Hints and Solutions for Review problems for Test 1

Problem #1 (Ch13):

A *5.7 kg* block attached to a spring executes simple harmonic motion on a frictionless horizontal surface. At time *t=0* s, the block has a displacement of *–0.2 m*, a velocity of *–0.8 m/s*, and an acceleration of *+4.1 m/s²*.

$$(1)\quad -0.2 = A\cdot\cos(\omega t+\varphi)$$
$$(2)\quad -0.8 = -A\omega\cdot\sin(\omega t+\varphi)$$
$$(3)\quad +4.1 = -A\omega^2\cdot\cos(\omega t+\varphi)$$

(b) Find angular frequency and oscillation period — ω=sqrt(4.1/0.2)=4.53 rad/s

(a) Find force constant of the spring — $k=\omega^2 m$=117 N/m

(d) Find initial phase of the motion — ~~$\varphi=\tan^{-1}(-4./\omega)=-0.723$ rad $=-41.4°$~~

correction (thanks to Craig Broerman): φ=arctan(–4./ ω)±πn where n=0,1,2,3...
note, that cos φ should be negative from eq. (1); that means that φ=–0.723 rad + π=2.418 rad

(c) Find amplitude of the motion — A=0.267 m

(e) Find displacement at *t=10 s* — ~~*x*=0.221 m~~ *x*= –0.221 m

(f) At what time amplitude is maximal — t=(nπ–φ)/ω, where n=0,1,2..

(g) At what time acceleration is maximal — at the same time

(h) Find maximum elastic potential energy — $U_{max}=kA^2/2$

Problem #2 (Ch 13)*

It has been suggested that a possible transportation system to connect two Cities such as e.g. Boston and Washington, D.C. might function like this: Drill a tunnel straight from Boston to Washington. A car released in Boston would fall under the influence of gravity, gaining speed, and then finally coasting back to the surface at Washington. The only expenditure of energy required would be that to overcome friction, and this could be reduced by using an evacuated tunnel or magnetic levitation of the train or other possible techniques. Assuming the density of the Earth is constant (it isn't), and using only the value of the acceleration of gravity at the Earth's surface (9.81 m/s^2) and the radius of the Earth (6380 km) determine how long it would take to travel between any two cities via a straight tunnel.

If position of car is x from the center of horizontal tunnel and it is moving in the direction of **$-x$** under the force $F(x)$. It we draw radius R from the center of Earth perpendicular to the center of horizontal tunnel and another line from Earth center to the car (its length will be less but ≈ R) , then car with position x will be at angle φ. We can see that $F/mg = \sin\varphi = x/R$ and $F = mgx/R$. This will be returning force in the 2-nd Newton's law:

$$m\ddot{x} = -\frac{mgx}{R} \quad \text{or} \quad \ddot{x} + \omega^2 x = 0 \quad \text{where} \quad \omega^2 = g/R$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

T is period of oscillations;
travel time will be half of that T/2 = 2,534 s ~ 42'

Problem #3 (Ch13):

A meter stick is freely pivoted about the 20 cm mark. Find the frequency of small oscillations.

Use parallel-axis theorem for momentum of inertia calculation in the textbook f-la (9.19): $I_P = I_{cm} + Md^2$ where d=0.3 m. $I_P = 0.17333\ ML^2$, where L=1m.

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{Mgd}{I_P}} = 0.656 Hz$$

Problem #1 (Ch 15)

A transverse wave is propagating in a string stretched along the x-axis.
The equation of the wave, in SI units, is given by:

$$y = 0.009 \cdot \cos \pi \left(87t - 16x\right)$$

(a) Find the amplitude of the wave. $A=0.009\ m$

(b) Find the frequency of the wave. $\omega=87\pi\ rad/s$

(c) Find the propagation constant (wave number k) of the wave $k= 16\pi\ rad/m$

(d) Find the wave speed of the wave. $v=\omega/k=5.44\ m/s$

(e) Find maximum velocity of a particle on the string

$$v_{\max} = \left|\dot{y}_{\max}\right| = A\omega = 0.009 \times 87\pi =$$

Problem #2 (Ch 15)

Which of the following is an accurate statement?

(a) A system like a vibrating string has only one resonant frequency.

(b) In order for a singer to break a wine glass by singing, she must adjust the amplitude of the sound she makes so that it is exactly equal to the amplitude of vibration of the wine glass.

(c) The resonant frequency of a system is the name given to the lowest possible frequency at which the system will naturally vibrate.

(d) An organ pipe has an infinite number of resonant frequencies.

(e) When an oscillatory system is driven by the sinusoidal force, the response amplitude of the system will be the same as the amplitude of the driving force.

Problem #3 (Ch 15)

Two violinists are trying to tune their instruments in an orchestra. One is producing the desired frequency 440 Hz. The other is producing a frequency of 448.4 Hz. By what percentage should the out of tune musician change the tension of his string to bring his instrument into tune at 440 Hz?

$$f_a = \frac{1}{2L}\sqrt{\frac{F_a}{\mu}}; \quad f_b = \frac{1}{2L}\sqrt{\frac{F_b}{\mu}};$$

$$F_b = F_a \cdot \left(\frac{f_b}{f_a}\right)^2 = F_a + \Delta F$$

$$\frac{\Delta F}{F_a} = \left(\frac{f_b}{f_a}\right)^2 - 1 = 0.0386$$

Problem #1 (Ch 16)

The howler monkey is the loudest land animal and can be heard up to a distance of 9.0 km. Assume the acoustic output of a howler to be uniform in all directions.

(a) What is total acoustic power emitted by the howler?

(b) A chorus of five howlers call at the same time.
At what largest distance they can be heard?

$$(a) P = I_{threshold} \times 4\pi R^2 = 10^{-12}\, W/m^2 \times 4\pi (9{,}000)^2 \cong 10^{-3} W$$

$$(b) R_5 = R_1 \times \sqrt{\frac{P_5}{P_1}} = 9{,}000 m \times \sqrt{5}$$

Problem #2 (Ch 16)

Two loudspeakers placed 6.0 m apart are driven in phase by an audio oscillator, whose frequency range is 400 Hz to 700 Hz. A point P is located 5.3 m from one loudspeaker and 3.6 m from the other. The speed of the sound is 344 m/s.
At some frequency f_0 a destructive interference occurs at the point P.

Find frequency f_0

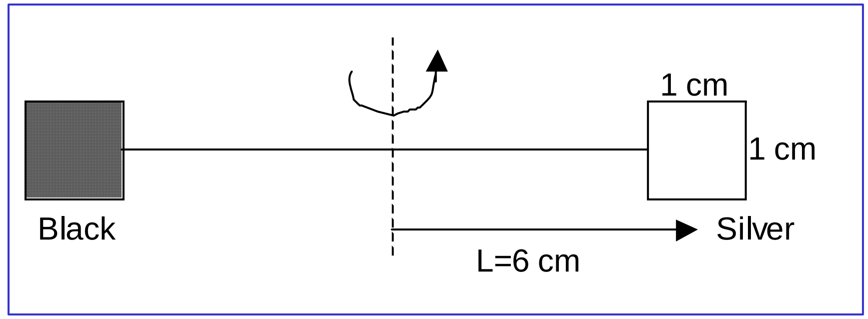
For distractive interference phase difference should be multiple of (lambda/2)*(1+2*n) where n=0,1,2,3…; lambda = v/f
L_1 – L_2 = (v/(2f))*(1+2*n) . Try different n to find one in the frequency range of the audio oscillator. You will find that n=2 fits with f_0 ~ 506 Hz

Problem #3 (Ch 16)

An open pipe, 0.9 m long, vibrates in the second overtone with a frequency of 602 Hz. What will be the length of the shortest stopped pipe that has the same resonant frequency?

L(stopped)=L(open)/6

Problem #1 (Ch 32)



A radiometer has two square vanes (1 cm by 1 cm), attached to a light horizontal cross arm, and pivoted about a vertical axis through the center. The center of each vane is 6 cm from the axis. One vane is silvered and it reflects all radiant energy incident upon it. The other vane is blackened and it absorbs all incident radiant energy. Radiant energy, which has an intensity of 300 W/m^2, is incident normally upon the vanes.

(a) Find radiant energy absorbed by blackened vane $= I\times(0.01\text{m})^2$ J/s

(b) Find radiation pressure on the blackened vane $p=10^{-6}$ N/m^2

(c) Find torque on the vane assembly due to radiation pressure (about the vertical axis)

(c) All right, you know pressures on both vanes. Pressure is Force/Area and you know areas of both vanes, so you know forces; you also know lever arms, so you know two opposite torques. Take vector sum of two.

Problem #2 (Ch 32)

A sinusoidal electromagnetic wave is propagating in vacuum.

(a) At a given point at a particular time the electric field is in the $+x$ direction and the magnetic field is in $-y$ direction. What is the direction of propagation of the wave?

$$-\hat{z}$$

(b) If the intensity of the wave is 0.94 W·m^{-2}, what is the electric field amplitude at the this point.
(use: $\mu_0=4\pi\times10^{-7}$ Wb/A/m ; $c=3\times10^{8}$ m/s)

$$E_{\max}^2 = I \cdot 2\mu_0 c \cong 709(V/m)^2$$

$$E_{\max} \cong 26.6 V/m$$

(c) Prove that dimension of the resultant electric field is [V/m]

$$E^2 = \frac{W}{m^2} \cdot \frac{Wb}{A \cdot m} \cdot \frac{m}{s} = \frac{J}{s \cdot m^2} \cdot \frac{T \cdot m^2}{A \cdot m} \cdot \frac{m}{s} = \frac{J \cdot T}{A \cdot s^2} =$$

$$= \frac{J}{A \cdot s^2} \cdot \frac{N \cdot m}{A \cdot m^2} = \frac{J^2}{(A \cdot s)^2 \cdot m^2} = \frac{(V \cdot C)^2}{C^2 \cdot m^2} = \left(\frac{V}{m}\right)^2$$

Problem #3 (Ch 32)

An electromagnetic wave polarized parallel to the *z-axis* is traveling in the positive direction along the *x-axis*. How can this wave best be detected using a straight wire antenna?

(a) Orient the antenna parallel to the *x-axis*.

(b) Orient the antenna parallel to the *y-axis*.

(c) Orient the antenna parallel to the *z-axis*.

(d) The antenna will work equally well for any of these orientations.

(e) This wave cannot be detected with a straight wire antenna.
A loop antenna must be used.

Problem #4 (Ch 32)

An electromagnetic wave is radiated by a straight wire antenna that is oriented vertically. Such a wave could be best detected by

(a) a loop antenna oriented with the plane of the loop horizontal.

(b) a loop antenna oriented with the plane of the loop vertical and parallel to the velocity vector of the wave.

(c) a loop antenna oriented with the plane of the loop perpendicular to the velocity vector of the wave.

(d) a straight wire antenna placed in a horizontal plane.

(e) more than one of these.

Problem #5 (Ch 32)

A 5.64×10^{14} Hz electromagnetic wave propagates in carbon tetrachloride with speed 2.05×10^{8} m/s.

(a) Find the wavelength of the wave in carbon tetrachloride. =363 nm

(b) Find the wavelength of the wave in vacuum. = 532 nm

(c) If relative magnetic permeability of carbon tetrachloride is 1.00, find the dielectric constant of carbon tetrachloride at that frequency.

K_ε=2.16 or n=1.47