Solutions

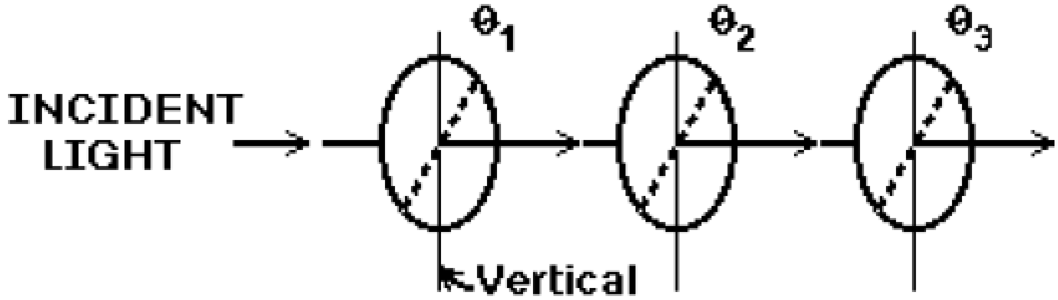
for test #2 covering Chapters 33-36

(each problem is 20 points; 100 points =100%)

① A ray in glass is incident onto a water-glass interface, at an angle of incidence equal to half the critical angle for that interface. The indices of refraction for water and the glass are 1.33 and 1.55, respectively. Find the angle of refraction in water in respect to the normal.

Critical angle for glass/water interface from formula (33.6): $\sin\theta_{crit} = n_b/n_a$. In our case n_a=1.55 and n_b=1.33, therefore $\theta_{crit} = \arcsin n_b/n_a = 59.1°$. Half of this angle is $\theta_a = \theta_{crit}/2 = 29.55°$. From Snell's law for refraction $n_a \sin\theta_a = n_b \sin\theta_b$ one can find angle of refraction θ_b in water: $\theta_b = \arcsin[(\sin 29.55° \cdot 1.55)/1.33] =$ **35.08°**

② In the Figure below, the orientation of the transmission axis for each of three polarizing filters is labeled relative to the vertical direction. A beam of light, polarized in the vertical direction, is incident on the first filter with an intensity of 1000 W/m^2. What is the intensity of the beam after it has passed through the three polarizing filters if θ_1 = 30°, θ_2 = 30°, and θ_3 = 60° ?



According to Malus's law (33.7) $I = I_0 \cos^2\phi$, where I_0 is intensity of incident polarized light and I is intensity after the polarizer with polarizer axis having angle ϕ with incident polarization direction. This angle is ϕ_1=30° for the first polarizer, ϕ_2=0° for second polarizer, and ϕ_3=30° for the third polarizer. Total intensity after three polarizers is $I = I_0 \cdot \cos^2\phi_1 \cdot \cos^2\phi_2 \cdot \cos^2\phi_3$ or $I = 1000 W/m^2 \cdot \cos^2 30° \cdot 1 \cdot \cos^2 30° =$ **562.5 W/m^2.**

③ The objective and the eyepiece of a refracting astronomical telescope have focal lengths of 320 cm and 4.0 cm, respectively. The telescope is used to view Neptune and the final image is set at infinity. The diameter of Neptune is 4.96 x 10^7 m and the distance from Earth at the time of observation is 4.4 x 10^{12} m. Find the angle subtended by the final telescopic image of Neptune, in *mrad*.

Angular magnification of refracting telescope is (34.24) $M = \theta'/\theta = -f_{obj}/f_{eyepc}$. Angular size of the Neptune is $\theta = diameter/distance = 4.96\times10^{7}\,m/4.4\times10^{12}\,m = 1.127\times10^{-5}$ radians. Now calculate θ_2: $\theta_2 = 1.127\times10^{-5}\times320cm/4cm = 9.02\times10^{-4}\,rad =$ **0.902 *mrad.***

④ Suppose you wanted to start a fire using sunlight and a mirror. Which of the following statements is most accurate?

☐ A) It would be best to use a plane mirror.

☐ B) It would be best to use a convex mirror.

☐ C) It would be best to use a concave mirror, with the object to be ignited positioned at the center of curvature of the mirror.

☑ D) It would be best to use a concave mirror, with the object to be ignited positioned halfway between the mirror and its center of curvature.

☐ E) One cannot start a fire using a mirror, since mirrors form only virtual images.

⑤ X-rays of wavelength 0.085 nm are scattered from the atoms of a crystal. The second-order maximum in the Bragg reflection occurs when the angle θ is 21.5°. What is the spacing between adjacent atomic planes in the crystal?

For Bragg's X-ray diffraction: $2\cdot d\cdot\sin\theta = m\lambda; d = \frac{m\lambda}{2\cdot\sin\theta} = \frac{2\times0.085nm}{2\times\sin21.5°} = 0.232\,nm =$ **2.32Å**

⑥ Consider a two-slit interference pattern producing an angular distribution of intensity (far from two sources). Let θ_m be the angular position of the *m*-th bright fringe, where the intensity is I_0. Assume that θ_m is small, so that $\sin\theta_m \approx \theta_m$. Let θ_m^+ and θ_m^- be the two angles on either side of θ_m for which $I=½I_0$. The quantity $\Delta\theta_m = |\theta_m^+ - \theta_m^-|$ is the half-width of the m-th fringe. Calculate $\Delta\theta_m$. How does $\Delta\theta_m$ depend on *m*?

Intensity depends on angle θ as

$$I(\theta) = I_0 \times \cos^2\left(\frac{\phi}{2}\right) = I_0 \times \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

with $\sin\theta_m$ being angular position of m-th bright fringe

$$\left(\sin\theta_m = \frac{\lambda}{d}m \quad \text{determines the direction to m-th bright fringe}\right)$$

Intensity is reduced by factor 2 for angles θ_m^+ and θ_m^-: $I(\theta_m^\pm) = \frac{1}{2}I_0$

when $\cos^2\frac{\phi}{2} = \frac{1}{2}$ or $\frac{\phi_m^\pm}{2} = \pm\arccos\left(\frac{\sqrt{2}}{2}\right) = \pm\frac{\pi}{4} \pm \pi m = \pm\frac{\pi}{4} + \frac{\phi_m}{2}$

where $\phi_m = \pm 2\pi m$ is phase angle; $\frac{\phi_m}{2} \approx \frac{\pi d}{\lambda}\theta_m$

$$\frac{\pi d}{\lambda}\left(\theta_m^+ - \theta_m\right) = \frac{\pi}{4}; \quad \frac{\pi d}{\lambda}\left(\theta_m^- - \theta_m\right) = -\frac{\pi}{4}$$

$$\Delta\theta_m = \theta_m^+ - \theta_m^- = \left(\frac{\lambda}{4d} + \theta_m\right) - \left(\theta_m - \frac{\lambda}{4d}\right) = \frac{2\lambda}{4d} = \frac{\lambda}{2d};$$

$\Delta\theta_m$ does not depend on m!