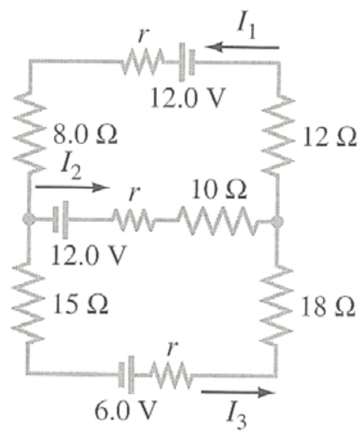
Problem 34 (Chapter 19)



Kirchhoff's Rules (see book pp. 564 – 567) can be used to determine currents I_1, I_2, I_3:

Junction rules (here for left and right junctions):

(1) $I_1 = I_2 + I_3$ for left junction
(2) $I_2 + I_3 = I_1$ for right junction

Since current is conserved at each junction, left and right junctions give the same equation

Loop Rules (here for upper and lower loops):

Note that *r = 1Ω*
Let follow around the loops in *counterclockwise direction*. If this *direction* coincides with the assigned direction of the current we will count voltage difference over the resistor as positive (negative othervise); if this *direction* crosses battery (inside) from **+** to **–** we will count *emf* of this battery $\mathcal{E}$ as positive (negative othervise) in the Loop Rule:

For upper loop starting from 8Ω resistor:

$$8\Omega \times I_1 - 12V + 1\Omega \times I_2 + 10\Omega \times I_2 + 12\Omega \times I_1 - 12V + 1\Omega \times I_1 = 0 \quad \text{or}$$

(3) $21 \times I_1 + 11 \times I_2 = 24$

For lower loop starting from 15Ω resistor:

$$15\Omega \times I_3 - 6V + 1\Omega \times I_3 + 18\Omega \times I_3 - 10\Omega \times I_2 - 1\Omega \times I_2 + 12V = 0 \quad \text{or}$$

(4) $34 \times I_3 - 11 \times I_2 = -6$

Now we have a system of three linear algebraic equations (1), (3), and (4):

(1) $I_1 = I_2 + I_3$

(3) $21\times I_1 + 11\times I_2 = 24$

(4) $34\times I_3 - 11\times I_2 = -6$

Let's first substitute (1) into (3). Also, let's resolve (4) expressing

$$I_3 = \frac{11}{34} I_2 - \frac{6}{34}$$

and substitute this into (3). We will get an equation for I_2:

$$I_2\left(33 + \frac{21\cdot 11}{34}\right) = 24 + \frac{21\cdot 6}{34}$$

We can find from these equations that:

$$I_2 = 0.696A$$
$$I_3 = 0.049A$$
$$I_1 = 0.745A$$

Now, terminal voltage of 6V battery can be found as:

$V = \mathcal{E} - I_3 \times r = 6V - 0.049A \times 1\Omega = 5.951V$