February 9, 2007, Instructor Yu. Kamyshkov

Solutions

Class P232

Test #1

① Radar working in "Ka-band" operates at frequency 38 GHz. Radar receiver indicates that a transmitted e-m pulse returns as an echo in 20 μs after transmission.

(a) How far away is the reflecting object?

Double distance $2L$ from radar to the object and back to the radar receiver takes $\Delta t = 20 \mu s = 2 \times 10^{-5}\ s$ for e-m wave propagating with velocity c.
Therefore, $2L = c \times \Delta t$ and
$L = c\Delta t/2 = 3 \times 10^{8}$ [m/s] $\times 2 \times 10^{-5}$ [s] $\times ½ = 3{,}000\ m = 3\ km$

(b) What is the wavelength of e-m waves emitted by radar?

$\lambda = c/f = 3 \times 10^{8}$ [m/s] / 38×10^{9} [Hz] $= 7.89 \times 10^{-3}\ m \approx 8\ mm$

② An 800 kHz radio signal is detected at a point 5.7 km distant from the transmitter tower. The electric field amplitude of the signal at that point is 120 mV/m. Assume that the signal power is radiated uniformly in all directions and that radio waves incident upon the ground are completely absorbed.

(a) Find the magnetic field amplitude of the signal at that point;

$E_0=cB_0$; $B_0=0.12$ [V/m] / 3×10^8 [m/s]= 4×10^{-10} [V·s/m^2]= 4×10^{-10} [T]

Check units: $\left[\frac{V\cdot s}{m^2}\right]=\left[\frac{(J/C)\cdot s}{m^2}\right]=\left[\frac{(N\cdot m)\cdot s}{C\cdot m^2}\right]=\left[\frac{N\cdot s}{(A\cdot s)\cdot m}\right]=\left[\frac{N}{A\cdot m}\right]=[T]$

(b) Find the average electromagnetic energy density at that point;

Instant energy density $u = \varepsilon_0 \cdot E^2$, where E is an instant value of electric field $E=E_0\cdot cos(kx-\omega t)$. Average energy density is an average of $cos^2(kx-\omega t)$ over time, that is $=½$.

$u_{av} = ½\varepsilon_0 \cdot E_0^2 = ½ \cdot 8.85\times10^{-12}$ [C^2/N/m^2] $\cdot (0.12)^2$ [V/m]^2 = 6.37×10^{-14}[J/m^3]

(c) Find the intensity of the radio signal at that point;

$I=½\varepsilon_0 \cdot c \cdot E_0^2 = c\cdot u_{av} = 3\times10^8$ [m/s] $\cdot 6.37\times10^{-14}$[J/m^3] = 1.91×10^{-5}[W/m^2]

(d) Find the average total power radiated by the transmitter.

$P(power)_{total}=I\cdot 4\pi\cdot r^2=1.91\times10^{-5}$[W/m^2]$\cdot 4\pi\cdot(5{,}700)^2$[m^2]$=7{,}805$ [W]≈7.8 [kW]

③ A *5.0 kg* block is attached to a spring whose force constant is *125 N/m*. The block is pulled from its equilibrium position at $x=0$ m to a position at $x=+0.687$ m and is released from rest. The block then executes damped oscillation along the *x*-axis. The damping force is proportional to the velocity. When the block first returns to the position $x=0$ m, its *x*-component of velocity is *–2.0 m/s* and its *x*-component of acceleration is $+5.6\ m/s^2$.

(a) Find the magnitude of the acceleration of the block upon release at $x=+0.687$ m.

Diff. equation that describes the damped motion is: $m\ddot{x} + b\dot{x} + kx = 0$,

When block is at position $x=+0.687$ m its velocity $\dot{x} = 0$ and

$a = \ddot{x} = -kx/m = -125$ [N/m]·*0.687* [m] / *5* [kg]= -17.18 [$m \cdot s^{-2}$] it is max. at this point

(b) Find damping coefficient *b*.

We can find it e.g. at $x=0$ from our diff. equation: $b = -m\ddot{x}/\dot{x}$

$b = -5 \cdot 5.6/(-2.0)$ [$kg \cdot m/s^2/(m/s)$]=*14* [kg/s]

(c) Find work done by the damping force during block travel from $x=+0.687$ m to $x=0$ m.

$W_{damping}$ = *ΔEnergy = P.E.(at x=+0.687 m) – K.E.(at x=0 m)* = $½kx^2 - ½mv^2$=

= ½·*125*[N/m] · $(0.687)^2$ [m^2] – ½·*5*[kg] · $(2)^2$[$m/s]^2$= *19.5* [J]

④ (Answer Yes/No for all questions; give additional explanations if find necessary). Tension of the free guitar string is suddenly changed while it sounds. Assume that the length L of the string did not change and the effect of oscillation damping is negligible.

Yes ☐ No ☑ (a) does the wavelength of the fundamental change?

no, since $\lambda = L/2$ for fundamental does not change.

Yes ☑ No ☐ (b) does the frequency of the fundamental change?

yes, since $f = \sqrt{F/\mu}/2L$,

where F is wire tension and μ is mass per unit length.

Yes ☐ No ☑ (c) does the energy of the string change?

no, since length of the string remains the same, change of the tension did not produce any work and energy of the string did not change.

Yes ☑ No ☐ (d) does the amplitude of the fundamental change?

yes, since the energy of the string that depends on both amplitude A and frequency (or tension) did not change, and when frequency changes, the amplitude must change too.

Yes ☐ No ☑ (e) does the phase of the fundamental change?

phase did not change if tension changed instantly

Yes ☑ No ☐ (f) does the wave velocity along the wire change?

yes, since velocity of transverse wave on a string: $v = \sqrt{F/\mu}$

Yes ☐ No ☑ (g) does the velocity of sound wave change?

no, since sound wave velocity depends only on temperature of air.

⑤ Two police cars have identical sirens that produce a frequency of $f=570Hz$. A stationary listener is standing between two cars. One car is parked and the other is approaching the listener and both have their sirens on. The listener notices 2.6 beats per second. Find the speed of the approaching police car (the speed of sound is $v=340\ m/s$).

Doppler-shifted frequency of approaching can be found from general formula for moving source and moving listener: $f_L = f \cdot (v+v_L)/(v+v_s)$ taking $v_L=0$ and changing the sign of v_s: $f_L = f \cdot v/(v-v_s)$.

But let’s deduce this formula from scratch.

Siren of the parked car generates frequency $f = 570\ Hz$ with the crests of sound wave emitted with period $T=1/f$ and wavelength $\lambda=v\cdot T$. Doppler effect for stationary listener and approaching siren takes place. Wavelength generated by moving siren is shorter than λ. For the same time intervals T: $\lambda_{mov}=v\cdot T-v_s\cdot T$, where v_s is velocity of approaching car (source). Listener will now receive crests of wave with λ_{mov} moving to him with velocity v (speed of sound in air) with time intervals $T_L=1/f_L$.

$$\lambda_{mov} = (v - v_{car})\cdot\frac{1}{f} = v\cdot\frac{1}{f_L} \quad \text{or} \quad f_L = f\cdot\frac{v}{v-v_s}$$

$f_{beat} = f_1 - f_2 = f_L$ (for moving siren) $- f_L$ (for stationary siren).

$$f_{beat} = f\frac{v}{v-v_s} - f = f\times\left(\frac{v}{v-v_s}-1\right) = f\times\left(\frac{v_s}{v-v_s}\right)$$

$$f_{beat}(v-v_s) = f\cdot v_s; \qquad v_s = \frac{f_{beat}\cdot v}{f_{beat}+f} = \frac{2.6}{2.6+570}\times 340 = 1.54\,[\text{m/s}] \approx 5.56\ [\text{km/h}]$$

⑥ What happens when a periodic driving force is applied to a vibrating system?

A) The system will stop vibrating and finally come to a stop.

B) The system will exhibit chaotic motion.

☑ C) The system will vibrate at the frequency of the driving force.

D) The system will vibrate at its natural frequency.

E) It will vibrate at some multiple of the driving frequency (call a harmonic or "overtone").