Government Subsidies and Incentives for R&D Collaboration

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Abstract

While both the determinants of R&D collaborations and the effects of government funding on private R&D expenditures have received considerable attention individually, the effect of their interaction on incentives for firm collaboration is not well understood. This paper develops a theoretical model which can explain a previously overlooked empirical finding: R&D collaborations that receive government funding are significantly smaller than those that don’t. The model finds that government R&D subsidies can crowd out large collaborations under very general conditions even if additional firms offer unique intellectual capital to research projects or if firm-specific intellectual capital and R&D funds are compliments, leading to a potential increase in market prices. While subsidies decrease the size of collaborations which would have occurred anyway, there is evidence that they could spur new R&D collaborations.

Keywords: Innovation, Self-Enforcing Agreements, Crowding Out, Research Joint Ventures

JEL Codes: O31, L22, L11, O33.

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1 Introduction

It is widely accepted that there can be a role for government in R&D insofar as there is a public good aspect to widely implementable basic R&D undertaken to reduce firm costs. In line with OECD averages, the United States economy spent almost 3% of GDP on R&D in 2009, with the government accounting for 27.1% of all R&D expenditures and private industry accounting for 67.3%.\footnote{Of total R&D expenditures, 72.6% of actual research was conducted by the private firms, 12.8% by universities and 10.6% by government agencies such as national laboratories.}\footnote{National laboratories are dedicated research institutions that have operating budgets decided at the national level. In the United States, for example, the Department of Energy’s National Labs operate on a budget of roughly $5 billion per year. National labs perform R&D to meet government objectives in addition to working both for and with private firms and universities.} These data and recent studies suggest that R&D is performed by various entities and governments fund a significant portion of research (Aschhoff and Schmidt (2008)).

The effects of increased government R&D expenditures on private R&D projects have been studied in some depth. There is mixed evidence that government R&D subsidies to individual private firms crowd out those firms’ own spending on R&D (David, Hall, and Toole (2000), Wallsten (2000), Almus and Czarnitzki (2003), Ciccotello, Hornyak, and Piwowar (2004), Toole (2007)). While the effects of exogenous government funding on R&D expenditures of individual firms has been studied in some depth, the effect of government funding on research collaborations between firms has received little attention.\footnote{One notable exception is Fan and Wolfseter (2008) which analyzes the incentives for different governments to compete on R&D subsidy rates in a globally competitive landscape where two domestic countries can cooperate. This paper focuses on firm incentives conditional on a subsidy regime. Cohen (1994) discusses how government subsidies could affect incentives for R&D collaborations but offers no rigorous model.} It is not clear that the incentives governing an individual firm’s use of government R&D grants are the same as the incentives of research collaboration member firms, the research collaboration as a whole, or a potentially collaborating firm.

R&D collaborations, commonly called Research Joint Ventures (RJVs), are arrangements in which multiple firms voluntarily enter into an agreement to spread R&D costs

\footnote{The remaining portion is categorized as other.}
and benefits with other RJV members but compete in the output market (Katz (1986), Kamien, Muller, and Zang (1992), Bloch (1995), Leahy and Neary (1997), Hernan, Marin, and Siotis (2003), Bourreau and Dogan (2010)). They have been legally permitted in many OECD countries for roughly 30 years. Hence, research collaborations are inherently self-enforcing agreements (Poyago-Theotoky (1995)). Recently, many governments including the United States, began earmarking R&D subsidies specifically for R&D collaborations between private firms. While legally permitting RJVs is widely accepted, passing legislation to subsidize them implies that there is some form of market failure in their provision. To that end, the introduction of government funds into research collaborations may increase the incentive of any one firm to enter into a collaboration. However, exogenous government funding of an R&D collaboration may also increase the incentive for firms to appropriate government funds for themselves sharing the gains from cost reducing R&D less widely.

This paper develops a simple theoretical model and an extended model explaining how government funding affects the incentives to form R&D collaborations. The model is motivated by datasets describing all RJVs registered in the US from 1985-2007 and all funded R&D collaborations in the US under one particular funding program. The data show that receipt of government grants is correlated with a significant decrease the size of R&D collaborations.

The theoretical model highlights how the presence of government R&D grants earmarked for collaborations between firms affect the incentives for firms to more or less cooperative in R&D with other firms. Since the empirical motivation of the paper is the observed number of firms in funded versus unfunded RJVs, the theoretical model focuses on how the equilibrium number of firms that enter into a collaboration is affected by the possibility of government subsidies. The number of firms that enter into a cost reducing collaboration is important: if a small number of firms capture a large amount of government funds, then those cost savings are not as widely distributed.

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4 One example is the Advanced Technology Program (ATP) which was administered by the National Institute of Standards and Technology (NIST) in the U.S.
throughout the industry. The simple model developed in this paper shows that this incentive to reduce RJV size can actually lead to increases in output market prices due to the implementation of a RJV subsidy program.⁵

There are two theoretical models developed in the paper which have the size of RJVs as endogenously chosen by participants in a competitive output market. In the first theoretical model, additional firms are valuable only insofar as they offer additional R&D funding. This first theoretical model extends the one described by Cohen (1994) in that firms have asymmetric costs, potentially affecting the relative incentives of firms to enter into - or prevent entry into- R&D collaborations. The second theoretical model extends the first by allowing firms to improve R&D productivity by contributing firm-specific intellectual capital directly to the collaboration through their involvement in additional to monetary resources.⁶ The extended model, then, creates added incentives to have large collaborations. Further, the most general form of the second model also permits firms to fund R&D collaboration at different levels. The goal of each model is to carefully identify how various collaborative and subsidy structures affect the incentives of collaborating firms.

Both models find that government R&D grants reduce equilibrium collaboration size of RJVs which would have formed anyway regardless of the government’s grant awarding process. In forming a RJV, members firms weigh the benefits of permitting an additional participant in terms of additional R&D funds and intellectual capital against the cost of diffusing any technological improvements to additional competitors. When government grants are introduced, the marginal net benefit in terms of addition funds from including another member is reduced. As a result, the models find that government grants to R&D collaborations will often reduce equilibrium collaboration size regardless of the government’s grant awarding process. This can have surprising

⁵Focusing on this one statistic, however, abstracts away from other important metrics. For example, the presence of government grants could affect bargaining over firm specific monetary contributions to a RJV and therefore total R&D expenditures. However, the number of firms is a sufficient statistic for the principle question of this paper: what is the effect of government R&D subsidies on incentives for R&D collaboration between firms?

⁶Instead of intellectual property, I refer to the intellectual capital of a firm as this may be embodied in the skills set of the workforce of individual firms.
effects in the market: equilibrium market prices can actually increase if RJVs are subsidized if the incentive to capture government R&D subsidies is large enough. This result can hold under very general conditions, even when firm-specific intellectual capital and R&D funds are compliments. While government grants will often decrease the size of RJVs which would have occurred anyway, the most general model finds suggestive evidence that subsidies might also increase the desirability of collaboration for firms previously not willing to collaborate. The strategic incentives created by government subsidized R&D collaborations is very important for several reasons. First, it creates a very significant problem in determining the additional value created by government R&D subsidies dedicated to collaborations. Specifically, there is both a direct effect (reduced cost) and indirect effect (a new RJV industry structure) that makes constructing a counterfactual equilibrium for RJVs without subsidies very difficult. Ignoring the strategic effect will lead to incorrect welfare implications of the subsidy policy. Second, if the size of research collaborations change upon the introduction of exogenous government funds (e.g., there is an impact on the extensive margin of collaboration size), then new lower cost proprietary technologies will not be used as widely in the economy, decreasing welfare relative to a situation where the strategic did not exist.\footnote{While beyond the scope of the current model, this effect would be exacerbated if government funds crowd out individual firms’ R&D levels on the intensive margin as well.} Given the magnitude of OECD governments’ R&D funding, these interactions can no doubt affect current and future distributions of welfare for both consumers and producers by affecting the distribution of costs across firms (Hopenhayn (1992)).\footnote{This is especially true when one considers that government R&D funding levels for collaborations do not endogenously respond to the quality and quantity of R&D grant applications.} As a result, this could be an important but previously unidentified source of crowding out from government grants.

This paper contributes to the literature in several ways. First, it highlights the subtle empirical finding that firm collaborations are smaller when they receive government subsidies than when they do not. Second, it develops a rigorous theoretical model which highlights the strategic incentives created when R&D collaborations are
eligible for government funding. Third, it shows that the strategic incentive created by subsidies for firms in permitting additional entry into collaborations is robust to a wide variety of settings. Lastly, it identifies the importance of strategic incentives for empirically identifying the additional benefit created by RJVs when R&D collaborations are eligible for government funding and therefore estimating welfare from such policies.

The paper is set up as follows: section 2 offers simplistic empirical motivation showing that RJVs receiving government funding are smaller than those which do not. Section 3 introduces the first theoretical model and discusses results. Section 4 introduces the second theoretical model and discusses results. Section 5 concludes.

2 Empirical Motivation

This section describes the composition of R&D collaborations in the United States using a dataset containing information on the characteristics of US RJVs from the mid-1980s to the late 2000s. This analysis is only descriptive and meant to highlight the relative size of RJVs that receive government funding. It is clear from simplistic analysis of the data that R&D collaborations receiving government grants are significantly smaller than those which do not. Further, it is clear that funding levels are significant and large.

The passage of the National Cooperative Research Act (NCRA) of 1984 relaxed anti-trust laws in the US as they pertained to cooperation between firms in R&D activities in the US. In order to qualify for anti-trust law indemnity under the act, all RJVs in the US must file with a Federal Registry. Data from the filings was collected from 1985-2007 in the Collaborative Research database (CORE). The CORE database includes a binary variable indicating whether the RJV was funded by a government grant at the time of filing. Many of the grants in the database are from the Advanced Technology Program (ATP) which was administered by the National Institute of Standards and Technology (NIST). This federal program is an important source of R&D

Table 1: US RJV Summary Statistics, 1985-2007

<table>
<thead>
<tr>
<th></th>
<th>Funded</th>
<th>Unfunded</th>
<th>All RJVs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>79</td>
<td>880</td>
<td>959</td>
</tr>
<tr>
<td>Ave. Number Members</td>
<td>6.11</td>
<td>13.92</td>
<td>13.28</td>
</tr>
<tr>
<td>Process Driven</td>
<td>38</td>
<td>450</td>
<td>488</td>
</tr>
<tr>
<td>Product Driven</td>
<td>35</td>
<td>353</td>
<td>388</td>
</tr>
</tbody>
</table>

Data from CORE database, 1985-2007.

Table 1 shows summary statistics from the CORE database by RJVs that received government R&D funding, those that don’t and the sum over all RJVs for all years in which data is available. RJVs receiving government funding are less than half the size of RJVs not receiving government funding and the difference is statically significant. There is no clear relationship between funding a RJV and whether a RJV is created to improve an industry’s product or an industry’s process implying there is no systematic difference, at a high level, between the types of projects collaborated upon generally and collaborated on which receive government funding. Appendix B offers a more in depth empirical treatment but these qualitative results from Table 1 hold.

Difference in means shows that receiving funding at time of formation is correlated with smaller sized R&D collaborations. An alternative way to summarize the relationship between government grants and RJV characteristics is to look at all RJV which receive government funding at any point in time. This is possible by using an alternative database summarizing all R&D collaborations which received NIST funding at any point in time from 1991-2007. The ATP database was scraped from the ATP website and is publically available. The ATP database has the universe of all R&D collaborations which received an NIST during the program’s duration. The CORE database and the ATP database are different because the CORE database lists all reg-

9RJVs in the ATP database can be partially matched to those in the CORE database but complete matching is impossible because of the lack of firm specific data in the CORE database. As a result, the complete list of R&D collaborations that were funded at any point cannot be juxtaposed relative to those that were not.
istered RJVs but notes that the collaboration received funding only when that funding was already secured at the time of filing whereas the ATP database lists only RJVs that received funding at any point during their existence. Brief analysis of the ATP database below shows that R&D collaborations that received government funding at any time where roughly the same size as those that received funding at the time of registration and that the leading firm in R&D collaborations if often larger firms.

The ATP dataset contains the size of the monetary award, the firm partnership contribution, the size of the RJV, the member firms, indication of which firm was the leading firm in the collaboration, and the type of award that was granted. In all, there were 185 funded RJVs between 1991 and 2007. The average award was $5.79 million and the average total match of member firms was $6.52 million meaning that the match rate for the total of all firms was a bit over 50%. This funding, then, is a significant subsidy to firms for being in R&D collaborations. Casually, then, one might expect more firms to want to participate in those RJVs which already would have existed in the absence of funding, thereby increase the size of RJV collaborations. There is also an additional incentive for firms to create new R&D collaborations if funding is present.

There are two more important aspects of the ATP dataset. First, the average size of all funded collaborations was 4.39 firms (standard deviation 3.64). Recalling that the average size of all RJVs in the CORE database that received a grant at the time of filing was 6.11, the difference between RJV size for collaborations receiving funding is not large. This evidence is further corroborated by the fact that the average size of RJV that did not have a grant at time of filing was 13.94 in the CORE database. Note that 13.94 is a lower bound and the actual average size for RJVs that never received a grant; the true average is higher since the the ATP database shows that the population mean of the federally registered RJVs which received grants after federal registration was smaller than the mean of those receiving grants at the time of registration.

Second, the leading firm in each collaboration is listed explicitly in the ATP database. The leading firms tend to be very large recognizable companies or subsidiaries of large
recognizable companies. A complete quantitative analysis of the relative size of leading firms is not possible since some of leading firms and other member firms are private. 45 of 185 funded collaborations have as the leading firm one of the following companies: Dow, Siemens, GE, GM, Ford, Caterpillar, Xerox, 3M, IBM, United Technologies or Honeywell. Roughly half of the funded RJVs have at least one of these firms as a member. The point to be made is that funded research collaborations often are led by an industry’s largest firms. Further, there is significant heterogeneity in the composition of funded RJVs member firms. The models below reflect this fact by allowing firm heterogeneity both with respect to firms specific cost and contributions to R&D funds.

3 Simple Theoretical Model

This section introduces the first theoretical model starting with firms’ cost structure followed by the competitive environment and finishing with the collaborative R&D structure. It then examines equilibrium in the collaborative R&D environment and output market introduced in the previous section and derives theoretical results. This model extends the discussion by Cohen (1994) by allowing asymmetric firm costs and is presented to develop intuition for the second model which follows. All proofs are in the appendix.

Each firm in the industry is assumed to produce a homogenous good. Assume that each firm’s marginal cost has two components. The first component is shared by all firms in the industry, \( \bar{c} \), and the other is idiosyncratic and firm specific, \( \eta_i \). Establishing common and idiosyncratic cost components is valuable for two reasons. First, it allows for a simple but empirically motivated RJV formation rule which we detail below. Second, it greatly simplifies the exposition of a brief welfare exercise at the end of this section.

Assume that \( \eta_i \sim f(\eta) \) for all firms \( i \). Thus, a firm’s total marginal cost is defined as \( c_i = \bar{c} + \eta_i \) where \( \bar{c} \) is a common cost component shared by all firms in the industry and \( \eta_i \) is a firm specific cost component. Note that in equilibrium \( \max(\eta_i) = P^* - \bar{c} \)
where \( P^* \) is the equilibrium price in the market. Put another way, the equilibrium price is the market is the maximum possible marginal cost for any firm.

Assume there are \( i = 1, 2, ..., N \) firms in Cournot market competition. Inverse industry demand is assumed linear:

\[
P = \alpha - \beta \sum_{i=1}^{N} q_i.
\] (1)

Each firm’s equilibrium Cournot profit is given by \( \pi_i = (q^*_i)^2 \beta \).\(^{10}\)

Assume that collaborations can reduce the common cost component, \( \bar{c} \), of only R&D collaboration member firms. For simplicity, assume that firms not in the R&D collaboration do not receive any benefit in the form of spillovers. Further, to highlight incentives for collaborations, we abstract from firm’s own R&D decisions in this model. Assuming perfectly functioning capital markets, all firm specific individual R&D projects will be funded in equilibrium. Given the perfect capital markets assumption, any interaction between firms’ own R&D decisions and collaborating with additional firms would be second order. Further, this modeling decision greatly simplifies the theoretical model to highlight the role of government grants on incentives to form R&D collaborations.

Cost reductions to firms in the R&D collaboration are a function of total monetary resources dedicated to the collaboration, \( R \). Specifically, upon entering a RJV, the common costs of all member firms are dictated by a convex function \( \bar{c}(R) \) where \( \bar{c}_R < 0 \) and \( \bar{c}_{RR} > 0 \).\(^{11}\) Total R&D resources \( R \) is assumed to be the sum of all RJV member firms’ contribution to the RJV, \( \phi_i \), plus any additional funds received from a government grant, \( \tilde{R} \), such that \( R = \sum_{i=1}^{n} \phi_i + \tilde{R} \).

Any government grant, \( \tilde{R} \), can take one of two forms. The first is a block grant. A block grant is a set amount of funding dedicated to the RJV. This assumption reflects that government grants often require R&D collaborations to apply for a set amount of government funding upon forming. We also consider a cost match grant structure.

\(^{10}\)Note that \( q^*_i = \frac{\alpha + \sum_{j \neq i} c_j - N c_i}{(N+1)\beta} \).

\(^{11}\)Note that \( \bar{c}(0) = \bar{c} \) and \( \lim_{R \to \infty} \bar{c}(R) = 0. \)
In a cost match structure, the government matches $s\%$ of each firm’s contribution to the R&D collaboration. This is precisely the structure of the ATP grants discussed above. Given that both types of granting structures occur in OECD countries, both are important to consider separately. Finally, for simplicity in this simple model, we assume that all firms pay the same entry fee such that $\phi_i = \phi_j = \phi$. While we relax this assumption in the extended model, throughout the paper we don’t allow firms to endogenously bargain over firm specification contributions. We instead focus on different cost sharing rules observed in practice, such as contributing as a proportion of expected firm gains. This serves to maintain focus on how the presence of government subsidies earmarked for RJV collaborations affect the incentives to engage in widely disseminated cost saving technologies.\footnote{Again, the number of firms that enter into a cost reducing collaboration is important: if a small number of firms capture a large amount of government funds, then those cost savings are not as widely distributed throughout the industry; therefore increasing market prices had the RJV size effect at the core of this paper not been present.}

The RJV formation rule in this model comprises a two stage game. The first stage is the RJV formation phase and the second stage is the production phase. In the formation phase, a leading firm starts a RJV of size one and firms are sequentially invited to enter the RJV one by one. The leading firm is always assumed to be the firm that benefits the most from the RJV. Put another way, the firm with the smallest idiosyncratic unit cost, $\eta_i$, is the leading firm since they produce the largest quantity in the production stage and thus have the greatest benefit of lower unit costs. This modeling decision reflects the programs discussed in the previous section: all NIST collaborations have a leading firm that is the point of contact for the federal agency.\footnote{It is often the case that leading firms in RJVs are also the largest firm in the collaboration. An NIST dataset highlighting this feature is available upon request.} Invited firms can accept or decline invitation. If a firm accepts invitation, then both firms must unanimously agree to invite an additional firm into the collaboration. One by one, invitations continue until all firms already in the RJV do not unanimously agree to offer further entry to additional firms.

A firm deciding whether or not to enter a RJV must decide whether to be a con-
tributing member of a RJV and pay the amount $\phi$ for entry in order to gain access to the lower unit costs via R&D collaboration. Assume that $\phi$ is always less than the benefit of joining the collaboration so that firms will always accept an offer to join a RJV when invited.\footnote{This assumption is for simplicity only. Proofs showing it can be relaxed at no qualitative loss both for this simple and the extended model are available upon request.} In the second stage all firms produce Cournot equilibrium quantities as a function of their post R&D marginal costs and earn subsequent profits. Firms are assumed to have perfect foresight of the production stage in the formation stage as is common in the literature (Poyago-Theotoky (1997)). We assume that only one RJV is allowed per industry. This assumption highlights the incentives of external funding on incentives to collaborate. In sum, an R&D collaboration is in equilibrium when either 1) one firm in the collaboration would suffer a decrease in profits from permitting an addition firm to enter, thereby blocking further entry or 2) no additional firms have an incentive to enter the collaboration. These equilibrium conditions highlight the self enforcing nature of R&D collaborations.

Noting that we restrict our analysis to collaborations which have an internal solution (e.g., not all firms in the industry are invited to enter the R&D collaboration), it is important to state three lemmas.

**Lemma 1:**

_A firm’s profits are identically effected symmetrically by a decrease in cost from any rival firm._

Intuitively, if there is a decrease in a particular firm’s costs, that firm will always produce more output. For a given cost reduction, the increase in output is the same regardless of initial costs of the firm. For example, if costs decrease by one unit, the increase in output will be the same for the lowest cost firm or the highest cost firm. In a linear Cournot game, when a firm increases their output, the market price falls thereby decreasing profits of other firms. Lemma 1 states that a firm’s profits fall by the same amount if a competitor firm’s costs decrease, regardless of the competitor firm.
Lemma 1 indirectly addresses the decisions of firms already in an R&D collaboration to invite in additional firms. Specifically, firms already in a RJV are indifferent over which firm to invite into the collaboration next from a strategic standpoint since the indirect effect on own profits of inviting additional firms into a RJV is not a function of the additional firm’s characteristics.

The next Lemma directly addresses the joint decisions of firms already in a collaboration for inviting in additional firms.

**Lemma 2:**

*Regardless of idiosyncratic costs, firms already in a R&D collaboration will always agree on when to stop inviting additional firms into the collaboration.*

Intuitively, Lemma 2 states that while the level of the change in profits from permitting additional entry is a function of a firm’s characteristics, the sign of the change in profits is not. Specifically, the reduction in costs to a firm entering the R&D collaboration symmetrically affects all firms already in the RJV as per Lemma 1. Similarly, the benefits on RJV expansion are shared amongst firms in the RJV thereby increasing competition. Therefore, all firms already in a RJV will agree on when to add and when to stop adding additional firms.

The final Lemma addresses the marginal benefit of adding additional firms to the R&D collaboration as the size of the collaboration increases.

**Lemma 3:**

*The marginal benefit of each additional firm entering the R&D collaboration to firms already in the collaboration is monotonically decreasing.*

There are two reasons why the marginal benefit of additional firms entering a R&D collaboration is decreasing. First, the direct decrease in cost to the entering firm grows as the total level of R&D funding grows making the entering firm relatively more competitive as RJV size grows. Second, there is a cost reduction for all firms already in the collaboration due to additional entry. As the number of firms in the collaboration increases the number of competitors receiving a benefit grows making the
industry more competitive.\textsuperscript{15}

The Lemmas and the RJV formation rule jointly lead to the main results from the first model. These results, and subsequent results, consider equilibrium R&D collaboration size when a collaboration forms privately and one that forms in the presence of an exogenously determined amount of government funding. This modeling decision is motivated by the nature of many R&D collaboration funding opportunities which set either a maximum possible block grant to the collaboration or, as is more common, a specific cost matching ratio. For example, the cost match ratio is set at roughly 50\% in the ATP dataset discussed above. We wish to determine if equilibrium collaboration size is larger, smaller or unchanged given the presence of such exogenous government funding.

**Proposition 1:**

R\&D collaborations receiving exogenous block grant funding are smaller than those not receiving funding.

\textsuperscript{15}This is similar to the results in Bloch (1995).
Corollary 1:

*R&D collaborations receiving exogenous cost match funding are smaller than those not receiving funding.*

The intuition behind both Proposition 1 and Corollary 1 is shown in Figure 1. In Figure 1, the marginal cost curve represents the marginal cost to firms already in a RJV of permitting additional firms to enter. The marginal benefit curve is defined similarly. Without funding, the equilibrium number of firms in a collaboration is $n^*$ with each of those $n^*$ having shared costs $\bar{c} - \bar{c}(\phi n^*)$ after the cost saving collaboration. The introduction of government funds to a R&D collaboration causes a discrete increase in the marginal cost associated with each entrant to firms already in the RJV. All competitors already in the collaboration have significantly lower costs due to the government funding, increasing the cost of permitting further entry into the R&D collaboration due to the competitive effect. Further, the marginal benefit associated with each additional entrant is reduced upon the introduction of exogenous funding because the function dictating returns to R&D funding is concave. Taken together, these two factors will always lead to a decrease in equilibrium collaboration size upon the introduction of government funds. Therefore, in the presence of grants for collaborative R&D, the equilibrium size decreases to $n^*_R < n^*$. We term the ratio of this size effect the “funds capture effect” (e.g., $\frac{n^*_R}{n^*} < 1$). The new costs of the firms which are in the funded collaboration are $\bar{c} - \bar{c}(\tilde{R} + \phi n^*_R)$. Note that due to the “funds capture effect”, it is possible for the cost reduction to firms in the funded RJV could be greater or less than those in the unfunded RJV. The magnitude of this relative cost reduction difference we term the “cost reduction effect”. This qualitative result holds whether the funding mechanism is a block grant or cost match.

This section developed a very simple theoretical model showing that if government funds are earmarked to subsidize collaborative R&D, it will decrease the equilibrium size of those collaborations. A related but different question is identifying the implications of this size reduction for welfare. For example, is it possible that due to the funds capture effect being sufficiently larger, welfare could actually decrease if R&D subsidies
were allocated to collaborations. A sufficient statistic to identify welfare effects is to examine the effect of implementing a program of R&D grants for collaborations on market price relative to not implementing such a program.

There are two relevant cases we consider: 1) the effect of unfunded R&D collaborations on market prices and 2) the effect of funded R&D collaborations on market prices. Market prices are proportional to the sum of all firms’ costs in Cournot (see equation (1) above). As a result, we would like to evaluate the total industry decrease in costs attributable to unfunded RJVs relative to those from funded RJVs. Since collaboration only affects the shared cost component in this model, we can ignore all idiosyncratic costs in the analysis, greatly simplifying exposition.\footnote{\textsuperscript{16}This model abstracts from spillovers. There is a very real question of the extent to which spillovers could matter in welfare analysis of these policies. This interesting extension is left to future work.}

As shown above, the decrease in cost to any one firm in an unfunded RJV after engaging in a cost reducing collaboration is $\bar{c}(\phi n^*)$. As a result, the total decrease in cost is $n^*\bar{c}(\phi n^*)$. Similarly, the decrease in cost to any one firm in a funded RJV after engaging in a cost reducing collaboration is $\bar{c}(\tilde{R} + \phi n^*_{\tilde{R}})$. Therefore, the total decrease in cost in the industry conditional on a funded RJV is $n^*_R\bar{c}(\tilde{R} + \phi n^*_{\tilde{R}})$. Comparing cost reductions across funded versus unfunded RJVs yields a couple of simple rules of thumb which indicate whether a policy of subsidizing R&D collaboration could actually cause an increase in market prices relative to not subsidizing them. Specifically, consider when total industry cost reductions from unsubsidized RJVs are larger than for subsidized RJVS:

\[ n^*\bar{c}(\phi n^*) > n^*_R\bar{c}(\tilde{R} + \phi n^*_{\tilde{R}}). \]

Rearranging terms:

\[ \frac{\bar{c}(\phi n^*)}{\bar{c}(\tilde{R} + \phi n^*_{\tilde{R}})} > \frac{n^*_R}{n^*}. \]

The expression on the right hand side is the “funds capture effect” and it is always
less than one.\textsuperscript{17} The expression on the left hand side will always be greater than one if the subsidy more than accounts for the decrease in funding from direct contributions of collaborating firms.\textsuperscript{18} Therefore, even in the case when the subsidy doesn’t account for the decrease in funding from the direct contributions of collaborating firms, this model finds that there is still plenty of scope for subsidizing R&D collaborations to actually increase market prices relative to not subsidizing them.

### 4 Extended Theoretical Model

In this section, we extend the model above by allowing firms to affect productivity of a RJV by both their monetary contributions and their intellectual capital. As a result, the model in this section provides a stronger incentive to maintain high RJV firm membership numbers than the previous model.

R&D collaborations still reduce the common cost component, \( \bar{c} \), of R&D collaboration member firms. Firms that are not in the R&D collaboration do not receive any benefit in the form of spillovers. As before, cost reductions to firms in the R&D collaboration are a function of total monetary resources dedicated to the collaboration, \( R \). Now, though, common costs of members of a R&D collaboration are assumed to also be a function of the number of RJV member firms directly, \( n \). Including the number of member firms directly in the R&D benefit function represents the potential benefits of the intellectual capital from each entering firm pertinent to the R&D collaboration. We assume below that benefits to R&D are increasing in number of member firms to reflect that each firm brings additional expertise to the collaboration that can benefit all collaborating firms. This is identical to assuming all firms have unique intellectual capital but that R&D benefits are concave in intellectual capital. As a result, each entering firm’s intellectual capital directly increases increase the productivity of R&D conditional on an amount of funding, a modeling assumption with support in the literature (Cassiman and Veugelers \( 2002 \) and Lopez \( 2008 \)).

\textsuperscript{17} In the data presented above, this is roughly 50%.

\textsuperscript{18} Recall that the function \( \bar{c}(\cdot) \) is decreasing in it argument.
The decrease in cost of a firm entering a RJV is \( \bar{c} - \bar{c}(n, R) \) where \( \bar{c}(n, R) \) is the common cost of firms that enter the RJV. Assume that the function \( \bar{c}(n, R) \) is decreasing and convex in both its arguments: \( \frac{\partial \bar{c}(n, R)}{\partial n} < 0, \frac{\partial^2 \bar{c}(n, R)}{\partial n^2} > 0, \frac{\partial \bar{c}(n, R)}{\partial R} < 0, \text{ and } \frac{\partial^2 \bar{c}(n, R)}{\partial R^2} > 0. \) For now, allow that the cross-partial of RJV size, \( n \), and RJV funding, \( R \), can be either positive (\( R \) and \( n \) are substitutes) or negative (\( R \) and \( n \) are complements). Each case will be analyzed separately below.

Each case considered in the extended model, that intellectual capital (represented by \( n \)) and monetary resources (represented by \( R \)) may be substitutes or complements, is reasonable. For example, with basic research large fixed costs investments in capital, such as laboratory equipment, are needed in order to fully take advantage of each firm’s intellectual capital. Indeed, this is a main motivation for RJVs legality. In this case, monetary resources and intellectual capital are complements. At the same it is also possible that monetary resources and intellectual capital are substitutes. For example, if monetary resources pay for information, such as data or experiments, then it could deem a particular firm’s intellectual capital unimportant.

The extended models also considers two regimes over R&D collaboration entry fees, \( \phi_i \). The first case assumes that \( \phi_i = \phi_j = \phi \) \( \forall \ i \neq j \) such that all firms are equal research partners. In this case, it is possible to embed monetary resources of the RJV, \( R \), in the number of RJV member firms, \( n \). As a result, define \( \tilde{R} = R - \sum_{i=1}^{n} \phi_i \) as the total resources used in the RJV not contributed by firms, such as resources from a government R&D grant. For notational ease, when a RJV does not receive a government grant the \( \tilde{R} \) argument will be dropped since \( \tilde{R} = 0 \) such that: \( \bar{c}(n, 0) = \bar{c}(n) \). The second regime for entry fees assumes that \( \phi_i \) is proportional to the benefit a firm receives from entry and will be discussed in detail below. Both regimes of collaboration fees are important to consider: in the ATP database discussed

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19 A final technical assumption is that \( | \frac{\partial \bar{c}(n, R)}{\partial n} | + | \frac{\partial \bar{c}(n, R)}{\partial R} | > | \frac{\partial^2 \bar{c}(n, R)}{\partial R \partial n} | \) implying that the sum of the direct effects of further entry is larger than the interaction of the two effects. This assumption is not needed for the results in this paper but eliminates two classes of uninteresting equilibria. This assumption eliminates the trivial cases for which every firm enters that RJV in the case of complements or no firms enter in the case of substitutes.

20 Entry fees are can be thought of as cost sharing rules.
above, RJVs were formed between only large firms and between both large firms and small firms. While symmetric entry fees may be expected from firms of similar size (or firms expected to gain the same amount from the collaboration), it may not be a reasonable assumption if there are firms of dramatically different sizes within the same collaboration. All proofs are in the appendix.

There are some drawbacks to this modeling approach. The first is that firms have no option for in house research. If capital markets are efficient then all productive firm-level research projects will be undertaken. Second, this model does not allow multiple RJV to form in a single industry. Third, this model does not consider allow for spillovers. These first three assumptions are made to focus the paper on the incentives created by exogenous government funding dedicated for R&D collaborations. For example, allowing in house research within the model will not affect the change in incentives for collaboration created by R&D subsidies dedicated to RJVs. Similarly, allowing multiple RJVs in an industry does not necessarily change the incentives for collaboration within any one particular RJV which are created by R&D subsidies. Lastly, spillovers will affect the opportunity cost of not being in a collaboration but will not necessarily change the incentives for collaboration within any one particular RJV.

Fourth, this model does not permit bargaining over firm specific entry fees $\phi_i$. Finally, the awarding process of government funds is not explicitly modeled here. Allowing bargaining over a firm specific $\phi_i$ or modeling the funding award process distracts from the focus of this paper, which is to find the effect of government funds on incentives to form R&D collaborations.

4.1 Symmetric Entry Fees

Consider the profits of a member $i$ in an RJV of size $n$ if all firms pay a symmetric R&D costs $\phi$. Firm $i$ will always accept another firm into the RJV as long as its profits increase when additional firms enter. The equilibrium invitation strategy is to invite firms until the marginal benefit of doing so is zero. This leads directly to proposition 2.
Proposition 2:

For any market size $N$ in which a private RJV is formed by at least two firms, there is number of member firms, $n < N$, already in the RJV for which member firms will exclude any additional members.

Intuitively, the benefit that accrues to a firm in an R&D collaboration when another firm enters is derived from both the additional firm’s contribution to funding, $\phi$, and the intellectual capital of additional firms. The cost associated with another firm entering is the decrease in price associated with the increases in quantity supplied to the market by the marginal firm. When these two effects cancel the firms already in the RJV will be opposed to allowing another entrant. Proposition 2 says that the two effects cancel for some $n < N$. The reason is the discrete fall in the entering firm’s cost relative to the diminishing effect on RJV member firm costs. Note that due to the assumption that all firms pay a common entry cost, the leading firm and firms already in the RJV are indifferent as to which firms to invite into the RJV. This assumption will be relaxed in the next subsection.

Proposition 2 states that for any RJV, there is a subset of firms that will block further entry. Of specific interest in this paper is how the existence of exogenous funds, such as government grants, will affect the incentives of firms already in a R&D collaboration to permit additional entry. There are two cases to consider: 1) when funding and intellectual capital from additional firms are substitutes ($\frac{\partial^2 c}{\partial n \partial \tilde{R}} > 0$) and 2) when they are complements ($\frac{\partial^2 c}{\partial n \partial \tilde{R}} < 0$). It is reasonable that in some cases firms and funding are substitutes: if additional firms decrease the appropriability of new technologies then the relative benefit of their RJV fee is ameliorated by additional funds. It is also reasonable that in some cases firms and funding are complements if additional funds increase the benefit of RJVs of any size. Consider first the case of substitutes in the following proposition:

Proposition 3:

If innovative capital and R&D funding are substitutes, a RJV with exogenous funding will always have fewer members than a RJV which doesn’t have exogenous funding.
In the case of substitutes, a R&D collaborations receiving exogenous funding will always be smaller than a RJV that doesn’t have exogenous funding because the monetary resources gained by adding the marginal firm are offset by the resources which are garnered by the exogenous funding. It implies that government grants to R&D will crowd out membership from marginal firms, a finding consistent with the data presented above. 

Figure 2 visually represents the effect of further firm entry to a firm’s profits that is already in a RJV when innovative capital and R&D spending are substitutes. In Figure 2, the marginal cost curve represents the marginal cost to firms already in a RJV of permitting additional firms to enter. The marginal benefit curve is defined similarly. In Figure 2, the marginal cost to firms already in RJV of further firm entry is proportional to the sum of the discrete decrease in costs of the entering firm ($\bar{c} - \bar{c}(n, R)$) plus the marginal decrease in cost received by the other firms already in the collaboration ($n\bar{c}_n(n, R)$). The marginal benefit to firms already in RJV of further firm entry is the decrease in own firm costs due to the marginal entrant ($-N\bar{c}_n(n, R)$).
When exogenous government funds are introduced, $\tilde{R} > 0$, the marginal cost of adding another firm increases for all RJV sizes thereby shifting up the MC curve. Meanwhile, the marginal benefit falls since funds and firms are substitutes and the MB curve shifts down. Therefore, it will always be the case the if government funds and research partners are substitutes then government funds will crowd out additional research partners.

Now consider the case when government funding and additional research partners are complements:

**Proposition 4:**

*If innovative capital and R&D funding are complements, a RJV with exogenous funding can have fewer members than one that doesn’t.*

It is somewhat surprising that even when R&D funding and number of members are complements, government grants can still crowd out entrants. Since funds and innovative capital are complements, addition government grants now increase the relative benefit of adding additional firms thereby shifting up the MB curve of additional firm entry for a RJV of any size. However, each firm already in the RJV has lower shared costs due to the government grant. This exacerbates the competitive costs of adding addition firms to the collaboration shifting the marginal cost curve of allowing additional firms to enter up. Further, the government grant decreases the direct effect of an incoming firm’s monetary resources. If the second two effects (exacerbated competitive effects and decreased benefit of incoming R&D dollars) dominate the first (interaction of incoming firms and exogenous funding), then the equilibrium RJV size will fall. More precisely, the exacerbated competitive effects due to government funding can be defined as the elasticity of the marginal decrease in costs due to additional members with respect to funding. The decreased benefit of incoming R&D dollars can be defined as the elasticity of the marginal decrease in costs due to funding with respect to additional members. Both of these elasticities must be more inelastic than both the elasticity of the marginal decrease in costs due to firms with respect to firms and the marginal decrease in costs due to R&D dollars with respect to R&D dollars. Whether
this is true or not is an empirical question that is outstanding.

Figure 3 visually represents the effect of further firm entry to a firm’s profits that is already in a RJV when firms and R&D spending are compliments. The main difference between Figure 2 and Figure 3 is the change of direction in the MB curve’s shift in Figure 3. In the case of complements, exogenous funding increases the benefit of additional firms. Note though that complementarity also increases the MC of new firm entry by more than in the case of substitutes. On net, the equilibrium size of RJVs receiving government grants can still decrease if the magnitude of the shift in the MC curve overwhelms the magnitude of the shift in the MB curve. This will occur as the direct effects from the increase in RJV and R&D overwhelm the interaction of the two, which is a restatement of the proof of Proposition 4.

\[
MC = \bar{c} - \bar{c}(n, 0) - n\bar{c}_n(n)
\]

\[
MB = -N\bar{c}_n(n, 0)
\]

\[
MB, MC
\]

\[
MC = \bar{c} - \bar{c}(n, \tilde{R}) - n\bar{c}_n(n)
\]

\[
MC = \bar{c} - \bar{c}(n, 0) - n\bar{c}_n(n)
\]

\[
MB = -N\bar{c}_n(n, \tilde{R})
\]

\[
MB = -N\bar{c}_n(n, 0)
\]

Figure 3: Effect of government funds on RJV size: complements

4.2 Asymmetric Entry Fees

In the asymmetric entry fee case assume that the leading firm in the R&D collaboration can demand a firm specific entry fee for all firms in the industry. This more
general model has the effect to make potential R&D partners heterogeneous in the RJV formation stage. With respect to the output market, linear cournot competition treats lowered cost of any competitor identically. As a result, the leading firms is indifferent as to which firm to invite to join a RJV on the competitive margin. However, asymmetric entry fees affect the relative benefit of different potential collaborators in so far as different firms have an asymmetric individual incentives to commit more or less resources to the collaboration. As a result, asymmetric entry fees affect the leading firm’s invitation strategy.

We’ve shown above that lower cost firms have a greater incentive to enter in cost reducing R&D collaborations due to the potential to increase profits on inframarginal units. Assume that firms pay a RJV entry fee proportional to their benefit of entry: \[ \delta \Phi_i \equiv \pi_i^{*,RJV} - \pi_i^{*,-RJV} \] where \( \delta \in (0, 1] \) where \( \delta \) is assumed to be the same across all firms. Note that in the case where the leading firm as all bargaining power, \( \delta = 1 \).

Allowing asymmetric entry fees effectively orders the relative value to the RJV of each unique candidate firm. Specifically, since \( \frac{\partial c}{\partial R} < 0 \), the most valuable individual firm for firms already in a RJV collaboration is the one with the highest entry fee \( \delta \Phi_i \). This creates a simple equilibrium invitation strategy for those already in a R&D collaboration:

**Lemma 4:**

For any distribution of firms indexed by heterogeneous costs with continuous support, the equilibrium invitation strategy is to sequentially invite the lowest cost firms.

Lemma 4 is a very intuitive result. Given that R&D dollars always lower costs and increase profits once a firm is in a R&D collaboration, it is always optimal to invite in those firms willing to pay the most into a RJV. Lemma 4 also has significant empirical implications. It implies that any R&D collaboration will be comprised of the industry’s lowest cost firms. This results is agrees with the data in the ATP dataset discussed above.

The unique equilibrium invitation strategy to sequentially invite low cost firms into a R&D collaboration facilitates extension of the results from the symmetric entry fee
case to the asymmetric case:

**Corollary 2:**

*In the asymmetric case, for any market size $N$, there is number of member firms, $n < N$, already in the RJV for which member firms will exclude any additional members.*

**Proposition 5:**

*With asymmetric cost sharing, if innovative capital and R&D funding are substitutes, a RJV with exogenous funding will always have fewer members than a RJV which doesn’t have exogenous funding.*

**Corollary 3:**

*With asymmetric cost sharing, if innovative capital and R&D funding are complements, a RJV with exogenous funding can have fewer members than one that doesn’t.*

The only difference between asymmetric entry fee and the symmetric entry fee case is that the benefit of entry to outside firms increases when a RJV is funded exogenously. As a result all outside firms all willing to pay more to enter the RJV. Due to the equilibrium invitation strategy, though, additional firms in the asymmetric entry fee case are always less valuable than in the symmetric entry fee case since firms that enter early pay more toward R&D than those that enter later. As a result, any crowding out of firms will be exacerbated if firms’ cost shares are proportional to their benefit of entry.

The robustness of the main results to asymmetric RJV entry fees highlights the drivers behind how government subsidies to collaborative R&D can crowd out incentives to increase membership. Specifically, it increases the discrete decrease in cost that any candidate firm receives from entry. This has the effect of increasing the marginal firm’s benefit from entry and thereby increases the quantity any firm will supply to the market. Further, the presence of government subsidies decreases the marginal value of an entering firm’s entry fee to those firms already in the RJV. Thus, the wedge created by exogenous funding increases the negative competitive effect of those firms already in the RJV to additional entry and decreases the benefit of any potential additional
firm’s monetary resources. Finally, the incentives highlighted in this model are always present regardless of the awarding process for the government grant or the number of R&D collaborations within an industry. Specifically, the crowding out effect of government grants will occur in every case due to strategic competitive effects in addition to the decreasing marginal productivity of R&D funding, whether from private firms or the government.

One final caveat to these results is that when asymmetric entry fees are allowed in the more general model and the government subsizes R&D collaborations, firms are willing to pay more to enter R&D collaborations than they previously were. As a result, this general model offers evidence that government subsidies could induce firms that previously did not have a profit motive to enter an R&D collaboration to enter one. As a result while R&D subsidies may decrease the size of RJVs which would have formed anyway, they may increase the number of RJVs:

**Corollary 4:**

*With asymmetric cost sharing, exogenous funding increases the willingness to pay of firms to collaborate.*

It is important to note that welfare calculations beyond the general rules of thumb described at the end of section 3 are exceedingly difficult in this setting for a few reasons. First, the function dictating the benefits of R&D, $\bar{c}(n,R)$, and the opportunity cost of exogenous government funding are highly uncertain. Regardless, not accounting for the incentives created by R&D grants for collaborative research artificially increases any welfare gains of such a government R&D subsidies program. Second, this model implies that the distribution of costs for firms within an industry will change as a result of the incentives created by grants for R&D collaboration. As a result, the long run competitive effects could overwhelm the short run effects [Hopenhayn (1992)]. Third, the relative decrease in R&D benefits relative to a potential increase in total R&D collaborations introduced in Corollary 4 must be estimated.

This section developed a theoretical model which makes predictions about the size and composition of RJVs depending on whether or not they receive government grants.
If firms can form an exclusive RJV, there is an equilibrium RJV size $n^*$ at which member firms block further entry. If firms can form an exclusive RJV that receives an exogenous funding through government grants, the RJV will have fewer firms even in some cases where funding and intellectual capital are complements. The reason for the last result is due to the effect from government funding making the resources contributed by a marginal firm less valuable overwhelming the complementarity of members and R&D funding. These results hold for both symmetric and asymmetric RJV entry costs. As a result R&D subsidies to collaborative efforts will often crowd out the marginal member and decrease the size of R&D collaboration.

5 Discussion and Conclusion

This paper has developed a simple theoretical model which shows how the introduction of exogenous federal can crowd out membership in cost reducing R&D collaborations. As a result, this paper builds two general models which show that government grants or subsidies can crowd out a form of private R&D: reduced sharing of jointly developed intellectual property. While the incentives for collaboration in the presence of government subsidies have been discussed in the literature [Cohen 1994], they have not been formally modeled in any published work nor have the incentives been considered in a model permitting firm heterogeneity or where firms have R&D enhancing innovative capital. The model can explain a previously unnoticed artifact of the data that RJVs receiving government funding are significantly smaller when receiving a R&D grant. Finally, there are simple rules of thumb which this model shows that subsidizing R&D collaborations could decrease welfare.

There are several avenues for future work. The model suggests that subsidizing collaborative R&D will lead to smaller, but potentially more, R&D collaborations. In the asymmetric Cournot model here, total quantity is a function of total industry-wide production costs. In this model, then, R&D subsidies will have the direct effect of

$\Sigma q_i^* = \frac{a + \sum_{j=1}^{N} c_j - Nc_i}{(N+1)\beta}$. Simplification shows that the distribution of costs

21Total industry quantity is $\Sigma q_i^* = \frac{a + \sum_{j=1}^{N} c_j - Nc_i}{(N+1)\beta}$. Simplification shows that the distribution of costs
increasing welfare by reducing costs. However, some of those welfare gains may be eroded as R&D benefits are distributed across fewer firms due to the strategic effect. The opposite result is also possible, though: if smaller R&D collaboratives were more productive (e.g., R&D benefits are very concave in R&D spending) then the marginal dollar used for a subsidy could increase welfare more specifically due to the strategic effect even though the benefits of R&D are distributed less widely. As a result, one interesting avenue for future research would be to address these counter-intuitive welfare effects of R&D subsidies.

There are other important future research as well. First, to the author’s knowledge, there is currently no well-accepted way to empirically evaluate the additional effect of government subsidies for R&D collaboration on collaborative activity. This model identifies an important strategic element regulators should use when evaluating subsidy programs. Second, both theoretically and empirically identifying the relative magnitudes of the flypaper effect relative to R&D expenditure crowding out on the intensive margin for RJV member firms is needed. Third, future theoretical research is needed to endogenize the funding decisions of R&D collaboration member firms and allow for a more flexible formation rule. Fourth, a model carefully analyzing the relative welfare effects of the crowding out effect and the increased incentive to collaborate due to government subsidies in the most general model is an important line of future research. Finally, empirical research is needed to identify the magnitude of the causal effect of federal grants to R&D collaborations.
References


6 Appendix

Lemma 1:

A firm’s profits are identically effected by a decrease in cost from any rival firm.

PROOF: Taking other firms’ costs parametrically, using the envelope theorem
finds that \( \frac{\partial \pi_i^*}{\partial c_j} = 2q_i^* \frac{1}{N+1} \). Similarly, \( \frac{\partial \pi_i^*}{\partial c_k} = 2q_i^* \frac{1}{N+1} \). As a result \( \frac{\partial \pi_i^*}{\partial c_j} = \frac{\partial \pi_i^*}{\partial c_k} \) for all \( j \) and \( k \). QED.

Lemma 2:

Regardless of idiosyncratic costs, firms already in a R&D collaboration will always agree on when to stop inviting additional firms into the collaboration.

PROOF: Consider a RJV of size \( n < N \) which is considering inviting another member to join. Noting that \( RJV_n \) denotes a collaboration of size \( n \), the sign of the change in profit to a firm \( i \) already in a RJV for allowing addition firm \( j \) to enter can be simplified to:

\[
\Pi_{n,i} = \pi_i(RJV_{n+1}) - \pi_i(RJV_n) = \beta \left( (q_i^*(RJV_{n+1}))^2 - (q_i^*(RJV_n))^2 \right)
\propto \frac{-\bar{c} + \bar{c}((n+1)\phi) + n\bar{c}_R(n\phi)\phi - N\bar{c}_R(n\phi)\phi}{(N+1)}
\]

Note that the firm specific cost component, \( \eta_i \), embedded in determining equilibrium quantity will cancel. The sign of equation [2] is determined by the numerator of the quotient. There are three effects on profits to permitting addition firm entry. First, the entering firm’s cost decrease \((-\bar{c} + \bar{c}((n+1)\phi))\). Second, the costs of competing firms already in the RJV decrease: \( n\bar{c}_R(n\phi)\phi \). Note that \( \bar{c}_R(n\phi) \) represents the derivative of costs for RJV member firms at a total level of R&D funding of \( n\phi \). We use \( \bar{c}_R(n\phi)\phi \) to approximate \( \bar{c}((n+1)\phi) - \bar{c}(n\phi) \). Third, the costs of the firm \( i \) decrease, increasing profit \((-N\bar{c}_R(n\phi)\phi > 0)\). No term in the numerator of the quotient is a function of firm \( i \). As a result, the sign of the change in profit from letting addition firms enter the RJV is wholly determined by the size of the entry fee and the function dictating returns from R&D spending. QED.

Lemma 3:

The marginal benefit of each additional firm entering the R&D collaboration to firms
already in the collaboration is monotonically decreasing.

PROOF: Note that

\[
\Pi_{n+1,i} - \Pi_{n,i} \propto \bar{c}(\phi(n+2)) - \bar{c}(\phi(n+1)) - (N - n)\phi(\bar{c}_R(\phi(n+1)) - \bar{c}_R(\phi n)) \tag{3}
\]

By convexity of \(\bar{c}(\cdot)\), \(\bar{c}(\phi(n+2)) - \bar{c}(\phi(n+1)) < 0\). Similarly, \(\bar{c}_R(\phi(n+1)) - \bar{c}_R(\phi n) > 0\) meaning that the entire last term is negative. Since \(n\) is arbitrary, iteration proves the result. QED.

**Proposition 1:**

R&D collaborations receiving exogenous block grant funding are smaller than those receiving funding in equilibrium.

PROOF: This proof proceeds by construction. Note that the change in profit to a firm already in a RJV of permitting further entry is proportional to the numerator in the quotient of (2). Compare the change in profit to a firm already in a RJV of permitting further entry with exogenous funding, \(\tilde{R} > 0\), to without:

\[
\Pi_{n,i}\big|\tilde{R} - \Pi_{n,i} \propto \frac{\bar{c}(n\phi + \tilde{R}) - \bar{c} - (N - n)\bar{c}_R(n\phi + \tilde{R})\phi}{(N + 1)\beta} - \frac{\bar{c}(n\phi) - \bar{c} - (N - n)\bar{c}_R(n\phi)\phi}{(N + 1)\beta} \tag{4}
\]

\[
\propto \left(\bar{c}(n\phi + \tilde{R}) - \bar{c}(n\phi)\right) - (N - n)\phi \left(\bar{c}_R(n\phi + \tilde{R}) - \bar{c}_R(n\phi)\right) \tag{5}
\]

As before, \(\bar{c}_R(X)\) is the marginal change in cost from additional R&D funding at a funding level of \(X\). By Lemma 1, invited firms can be randomly selected by firms already in the RJV. By strict concavity of the function \(\bar{c}(\cdot)\) and \(\tilde{R} > 0\), the first term in equation (5) is negative. The second term in the brackets, \(\bar{c}_R(n\phi + \tilde{R}) - \bar{c}_R(n\phi)\) is positive since \(\bar{c}_R(n\phi) < \bar{c}_R(n\phi + \tilde{R})\) by concavity of \(\bar{c}(\cdot)\). The negative sign in front of the second term implies the entire expression is negative. Therefore, for the RJV size \(n^*\) such that equation (2) is zero (e.g., no additional firms invited), \(\Pi_{n,i}|\tilde{R} < 0\). As a result, by Lemma 2, \(n^*\), cannot be equilibrium resource size when funded. By Lemma 3, the equilibrium RJV size when there is exogenous funding, \(n^*_R > n^*\). QED.

**Corollary 1:**

R&D collaborations receiving exogenous cost match funding are smaller than those
PROOF: The analog of equation (5) with cost matching is:

\[
\Pi_{n,i} |\tilde{R} - \Pi_{n,i} \propto (\tilde{c}(n\phi(1+s)) - \tilde{c}(n\phi)) - (N-n)\phi(1+s) (\tilde{c}_R(n\phi(1+s)) - \tilde{c}_R(n\phi)) \quad (6)
\]

where \( s > 0 \) is the percent of the cost match. All results from Proposition 1 immediately follow. QED.

Proposition 2:

For any market size \( N \) in which a private RJV is formed by at least two firms, there is number of member firms, \( n < N \), already in the RJV for which member firms will exclude any additional members.

Proof: Consider the change in profits to a firm \( i \) that is a founding member of a RJV caused by increasing the number of RJV members by one. Treating \( n \) as a continuous variable, the chain rule implies that

\[
\frac{\partial \pi^*_{RJV}}{\partial n} = 2q^*_{i,RJV} \left\{ \frac{\partial q^*_{i,RJV}}{\partial n} + \frac{\partial q^*_{i,RJV}}{\partial R} \phi \right\}
\]

since each additional firm enters by paying an entry fee \( \phi \). The entire expression’s sign is a function of bracketed portion which allows it to be worked on independently. Working with the derivative of quantity with respect to RJV size \( n \) directly, the derivative can be expressed as

\[
\frac{\partial q^*_{i,RJV}}{\partial n} + \frac{\partial q^*_{i,RJV}}{\partial R} \phi = \frac{\tilde{c}(n) - \tilde{c} - (N-n) \frac{\partial \tilde{c}(n)}{\partial R} \phi - (N-n) \frac{\partial \tilde{c}(n)}{\partial n}}{(N+1)\beta}.
\quad (7)
\]

Equation (7) is proportional to the net marginal benefit to firm \( i \) of a new entrant. It reflects the direct effects of the additional firm, the incoming firm’s entry fee, \( \phi \), and the interaction of the two. When (7) is greater than zero, firm \( i \) will prefer an additional firm to enter. The number of firms in a rationalizable collaboration is determined when equation (7) equals zero.

Equation (7) always is signed by

\[
\text{sign} \left[ \frac{\partial q^*_{i,RJV}}{\partial n} (n) + \frac{\partial q^*_{i,RJV}}{\partial R} (N) \phi \right] = \text{sign} \left[ \tilde{c}(n) - \tilde{c} - (N-n) \frac{\partial \tilde{c}(n)}{\partial R} \phi - (N-n) \frac{\partial \tilde{c}(n)}{\partial n} \right]
\quad (8)
\]

Choose \( n = N \) such that the R&D collaboration would have all firms. Equation

not receiving funding.
evaluated at $n = N$ is negative by inspection. Evaluating at $n = N - 1$ is also negative by convexity of the R&D returns function $\bar{c}(n, R)$ in its arguments and by earlier assumption $|\frac{\partial (n, R)}{\partial n}| + |\frac{\partial (n, R)}{\partial R}| > |\frac{\partial^2 (n, R)}{\partial R \partial n}|$.

Checking second order conditions, the second derivative of profits with respect to the number of RJV member firms:

$$\frac{\partial^2 \pi^*, RJV}{\partial n^2} = 2 \left( \frac{\partial^2 q_i, RJV}{\partial n^2} + \left( \frac{\partial q_i, RJV}{\partial n} \right)^2 \right) \propto \frac{2 \frac{\partial \epsilon(n)}{\partial n} + \frac{\partial \epsilon(n)}{\partial R} \phi - (N - n) \left( \frac{\partial^2 \epsilon(n)}{\partial n^2} + \frac{\partial^2 \epsilon(n)}{\partial n \partial R} \phi \right)}{(N + 1) \beta} \tag{10}$$

By inspection, equation $\tag{10}$ is negative $\forall n$ if firms and R&D dollars are substitutes ($\epsilon_{n, R} > 0$). As a result, in the case of substitutes, a RJV will never contain all members, nor all members but one and there is some critical level firm $n^* < N$ for which further entry is not offered.

In the case of complements equation $\tag{10}$ is negative for $n = N$ and $n = N - 1$ by convexity of the R&D returns function $\bar{c}(n, R)$. Thus, if a research collaboration is rationalizable, there is number of member firms, $n < N$, already in the RJV for which member firms will exclude any additional members regardless if firms and R&D dollars are complements or substitutes. QED.

Proposition 3:
If innovative capital and R&D funding are substitutes, a RJV with exogenous funding will always have

Proof: The equilibrium condition for exclusive RJVs is when the numerator of the right hand side of equation $\tag{7}$ equals zero. If research partners and R&D funding are substitutes then $\frac{\partial^2 \epsilon}{\partial n \partial R} > 0$. Use the implicit function theorem to find $\frac{\partial n^*}{\partial R}$:

$$\frac{\partial n^*}{\partial R} = -\frac{\frac{\partial F}{\partial R}}{\frac{\partial F}{\partial n} = -\frac{- (N - n) \left( \frac{\partial^2 \epsilon(n)}{\partial n^2} + \frac{\partial^2 \epsilon(n)}{\partial R \partial n} \phi \right) + \frac{\partial \epsilon}{\partial R}}{2 \frac{\partial \epsilon(n)}{\partial n} + \frac{\partial \epsilon(n)}{\partial R} \phi - (N - n) \left( \frac{\partial^2 \epsilon(n)}{\partial n^2} + \frac{\partial^2 \epsilon(n)}{\partial n \partial R} \phi \right)}. \tag{11}$$

Both the numerator and denominator of equation $\tag{11}$, are signed negative given that $\frac{\partial^2 \epsilon}{\partial n \partial R} > 0$ meaning the entire quotient is positive. The negative sign in front means that equation $\tag{11}$ will always be negative in equilibrium, giving the desired result. QED.
Proposition 4:
If innovative capital and R&D funding are complements, a RJV with exogenous funding can have fewer members than one that doesn’t.

Proof: Examine equation (11) under the assumption that $\frac{\partial^2 \bar{c}}{\partial n \partial R} < 0$. Conditions must be derived under which the portions of the numerator and denominator contained the cross partial maintain their sign. Specifically, it must be the case that

$$
\begin{pmatrix}
\bar{c}_{n,n} & \bar{c}_{n,R} \\
\bar{c}_{n,R} & \bar{c}_{R,R}
\end{pmatrix}
\begin{pmatrix}
1 \\
\phi
\end{pmatrix}
> 
\begin{pmatrix}
0 \\
0
\end{pmatrix}
$$

(12)

The conditions in (12) can be arranged in terms of elasticities: $|\epsilon_{\bar{c},n,R}| < |\epsilon_{\bar{c},n,n}|$ and $|\epsilon_{\bar{c},R,n}| < |\epsilon_{\bar{c},R,R}|$, which gives the desired result. QED.

Lemma 4:
For any distribution of firms indexed by heterogeneous costs with continuous support, the equilibrium invitation strategy is to sequentially invite the lowest cost firms.

Proof: Assume not. Then for a set of $n$ firms called $\hat{n}$ with total R&D equals $\hat{R} = \delta \Sigma_{\{\hat{n}\}} \Phi_{\hat{n}}$ there exists an alternative set of $n$ firms called $\hat{n}^*$ with total R&D equals $\hat{R}^* = \delta \Sigma_{\{\hat{n}^*\}} \Phi_{\hat{n}}^*$ where $\hat{R}^* > \hat{R}$. Given that the derivative of equilibrium profits with respect to R&D dollars, $R$, is positive ($\frac{\partial \pi^{*,RJV}}{\partial R} > 0$), the strategy of inviting any firms but those with the highest entry fees, $\delta \Phi$, not optimal for any $n$. Therefore it is always optimal to invite the firms with the highest $\Phi$s, or lowest costs, sequentially giving the desired result. QED.

Corollary 2:
In the asymmetric case, for any market size $N$, there is number of member firms, $n < N$, already in the RJV for which member firms will exclude any additional members.

Proof: Follows directly from applying Lemma 2 to Proposition 2.

Proposition 5:
With asymmetric cost sharing, if innovative capital and R&D funding are substitutes, a RJV with exogenous funding will always have fewer members than a RJV which doesn’t have exogenous funding.

Proof: Note that $\hat{R}^* = \delta \Sigma_{\{\hat{n}^*\}} \Phi_{\hat{n}}^*$ by Lemma 2. If research partners and R&D funding are substitutes then $\frac{\partial^2 \bar{c}}{\partial n \partial R} > 0$. If total R&D dollars increases exogenously
it increases any firm $i$’s willingness to pay to enter the RJV and their subsequent entry fee $\delta \Phi_i$. As a result, entry fees can be defined as functions of levels of exogenous funding $\Phi_i(\tilde{R})$. The ordering of all $\Phi_i$ is preserved, however. Using the implicit function theorem to find $\frac{\partial n^*}{\partial \tilde{R}}$ we have:

$$\frac{\partial n^*}{\partial \tilde{R}} = -\frac{\frac{\partial F}{\partial \tilde{R}}}{\frac{\partial F}{\partial n} + 2\frac{\partial \tilde{c}(n)}{\partial n} \phi - (N - n) \left( \frac{\partial^2 \tilde{c}(n)}{\partial n^2} \phi + \frac{\partial^2 \tilde{c}(n)}{\partial n \partial R} \right)}.$$

(13)

Bringing the negative sign into the numerator of equation (13), the numerator is signed positive given that $\frac{\partial^2 \tilde{c}}{\partial n \partial \tilde{R}} > 0$. Thus, the sign of equation (13) is determined by the sign of the denominator. Simplifying notation, the denominator will always be negative by convexity of $\tilde{c}(n, \tilde{R})$ in $n$. Note that the increase in $\tilde{R}$ increases $\Phi(\tilde{R})$ as well. Since $R = \tilde{R} + \Sigma \Phi(\tilde{R})$ this further exacerbates the effect, giving the desired result. QED.

**Corollary 3:**

With asymmetric cost sharing, if innovative capital and R&D funding are complements, a RJV with exogenous funding can have fewer members than one that doesn’t.

Proof: Follows directly from applying Lemma 2 to Proposition 4.

**Corollary 4:**

With asymmetric cost sharing, exogenous funding increases the willingness to pay of firms to collaborate.

Proof: Follows directly from Lemma 4.
7 Appendix B: Detailed Empirical Analysis

To further examine the composition of R&D collaborations consider the following econometric specification of controlled difference in means. Causal inference is not identified in this specification and the goal is to establish further evidence of the form of the correlation between RJV size and the presence of government funding. The estimation is a difference of means with controls:

\[
size_{ijt} = \alpha + \sum_{j=1}^{J} industry_j \delta_j + NIST_i \beta_2 + x'_it \psi + \epsilon_{it}. \quad (14)
\]

Equation (14) is a cross-sectional econometric specification to further investigate the form of RJVs where \( i \) indexes a federally registered RJV. In equation (14) the \( industry \) variable is a binary variable indicating the two-digit sic code of the RJV. The variable \( NIST_i \) is a binary variables which equal one if RJV \( i \) was funded by a NIST program. A vector \( x_i \) controls for various traits of the RJV such as process and/or product based research in addition to what type of R&D projects the RJV undertakes (e.g., energy, government mandates, etc...). The coefficients of interest are the intercept term and the coefficient on \( NIST_i \). Both OLS and tobit models were performed in the presence and absence of year fixed effects were executed.

The dependent variable in all specifications (1)-(4) is the number of firms registered in the RJV.\(^{22}\) Regressions (1) and (2) are specifications showing estimates of standard OLS with robust errors without and with year fixed effects respectively. Specifications (3) and (4) are specifications showing estimates of a tobit model with robust errors without and with year fixed effects respectively. In all specifications, the coefficient on NIST is significant and negative. The preferred specification is (2) which is OLS with year fixed effects. Controlling for industry, year and other RJV characteristics, the CORE database shows that R&D collaborations that received government funding where roughly half the size of those that did not and that the difference was highly significant.

\(^{22}\)When RJVs of size larger than 100 are excluded, all coefficients of interest maintain their significance and sign. The same is true in general when the LHS variable is the ln of RJV size.
Table 2: Regression Results.

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Data from CORE database, 1985-2006.

*** = significant at 1%, ** = sig at 5%, * = sig at 10%.
LHS variable is the number of firms in the R&D collaboration.
Robust errors in parentheses. Specification 1 and 2 are OLS.
Specifications 3 and 4 are Tobit models controlled for LHS censoring of the number of firms in the RJV.