & \text{if } w \leq 4 \\
0.5, & \text{otherwise.}
\end{cases}
\]

which are exactly the curves we plotted.

---

A Manufacturing Law

**Little’s Law:** The fundamental relation between WIP, CT, and TH over the long-term is:

\[
WIP = TH \times CT
\]

units = \( \frac{\text{units}}{\text{hr}} \times \text{hrs} \)

**Examples:**
- Checking WIP levels in cash flow analysis.
- Measure of cycle time (e.g., what is cycle time for an automobile?)
- FGI and planned inventory.
**Penny Fab Two**

<table>
<thead>
<tr>
<th>Station Number</th>
<th>Number of Machines</th>
<th>Process Time</th>
<th>Station Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2 hr</td>
<td>j/hr</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>5 hr</td>
<td>j/hr</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>10 hr</td>
<td>j/hr</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3 hr</td>
<td>j/hr</td>
</tr>
</tbody>
</table>

\[
r_s = \quad T_0 = \quad W_0 =
\]

---

**Penny Fab Two**

<table>
<thead>
<tr>
<th>Station Number</th>
<th>Number of Machines</th>
<th>Process Time</th>
<th>Station Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2 hr</td>
<td>j/hr</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>5 hr</td>
<td>j/hr</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>10 hr</td>
<td>j/hr</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3 hr</td>
<td>j/hr</td>
</tr>
</tbody>
</table>

\[
r_s = \quad T_0 = \quad W_0 =
\]
Worst Case

Observation: The Best Case yields the minimum cycle time and maximum throughput for each WIP level.

Question: What conditions would cause the maximum cycle time and minimum throughput?

Experiment:
- set average process times same as Best Case (so $r_b$ and $T_0$ unchanged)
- follow a marked job through system
- imagine marked job experiences maximum queueing

Worst Case Penny Fab

Time = 0 hours
Worst Case Penny Fab

Time = 8 hours

Worst Case Penny Fab

Time = 16 hours
Worst Case Penny Fab

Time = 24 hours

Note:
\[ CT = 32 \text{ hours} \]
\[ = 4 \times 8 = wT_0 \]
\[ TH = 4/32 = 1/8 = 1/T_0 \]
TH vs. WIP: Worst Case

CT vs. WIP: Worst Case
Worst Case Performance

**Worst Case Law:** The worst case cycle time for a given WIP level, \(w\), is given by,

\[
CT_{\text{worst}} = w T_0
\]

The worst case throughput for a given WIP level, \(w\), is given by,

\[
TH_{\text{worst}} = 1 / T_0
\]

Randomness? None - perfectly predictable, but bad!

Practical Worst Case

**Observation:** There is a **BIG GAP** between the Best Case and Worst Case performance.

**Question:** Can we find an intermediate case that:
- divides “good” and “bad” lines, and
- is computable?

**Experiment:** consider a line with a given \(r_b\) and \(T_0\) and:
- single machine stations
- balanced lines
- variability such that all WIP configurations (states) are equally likely
**PWC Example – 3 jobs, 4 stations**

<table>
<thead>
<tr>
<th>State</th>
<th>Vector</th>
<th>State</th>
<th>Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(3,0,0,0)</td>
<td>11</td>
<td>(1,0,2,0)</td>
</tr>
<tr>
<td>2</td>
<td>(0,3,0,0)</td>
<td>12</td>
<td>(0,1,2,0)</td>
</tr>
<tr>
<td>3</td>
<td>(0,0,3,0)</td>
<td>13</td>
<td>(0,0,2,1)</td>
</tr>
<tr>
<td>4</td>
<td>(0,0,0,3)</td>
<td>14</td>
<td>(1,0,0,2)</td>
</tr>
<tr>
<td>5</td>
<td>(2,1,0,0)</td>
<td>15</td>
<td>(0,1,0,2)</td>
</tr>
<tr>
<td>6</td>
<td>(2,0,1,0)</td>
<td>16</td>
<td>(0,0,1,2)</td>
</tr>
<tr>
<td>7</td>
<td>(2,0,0,1)</td>
<td>17</td>
<td>(1,1,1,0)</td>
</tr>
<tr>
<td>8</td>
<td>(1,2,0,0)</td>
<td>18</td>
<td>(1,1,0,1)</td>
</tr>
<tr>
<td>9</td>
<td>(0,2,1,0)</td>
<td>19</td>
<td>(1,0,1,1)</td>
</tr>
<tr>
<td>10</td>
<td>(0,2,0,1)</td>
<td>20</td>
<td>(0,1,1,1)</td>
</tr>
</tbody>
</table>

*Note: average WIP at any station is 15/20 = 0.75, so jobs are spread evenly between stations.*

**Practical Worst Case**

Let \( w = \) jobs in system, \( N = \) no. stations in line, and \( t = \) process time at all stations:

\[
CT(\text{single}) = \left(1 + \frac{w-1}{N}\right)t \\
CT(\text{line}) = N \left[1 + \frac{(w-1)}{N}\right] t \\
= Nt + (w-1)t \\
= T_0 + (w-1)/r_b \\
TH = \frac{WIP}{CT} \quad \text{From Little’s Law} \\
= \frac{w}{(w+W_0-1)}r_b
\]
Practical Worst Case Performance

**Practical Worst Case Definition:** The practical worst case (PWC) cycle time for a given WIP level, $w$, is given by,

$$CT_{PWC} = T_0 + \frac{w - 1}{r_b}$$

The PWC throughput for a given WIP level, $w$, is given by,

$$TH_{PWC} = \frac{w}{W_0 + w - 1} r_b,$$

where $W_0$ is the critical WIP.

**TH vs. WIP: Practical Worst Case**

![Graph showing TH vs. WIP for Best Case, PWC, and Worst Case scenarios.](http://factory-physics.com)
CT vs. WIP: Practical Worst Case

Penny Fab Two Performance

Note: process times in PF2 have var equal to PWC.

But… unlike PWC, it has unbalanced line and multi machine stations.
Penny Fab Two Performance (cont.)

Back to the HAL Case - Capacity Data

<table>
<thead>
<tr>
<th>Process</th>
<th>Rate (p/hr)</th>
<th>Time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamination</td>
<td>191.5</td>
<td>1.2</td>
</tr>
<tr>
<td>Machining</td>
<td>186.2</td>
<td>5.9</td>
</tr>
<tr>
<td>Circuitize</td>
<td>150.5</td>
<td>6.9</td>
</tr>
<tr>
<td>Optical Test/Repair</td>
<td>157.8</td>
<td>5.6</td>
</tr>
<tr>
<td>Drilling</td>
<td>185.9</td>
<td>10.0</td>
</tr>
<tr>
<td>Copper Plate</td>
<td>136.4</td>
<td>1.5</td>
</tr>
<tr>
<td>Procoat</td>
<td>146.2</td>
<td>2.2</td>
</tr>
<tr>
<td>Sizing</td>
<td>126.5</td>
<td>2.4</td>
</tr>
<tr>
<td>EOL Test</td>
<td>169.5</td>
<td>1.8</td>
</tr>
<tr>
<td>$r_0$, $T_0$</td>
<td>126.5</td>
<td>33.1</td>
</tr>
</tbody>
</table>

http://factory-physics.com
HAL Case - Situation

Critical WIP: \( r_b T_0 = 126.5 \times 33.1 = 4187 \)

Actual Values:
- CT = 34 days = 816 hours
- WIP = 37400 panels
- TH = 45.8 panels/hour

Conclusions:
- Throughput is 36% of capacity
- WIP is 8.9 times critical WIP
- CT is 24.6 times raw process time

HAL Case - Analysis

WIP Required for PWC to Achieve TH = 0.36\(r_b\):

\[
TH = \frac{w}{w + W_0 - 1} \quad r_b = 0.36r_b
\]

\[
w = \frac{0.36}{0.64} (W_0 - 1) = \frac{0.36}{0.64} (4,187 - 1) = 2,355
\]

Much lower than actual WIP!

TH Resulting from PWC with WIP = 37,400:

\[
TH = \frac{w}{w + W_0 - 1} \quad r_b = \frac{37,400}{37,400 + 4,187 - 1} \quad 126.5 = 113.8
\]

Much higher than actual TH!

Conclusion: actual system is much worse than PWC!
**Labor Constrained Systems**

**Motivation:** performance of some systems are limited by labor or a combination of labor and equipment.

**Full Flexibility with Workers Tied to Jobs:**
- WIP limited by number of workers \( n \)
- capacity of line is \( n/T_0 \)
- Best case achieves capacity and has workers in “zones”
- *ample capacity* case also achieves full capacity with “pick and run” policy
Labor Constrained Systems (cont.)

Full Flexibility with Workers Not Tied to Jobs:
• TH depends on WIP levels
• TH_{cw}(n) \leq TH(w) \leq TH_{cw}(w)
• need policy to direct workers to jobs (focus on downstream is effective)

Agile Workforce Systems
• bucket brigades
• kanban with shared tasks
• overlapping zones
• many others

Factory Dynamics Takeaways

Performance Measures:
• throughput
• WIP
• cycle time
• service

Range of Cases:
• best case
• practical worst case
• worst case

Diagnostics:
• simple assessment based on \( r_p, T_0, \) actual WIP, actual TH
• evaluate relative to practical worst case