Review #3 problems with solutions for Chapters 37-40

Review problems

Chapter 37 #1

Two fixed navigation beacons mark the approach lane to a star. The beacons are in line with the star and are 28 Mm apart in a reference frame of the star. A spaceship approaches the star with a relative velocity of 0.8c and passes the beacons. The passage of the ship between the the beacons is timed by an observer on the ship. What time interval he measures?

$\Delta t = 0.07$ s

Chapter 38 #1

A 100 W arc lamp operates with an efficiency of 5% for emitting light. Assume it emits light of wavelength 657 nm. How many photons per second does it emit?

$\sim 1.7\times10^{19}$ γ/s

Review problems

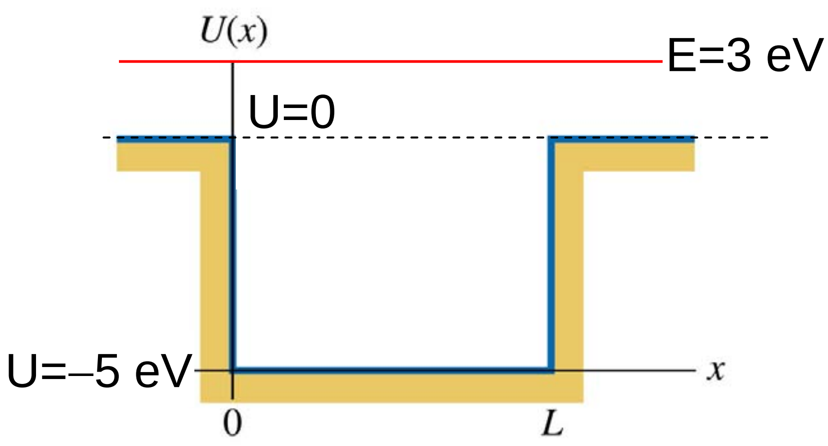
Chapter 39 #1

An electron has a kinetic energy of 5.8 eV. Find the energy of a photon that has the same de Broglie wavelength as the electron.

$E_{\gamma} \sim 2.4$ keV

Chapter 40 #1

A free electron which has a kinetic energy of 3.0 eV is incident on a square-well potential. The depth of the well is 5.0 eV and the width is 4.0 nm. Find the wavelength of the wave function of electron (a) outside the well; (b) inside the well.



Read also bottom of page 1528 of text book.
Electron is flying over the well. At infinite distance from the well KE=3 eV, potential energy U=0, and total energy E=+3eV.
Inside the well the total energy E will be conserved, but potential energy will change to U=–5 eV.

$$Solution \;\; \psi = Ce^{+ikx} + De^{-ikx}$$

$$where \;\; k = \frac{p}{\hbar} = \frac{\sqrt{2m(E-U)}}{\hbar}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi\hbar}{\sqrt{2m(E-U)}} = \frac{h}{\sqrt{2m(E-U)}}$$

outside well (E–U)=3 eV and $\lambda \approx 708\times10^{-12}$ m=708 pm
inside well (E–U)=(3 – (– 5))=8 eV and $\lambda \approx 433\times10^{-12}$ m=433 pm

λ(outside) ~ 708 pm
λ(inside) ~ 433 pm

Review problems

Chapter 37 #2

Two fixed navigation beacons mark the approach lane to a star. The beacons are in line with the star and are 78 Mm apart in a reference frame of the star. A spaceship approaches the star with a relative velocity of 0.5c and passes the beacons. As the ship passes the first beacon the ship emits a short radar pulse towards the second beacon, and the radar echo is received at the ship. Find time interval between the emission of the radar pulse and the reception of the radar echo measured by the observer on the board of spaceship.

(see next page for solution)

Possible solution (a) Consider the problem in the reference frame of moving ship. The beacon #2 is moving towards the observer inside the ship with velocity $v = \beta \cdot c$, where $\beta = 0.5$. Distance between beacons for this observer is contracted:

$$l_{12} = \frac{l_{12}^0}{\gamma}, \text{ where } l_{12}^0 = 78Mm \text{ and } \gamma = \frac{1}{\sqrt{1\text{-}v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}} = 1.155$$

For observer inside the ship the radar pulse will hit the mirror of the beacon #2 at some distance x from the observer and after reflection will return to the observer over the same distance x. It will move with velocity c in both directions. So, the signal will travel distance x both ways and will return to the ship after Δt

$$\Delta t = \frac{\vec{x}}{c} + \frac{\overleftarrow{x}}{c} = \frac{2x}{c}$$

Before radar pulse will hit beacon #2 the latter will travel towards the observer with velocity $v = \beta c$ for time Δ Δt/2 rough the path $(l_{12} - x) = v \cdot \Delta t/2$:

Thus, $(l_{12} - x) = v \cdot \frac{x}{c}$, solving this equation with respect to x we will find:

$$x = \frac{l_{12}}{(1+\beta)} = \frac{l_{12}^0}{\gamma \cdot (1+\beta)} \text{ and } \Delta = \frac{2x}{c} = \frac{2l_{12}^0}{\gamma \cdot c \cdot (1+\beta)} = \frac{2 \cdot 78 \cdot 10^6 \, m}{3 \cdot 10^8 \, m/s} \cdot \frac{\sqrt{1-0.25}}{1.5} \approx 0.30sec$$

Possible solution (b) Let's start in the reference frame of the star (or beacons). Time interval $\Delta t'$ needed for signal to reach beacon #2 and after the reflection return to starship:

$$\Delta t' = \frac{l_{12}^0}{c} + \frac{(l_{12}^0 - x)}{c}$$ *,where x is the distance travelled by the ship for time $\Delta t'$: $x = \Delta t' \cdot \beta c$.*

Combining these two equations we will have:

$$\beta l_{12}^0 + \beta(l_{12}^0 - x) = x \quad or \quad x = \frac{2\beta \cdot l_{12}^0}{1+\beta}. \quad Then, \; \Delta t' = \frac{x}{\beta c} = \frac{2l_{12}^0}{c \cdot (1+\beta)} = \frac{2 \cdot 78 \cdot 10^6 \, m}{1.5 \times 3 \cdot 10^8 \, m/s} \approx 0.35s$$

Time intervals Δals (a) and $\Delta t'$ in (b) are different because we consider time interval between two events from different reference frames. However, $\Delta t'$ in star's reference frame is related with Δith the reference frame of the starsip by time dilation as Δn= $\Delta t'/\gamma$, that then will coincides with the result from (a).

Chapter 37 #3

The special theory of relativity predicts that there is an upper limit *c* to the speed of a particle. It thus follows that there is also an upper limit on the following property of a particle (select one correct statement):

(a) The kinetic energy
(b) The total energy
(c) The linear momentum
(d) More than one of these
(e) None of these (← correct statement)

Total energy $E = \dfrac{mc^2}{\sqrt{1\text{-}\beta^2}}$

Kinetic energy $KE = \dfrac{mc^2}{\sqrt{1\text{-}\beta^2}} - mc^2$

Momentum $p = \dfrac{m\beta c}{\sqrt{1\text{-}\beta^2}} = \dfrac{mc}{\sqrt{1/\beta^2 - 1}}$

$When\ v \to c\ \ or\ \ \beta \to 1.0$

$E \to \infty\ m\ KE \to \infty\ and\ p \to \infty$

Review problems

Chapter 38 #2

A large number of *30.0 pm* photons are scattered twice by stationary electrons. Find the range of wavelengths of the doubly-scattered photons in units of *pm*.

When in Compton scattering the electrons are scattered one time the range of the resulting wavelengths can be found from (see book equation 38.23):

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\varphi) \text{ taking into account that } \cos\varphi \text{ varies by two units between 1 and } -1:$$

$$\lambda' \in [\lambda, \lambda + 2h/mc] \;,\quad 2h/mc = (2 \times 6.63 \times 10^{-34}\, J \cdot s)/(9.1 \times 10^{-31}\, kg \times 3 \times 10^{8}\, m/s) = 4.857pm$$

$$\lambda' \in [30pm,\ 34.857\ pm]$$

After second Compton scattering the range of scattered wavelengths will be extended by factor of 2, that is $\lambda_2' \in [30pm,\ 39.714\ pm]$

Chapter 38 #3

An electric current through a tungsten filament maintains its temperature at *2800 K*. Assume the tungsten filament behaves as an ideal "black body" radiator at that temperature and the effective radiating area of the filament is 2.0×10^{-6} m^2

(a) Find the wavelength at which the maximum in spectral emittance occurs;

(b) Find the total power radiated by the filament.

(a) From Wien's displacement law (38.30):

$$\lambda_m = \frac{hc}{4.965kT} = \frac{2.90\times10^{-3}\,m\cdot K}{T} = \frac{2.90\times10^{-3}\,m\cdot K}{2800K} \approx 1.036\times10^{-6}\,m = 1{,}036nm$$

(b) Total radiated intensity can be found from Stefan-Boltzmann low for a black body (38.28):

$$I = \sigma\times T^4 \quad \text{where} \quad \sigma = 5.67\times10^{-8}\,\frac{W}{m^2K^4}$$

Total radiated power can be found by multiplying intensty by total radiating area $A = 2.0\times10^{-6}\,m^2$

$$P = A\times I = A\times\sigma\times T^4 = 2.0\times10^{-6}\,m^2\times5.67\times10^{-8}\,\frac{W}{m^2K^4}\times\left(2800K\right)^4 \approx 6.97W$$

Review problems

Chapter 39 #2

Find (a) the shortest and (b) the longest wavelength of a photon that can be emitted by a hydrogen atom, for which the final state is *n=4*.

Hydrogen can be in the final state with n = 4 if transition occurs from excited states with n = 5,6,7,...∞. Energy levels of hydrogen in the quantum state n are given by e.g.(38.9)

$$E_n = -\frac{hcR}{n^2} = -\frac{13.6eV}{n^2}.$$

One can also use fomula for Brackett's series (on page 1455).

(a) the transition with shortest wavelength will correspond to n changing from ∞ to n = 4

$$\text{or } \Delta E = E_{n=\infty} - E_{n=4} = \left(-\frac{13.6eV}{\infty^2}\right) - \left(-\frac{13.6eV}{4^2}\right) = 0.85eV \text{ and corresponding wavelength :}$$

$$\text{from } \Delta E = \frac{hc}{\lambda}; \quad \lambda = \frac{hc}{\Delta E} = \frac{6.63\times10^{-34}\,J\cdot s\times3\cdot10^{8}\,m/s}{0.85\,eV\times1.6\times10^{-19}\,J/eV} = 1{,}462\,nm$$

(b) longest wavelength corresponds to the transition from n = 5 to n = 4 :

$$\Delta E = E_{n=5} - E_{n=4} = \left(-\frac{13.6eV}{5^2}\right) - \left(-\frac{13.6eV}{4^2}\right) = 0.306eV \text{ and corresponding wavelength :}$$

$$\text{from } \Delta E = \frac{hc}{\lambda}; \quad \lambda = \frac{hc}{\Delta E} = \frac{6.63\times10^{-34}\,J\cdot s\times3\cdot10^{8}\,m/s}{0.306\,eV\times1.6\times10^{-19}\,J/eV} = 4{,}063\,nm$$

Chapter 39 #3

An unstable particle produced in a high-energy collisions is measured to have an energy of 483 MeV and an uncertainty in energy of 39 keV. Use the Heisenberg uncertainty principle to estimate the lifetime of this particle.

$$Heisenberg \quad uncertainty \quad principle: \; \Delta E \times \Delta t \geq \hbar$$

$$\Delta t \geq \frac{\hbar}{\Delta E} = \frac{1.055 \times 10^{-34} \, J \cdot s}{3.9 \cdot 10^{4} \, eV \times 1.6 \times 10^{-19} \, J/eV} = 1.69 \times 10^{-20} \, s$$

Review problems # Chapter 40 #2

Wave function is given by

$$\psi(x) = 0 \qquad x < 0$$
$$\psi(x) = A \cdot x \cdot (L - x) \qquad 0 \le x \le L$$
$$\psi(x) = 0 \qquad x > L$$

Find normalization coefficient A for this wave function.

$$Normalization\ \ condition: \int_{-\infty}^{+\infty} |\psi(x)|^2 \cdot dx = \int_0^L A^2 \cdot x^2 \cdot (L - x)^2 dx = 1$$

$$1 = A^2 \int_0^L \left(x^2 L^2 - 2x^3 L + x^4\right) \cdot dx = A^2 \times \left| \frac{x^3 L^2}{3} - \frac{2x^4 L}{4} + \frac{x^5}{5} \right|_{x=0}^{x=L} = A^2 \times \left(\frac{L^5}{3} - \frac{L^5}{2} + \frac{L^5}{5} \right) =$$

$$= A^2 \times L^5 \times 0.0333 \qquad or \qquad A = \sqrt{\frac{30}{L^5}}$$

Chapter 40 #3

Consider a particle of mass *m* in a box of width *L* (from $x = 0$ to $x = L$) and let the particle be in a state n=11. What is the first value of *x*, larger than *0*, where the probability of finding the particle is highest?

For a particle in a box of size L in a state with n = 11 wave function can be found from eq.(40.13)

$$\psi_n(x) = \sqrt{\frac{2}{L}} \times \sin\frac{11\cdot\pi\cdot x}{L}.$$

Probability to find particle at given x within interval dx is given by $|\psi_n(x)|^2 dx$

and will go to max (at $x > 0$*) first time when* $\sin\left(\frac{11\cdot\pi\cdot x}{L}\right) = 1.$

That means that $\frac{11\cdot\pi\cdot x}{L} = \frac{\pi}{2}$ *and* $x = \frac{L}{22}$