Home Work Chapter 24. Force between Capacitor Plates: Find F(V), the magnitude of the force each plate experiences due to the other plate as a function of V, the potential drop across the capacitor. Express your answer in terms of A, d and ε_0.

Solution:

For uniform field $F = E_1 \cdot (-q)$, where E_1 is the field created by first plate (say, with charge $+q$) at the position of the second plate (with charge $-q$). To calculate field E_1 one needs to remove the plate with charge $-q$ to infinity. Then uniform field E_1 will exist on both sides from the plate 1. Its magnitude can be found from Gauss's theorem (see Example 22.7 on the page 852 of the book):

$$2 \cdot E_1 \cdot \mathrm{A} = \frac{+\mathrm{q}}{\varepsilon_0} \quad \text{or} \quad E_1 = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2A\varepsilon_0}$$

Then force can be found from $\vec{F} = \vec{E}_1 \cdot (-q) = -\frac{q^2}{2A\varepsilon_0}$. Minus sign indicates that vector $\vec{F}$ and vector $\vec{E}_1$ have opposite directions. For force magnitude calculation we can omit the minus sign.

Further, the q can be found as $q = C \cdot V$ and C as $C = \varepsilon_0 \frac{A}{d}$.

Then $F = \frac{q^2}{2A\varepsilon_0} = C^2 \cdot \frac{V^2}{2A\varepsilon_0} = \frac{\varepsilon_0{}^2 A^2}{d^2} \cdot \frac{V^2}{2A\varepsilon_0} = \frac{\varepsilon_0 A}{2} \cdot \left(\frac{V}{d}\right)^2$

$$F = \frac{\varepsilon_0 A}{2} \cdot \left(\frac{V}{d}\right)^2$$