Frame A:
Cash receipt at the END of year ...

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td></td>
<td></td>
<td></td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
<td>(e)</td>
<td>(f)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Frame B:
Cash receipt at the END of year ...

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td></td>
<td></td>
<td></td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>R</td>
<td>$</td>
<td>^</td>
<td>^</td>
<td>^</td>
</tr>
<tr>
<td></td>
<td>(g)</td>
<td>(h)</td>
<td>(i)</td>
<td>(j)</td>
<td>(k)</td>
<td>(l)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Assume that the time lines in Frames A and B (above) depict annual cash flows of R dollars at the ends of the periods indicated. If the appropriate compound annual interest rate is 8 percent, what dollar value does R take on if the present value of the cash flow stream depicted in Frame A or Frame B equals $1,000 as of point (a)? point (b)? point (c)? point (g)? point (h)? point (i)?

2. Assume that the time lines in Frames A and B (above) depict annual cash flows of R dollars at the ends of the periods indicated. If the appropriate compound annual interest rate is 8 percent, what dollar value does R take on if the future value of the cash flow stream depicted in Frame A or Frame B equals $1,000 as of point (d)? point (e)? point (f)? point (j)? point (k)? point (l)?

Answers:
Question 1 -- When the present value of the cash flow stream depicted in Frame A or Frame B equals $1,000 at point _____, R equals ______.
(a), $292.23; (b), $250.44; (c), $231.91; (g), $364.83; (h), $312.60; (i), $289.44

Question 2 -- When the future value of the cash flow stream depicted in Frame A or Frame B equals $1,000 at point _____, R equals ______.
(d), $170.44; (e), $157.83; (f), $135.28; (j), $212.72; (k), $197.01; (l), $168.83
$1,000 = R(PVIFA_{8\%}, 5) \times (PVIF_{8\%}, 2) = R(3.993) \times (.857) = R(3.422)

R = $1,000/3.422 = \underline{292.23} \text{ when the present value of the cash flow stream equals $1,000 at point (a)}

$1,000 = R(PVIFA_{8\%}, 5)

R = $1,000/3.993 = \underline{250.44} \text{ when the present value of the cash flow stream equals $1,000 at point (b)}

$1,000 = R + R(PVIFA_{8\%}, 4)

R = $1,000/4.312 = \underline{231.91} \text{ when the present value of the cash flow stream equals $1,000 at point (c)}
\[
\begin{array}{cccccccccccc}
\text{time} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\$ & R & R & R & R & R & R & R & R & R & R & R \\
\hline
\end{array}
\]

\[\$1,000 \quad \text{(d)}\]

\[\$1,000 = R(FVIFA_{8\%,5}) = R(5.867)\]

\[R = \frac{\$1,000}{5.867} = \$170.44\] when the future value of the cash flow stream equals \$1,000 at point (d)

\[
\begin{array}{cccccccccccc}
\text{time} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\$ & R & R & R & R & R & R & R & R & R & R & R \\
\hline
\end{array}
\]

\[\$1,000 \quad \text{(e)}\]

\[\$1,000 = R(FVIFA_{8\%,5}) \times (1.08)\]

\[\$1,000 = R(5.867) \times (1.08) = R(6.336)\]

\[R = \frac{\$1,000}{6.336} = \$157.83\] when the future value of the cash flow stream equals \$1,000 at point (e)

\[
\begin{array}{cccccccccccc}
\text{time} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\$ & R & R & R & R & R & R & R & R & R & R & R \\
\hline
\end{array}
\]

\[\$1,000 \quad \text{(f)}\]

\[\$1,000 = R(FVIFA_{8\%,5}) \times (FVIF_{8\%,3})\]

\[\$1,000 = R(5.867) \times (1.260) = R(7.392)\]

\[R = \frac{\$1,000}{7.392} = \$135.28\] when the future value of the cash flow stream equals \$1,000 at point (f)
The above pattern is equivalent to

\[ R \times PVIFA_{8\%,5} < \text{R} \]

minus

\[ R \]

\[ $1,000 = [R(PVIFA_{8\%,5}) \times (PVIF_{8\%,2})] - R(PVIF_{8\%,5}) \]
\[ $1,000 = [R(3.993) \times (.857)] - R(.681) \]
\[ $1,000 = R(3.422) - R(.681) = R(2.741) \]

\[ R = $1,000/2.741 = $364.83 \]

when the present value of the cash flow stream equals $1,000 at point (g)
The above pattern is equivalent to

\[ \text{R} \quad \text{R} \quad \text{R} \quad \text{R} \quad \text{R} \]

\[ \text{minus} \]

\[ \text{R} \]

\[ \$1,000 = \text{R}(\text{PVIFA}_{8\%}, 5) - \text{R}(\text{PVIF}_{8\%}, 3) \]

\[ \$1,000 = \text{R}(3.993) - \text{R}(.794) = \text{R}(3.199) \]

\[ \text{R} = \frac{\$1,000}{3.199} = \$312.60 \] when the present value of the cash flow stream equals $1,000 at point (h)

The above pattern is equivalent to

\[ \text{R} \quad \text{R} \quad \text{R} \quad \text{R} \quad \text{R} \]

\[ \text{minus} \]

\[ \text{R} \]

\[ \$1,000 = \text{R} + \text{R}(\text{PVIFA}_{8\%}, 4) - \text{R}(\text{PVIF}_{8\%}, 2) \]

\[ \$1,000 = \text{R} + \text{R}(3.312) - \text{R}(.857) = \text{R}(3.455) \]

\[ \text{R} = \frac{\$1,000}{3.455} = \$289.44 \] when the present value of the cash flow stream equals $1,000 at point (i)
The above pattern is equivalent to

\[ \begin{array}{ccccccc}
R & R & R & R & R \\
\end{array} \]

minus

\[ R \]

\[ \$1,000 = R(FVIFA_{8\%}, 5) - R(FVIF_{8\%}, 2) \]

\[ \$1,000 = R(5.867) - R(1.166) = R(4.701) \]

\[ R = \frac{\$1,000}{4.701} = \$212.72 \] when the future value of the cash flow stream equals $1,000 at point (j)
The above pattern is equivalent to

\[ R \quad R \quad R \quad R \quad R \]

\[ \rightarrow \]

\[ \text{minus} \]

\[ R \]

\[ \rightarrow \]

\[ \$1,000 = [R(FVIFA_{8\%, 5}) \times (1.08)] - R(FVIF_{8\%, 3}) \]

\[ \$1,000 = [R(5.867) \times (1.08)] - R(1.260) \]

\[ \$1,000 = R(6.336) - R(1.260) = R(5.076) \]

\[ R = \$1,000/5.076 = \$197.01 \] when the future value of the cash flow stream equals \$1,000 at point (k)
The above pattern is equivalent to

$$\text{R} \quad \text{R} \quad \text{R} \quad \text{R} \quad \text{R}$$

$$\rightarrow \text{R(FVIFA}_{8\%}, 5)$$

minus

$$\text{R}$$

$$1,000 = [\text{R(FVIFA}_{8\%}, 5) \times (\text{FVIF}_{8\%, 3})] - \text{R(FVIF}_{8\%}, 5)$$

$$1,000 = [5.867 \times 1.260] - 1\text{.469}$$

$$1,000 = 7\text{.392} - 1\text{.469} = 5\text{.923}$$

$$\text{R} = \frac{1,000}{5\text{.923}} = 168.83$$ when the future value of the cash flow stream equals $1,000 at point (1)