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Abstract: This study examines how behavior in inter-group contests is altered when players have incomplete information on their opponent. We model a Tullock contest where there are two possible types of groups that are heterogeneous in the incentives they face, and players only know the probability their opponent is a particular group type. Relative to a contest with complete information, we find theoretically that incomplete information lowers contest-level effort in (even) contests between groups of the same type, whereas it increases effort in uneven contests. Through an experiment, we compare three sources of heterogeneity – differences in cost-of-effort, prize value, and group size. For the cost and value treatments, we find that incomplete information increases effort in uneven contests but has no effect in even contests. For the group size treatments, incomplete information has no effect. A theory that assumes players are altruistic towards group members, rather than purely self-interested, is much better at predicting outcomes.

JEL classifications : C72; C92; D74; D82; D91; H41

Keywords: inter-group competition; heterogeneous contests; Tullock contests; incomplete information; public goods; group size paradox; experiments; altruism

1. Introduction

In many settings, entities are engaged in inter-group competitions where opponents may be at an advantage or disadvantage. Advantages in these "uneven" contests may arise from various sources such as talent differentials across groups that can be thought of as decreasing the relative (effort) cost associated with attaining a level of output. R&D firms or legal teams may face opponents that have fewer researchers or employees, thus leading to a potential advantage due to the increased opportunity to put forth productive effort. Some public or private organizations have a larger resource base or otherwise can better incentivize desirable actions from their members through bonuses, thus increasing the marginal returns from effort. While competing teams are rarely on equal footing in all important dimensions, also endemic to group contests is that the agents in one group have uncertainty over the extent of their disadvantage or advantage over the competing team. An academic research lab applying for a grant may know who their main competition is, but is unlikely to precisely know another lab's talent allocation, number of researchers (which can decrease the time to complete the project), or the motivation (such as from salary raises tied to external funding) another lab has to obtain the grant. This study uses theory and an experiment to examine how behavior in potentially uneven group contests is altered when players have incomplete information on the incentives facing their opponent. We further compare three different sources of potential advantage.

Our theoretical framework builds on Tullock's (1980) canonical model of a rent-seeking contest, Katz, Nitzan, and Rosenberg (1990) who model a group Tullock contest setting with inter-group heterogeneity, and Malueg and Yates (2004) who study individual contests with incomplete information over the prize value. In particular, we model a Tullock contest between two groups that are potentially heterogeneous in the terms of the incentives they face – teams may differ in their cost of effort, the value of the prize for winning team members, or group size –, and while players know the type of their own group, they only know the probability that the opponent is of a particular group type. Following common assumptions in the literature, players within a group are identical, group-level effort is an additive function of individual efforts (perfect substitutes), individual efforts are not directly observable to teammates, and the winning team receives a prize that is evenly split. To our knowledge, the only prior theoretical treatment of group contests with incomplete information is Eliaz and Wu (2018), who study uneven all-pay contests where two teams may differ in size and have incomplete information on the value of the other team's prize.

Motivated by prior experimental results, we consider a model wherein players are altruistic towards other group members. A stylized fact in the group contest literature is that effort far exceeds what is predicted from the self-interest model (e.g., Cason, Sheremeta and Zhang 2012; Leibbrandt and Sääksvuori 2012). Altruism is one leading explanation for this over-expenditure (Sheremeta 2018), and our model confirms that altruism can significantly increase effort. Further, the within-group altruism model predicts that group-level effort increases with group size, which is consistent with prior evidence, in particular Abbink et al. (2010), Ahn et al. (2011), and Ke (2013). The standard model, in contrast, predicts that group size has no effect on group-level effort.

Our experiment varies whether teams have complete or incomplete information on their opponent's type, and the potential source of advantage (cost, value, or group size). In doing so, we make three contributions to the literature. This is the first experiment to study the effect of incomplete information in an inter-group contest. In addition, we test whether the source of the potential advantage matters (cost-of-effort, prize value or group size). While these sources of advantage have been studied in group contest settings to a limited degree, prior work has examined them in isolation. As the self-interest model predicts group-level effort is invariant to group size, but that effort is a function of cost-of-effort and prize value, this provides an additional channel from which to gauge the predicative validity of the standard and within-group altruism theories.

While experimentally examining incomplete information in a group contest setting is novel, we note that prior studies have used experiments to examine the effects of uncertainty in lottery contests and all-pay auctions involving competition between individuals (see Dechenaux, Kovenock, and Sheremeta 2015). In many of these studies, players are *ex ante* symmetric, but a parameter value (e.g., corresponding to per-unit effort cost) for an individual is determined by taking an independent draw from a common uniform distribution. For example, Brookins and Ryvkin (2014), for a contest among four players, find theoretically and experimentally that incomplete information increases effort among advantaged players, and decreases effort among disadvantaged players. Also relevant is Boosey, Brookins and Ryvkin (2017), who introduce uncertainty in the number of players in a contest by varying the (independent) probability a player enters the contest, along with the maximum number of possible participants. They find that when the participation probability is low, an increase in maximum possible number of competitors increases effort while the opposite is true when the participation probability is high.

In his survey of the group contest literature, Sheremeta (2018) notes that few studies incorporate heterogeneity between groups. As exceptions, Heap et al. (2015) test the effects of providing teams with unequal endowments, which allows the advantaged team to contribute more towards winning the competition, and Bhattacharya (2016) examines contests between groups that differ in either the probability of winning (when groups expend equal effort) or their effort cost. These investigations, along with related ones involving symmetric groups with intra-group heterogeneity, generally find that advantaged players contribute relatively more effort.[1]

[1] The same result is evident from experiments involving heterogeneous contests between individuals (see Dechenaux, Kovenock, and Sheremeta 2015).

A few studies examine heterogeneity in group contests in the form of different group sizes. Rapoport and Borenstein (1989) and Kugler, Rapoport and Pazy (2010) examine contests between three- and five-player groups and find that even in cases where theory predicts the smaller team should expend more collective effort, larger groups are instead more likely to win. This theoretical result that, under some circumstances, a smaller group is more likely to win is often referred to as the "group size paradox". As highlighted by Sheremeta (2018), while comparative statics predictions of self-interest theory generally comport with experimental findings, the failure of experiments to support the group size paradox prediction is an important exception.

One distinction in our design is that, similar to Abbink et al. (2010) and Ahn, Isaac and Salmon (2011), who study contests between an individual and a group of four, we fix the value of the winning prize for an individual. In doing so, the difference in group size is the only potential source of advantage across competing groups. And, theoretically, if players are only motivated by self-interest, being a member of a larger team in this setting is not an "advantage" in the sense that an advantaged team is no more likely to win.

For each of the sources of advantage we study, the within-group altruism model predicts that incomplete information lowers group-level (and contest-level) effort in even contests, but increases contest-level effort in uneven contests. These results hold if players are purely selfish, but only in the cost-of-effort or prize value heterogeneity cases. The self-interest model predicts that a larger group holds no advantage and, intuitively, whether the size of the opposing group is known is irrelevant. Turning to experimental results, for the cost and value treatments, we find that incomplete information increases effort in uneven contests, but has no effect (or possibly a positive effect) in even contests. For the group size treatments, incomplete information has no effect on average, which is predicted only by the standard self-interest model. Nevertheless, the group-level efforts for all treatments are more closely aligned with the point predictions from the within-group altruism model, conditional on players having a reasonably high level of altruism. Moreover, as unilaterally predicted by the within-group altruism model, when groups of different size compete, effort is significantly higher for the larger group. Comparisons across contests that differ in the form of heterogeneity reveal that effort is similar across the cost-of-effort and prize value treatments, which is consistent with theory. Contest-level efforts are significantly higher for group size treatments. The study also finds risk-averse individuals, players in uneven contests, and persons in possibly heterogenous group-size contests are more likely to free-ride.

2. Theoretical Framework

Our theoretical framework builds on Tullock's (1980) canonical model of a rent-seeking contest, Katz, Nitzan, and Rosenberg (1990) who model a group Tullock contest setting with inter-group heterogeneity, and Malueg and Yates (2004) who study individual contests with incomplete information over the prize value. We solve for the symmetric equilibrium of the heterogenous group Tullock contest under complete and incomplete information. Moreover, for reasons given above we consider the case where players are altruistic towards the members of their own group. The standard model where players are only motivated by self-interest arises as a special case.

Consider a contest between two groups. Group g consists of N_g risk-neutral players, and groups compete to win a prize. Regardless of their individual actions, the value of the prize to each player on the winning team is v_g, and there is no prize for the losing team. This may, for instance, characterize a setting where the group prize is a (local) public good that is non-excludable and non-rival in consumption. All players simultaneously and independently expend effort x_{ig} at a (constant) per-unit cost of c_g. Player efforts within a group are perfect-substitutes, such that group-level effort, X_g, is simply the sum of player efforts; i.e., $X_g = \sum_{i=1}^{N_g} x_{ig}$. The probability of winning, p_g, depends on the relative effort of the competing teams. In particular, we use the contest success function (CSF) of Tullock (1980) for the standard lottery case:

[1] $$p_g = \frac{X_g}{X_g + X_{-g}}.$$

When both teams expend the same collective effort, each has a win probability of ½. Otherwise, with this CSF, the team that exerts more effort has a higher probability of winning.

Throughout the analysis we assume all players within a group are identical with respect to cost and value parameters, but there may be heterogeneity in either the cost, value, or group size across competing groups. We limit the analysis here to settings where there is at most one source of heterogeneity .[2] We consider two information conditions. In the complete information condition, each player has perfect knowledge of the incentives (i.e., parameters) facing their team as well as their opponent. In the incomplete information condition, players do not know with certainty one of the parameters that the opposing team faces.

A. Complete information

When all players know the parameters (cost, value, and group size) characterizing their own group as well as their opponent, the expected payoff of player i in group g is:

[2] $$\pi_{ig} = p_g v_g - c_g x_{ig} = \frac{X_g}{X_g + X_{-g}} v_g - c_g x_{ig}.$$

An altruistic, risk-neutral individual is postulated to have increasing utility in the gains of others:

[3] $$U_{ig} = \pi_{ig} + \alpha \sum_{j \neq i} \pi_{jg} = \left[1 + \alpha(N_g - 1)\right] \frac{X_g}{X_g + X_{-g}} v_g - c_g x_{ig} - \alpha \sum_{j \neq i} c_g x_{jg},$$

[2] The theory logistically extends to settings where a team has an advantage in multiple dimensions. Ambiguity in the various comparisons of course arises if a group holds an advantage in one dimension, but a disadvantage in another.

where $\alpha \geq 0$ denotes the weight placed on the payoffs of others. In the special case of $\alpha = 0$ this results in a model based purely on self-interest. Maximizing [3] with respect to player i's effort yields the first-order condition:

$$[4] \qquad \left[1 + \alpha(N_g - 1)\right] \frac{X_{-g}}{(X_g + X_{-g})^2} v_g - c_g = 0.$$

The maximization problem for a representative player from team $-g$ is of course symmetric, with the first-order condition:

$$[5] \qquad \left[1 + \alpha(N_{-g} - 1)\right] \frac{X_g}{(X_{-g} + X_g)^2} v_{-g} - c_{-g} = 0.$$

The first-order conditions include only group-level efforts, and so the theory is silent about individual effort. Assuming an interior solution, the Nash equilibrium is attained by solving [4] and [5] simultaneously for X_g and X_{-g}. The equilibrium is:

$$[6] \qquad X_g^* = \frac{c_{-g} v_g^2 v_{-g} \left[1+\alpha(N_g-1)\right]^2 \left[1+\alpha(N_{-g}-1)\right]}{\left(c_g v_{-g}\left[1+\alpha(N_{-g}-1)\right] + c_{-g} v_g \left[1+\alpha(N_g-1)\right]\right)^2}$$

and

$$X_{-g}^* = \frac{c_g v_{-g}^2 v_g \left[1+\alpha(N_{-g}-1)\right]^2 \left[1+\alpha(N_g-1)\right]}{\left(c_{-g} v_g\left[1+\alpha(N_g-1)\right] + c_g v_{-g} \left[1+\alpha(N_{-g}-1)\right]\right)^2}.$$

In terms of group-level efforts, this equilibrium is unique. In terms of individual effort, there is one symmetric equilibrium and multiple asymmetric equilibria in which the sum of the individual efforts equals the group-level equilibrium (Baik 1993). This is the result of assuming constant marginal effort costs and the same prize value to each team member.

In the special case of pure self-interest, which we will refer to as the "self-interest model" the equilibrium effort for group g is simply:

$$[7] \qquad X_g^* = \frac{c_{-g} v_g^2 v_{-g}}{\left(c_g v_{-g} + c_{-g} v_g\right)^2},$$

and group-level effort is not a function of group size. When players are altruistic, however, group-level effort is strictly higher, and is further increasing in the size of the player's group.

Using [6] we can obtain predictions for homogenous and heterogenous contests. To facilitate this, let $g \in (A, D)$, where A denotes that a group is of the type "advantaged" and D denotes the group type "disadvantaged". These labels are meant to identify group type and to be clear, when a group is "advantaged" (or "disadvantaged") this does not necessarily mean that this group has an advantage (or disadvantage) relative to their opponent. Contests where both groups are of the same type – either both advantaged or both disadvantaged – are referred to as "even" contests; otherwise, contests between types are "uneven". In contests with potential cost heterogeneity, the advantaged group is one where the per-unit cost of effort is lower, such that $c_A < c_D$. In a similar vein, advantaged and disadvantaged groups in the context of value and group size heterogeneity are defined by $v_A > v_D$ and $N_A > N_D$, respectively.

Equilibria for complete information contests are provided in Table 1. When both teams have a cost or value advantage, group effort is strictly higher when compared to an even contest between two disadvantaged teams. Moreover, in an uneven contest, the advantaged team exerts relatively more effort than their opponent. Interestingly, the effort of a disadvantaged team is lower when in an uneven contest relative to an even contest. This can be labelled a discouragement effect as the fact that they are playing against an advantaged opponent lowers the chance they win, which disincentivizes effort. In the special case where players are only self-interested, group size differences do not matter; i.e., $X_A^* = X_D^*$ for both even and uneven contests.

B. Incomplete information

In the incomplete information setting, players do not know with certainty the parameters that the opposing team faces. Instead, they know the opponent will either be of the advantaged or disadvantaged type with some probability. Let $0 < r < 1$ denote the probability that any contest opponent is advantaged, which is common knowledge. A player on team g then forms an expectation over profits based on the probability r the opponent is advantaged and the probability $1 - r$ that the opponent is disadvantaged. The maximization problem is:

$$[8] \qquad \max_{x_{ig}} U_{ig} = \left(r(p_g|A) + (1-r)(p_g|D)\right) v_g\left[1 + \alpha(N_g - 1)\right] - c_g x_{ig} - \alpha \textstyle\sum_{j \neq i} c_g x_{jg}$$

$$= \left(r \frac{X_g}{X_g + X_A} + (1-r) \frac{X_g}{X_g + X_D}\right) v_g\left[1 + \alpha(N_g - 1)\right] - c_g x_{ig} - \alpha \textstyle\sum_{j \neq i} c_g x_{jg}$$

The associated first-order condition is:

$$[9] \qquad \left(r \frac{X_A}{(X_g + X_A)^2} + (1-r) \frac{X_D}{(X_g + X_D)^2}\right) v_g[1 + \alpha(N_g - 1)] - c_g = 0.$$

This gives rise to two equations based on whether $g = A$ or $g = D$. This in turn leads to a system of two equations and two unknowns. We provide the closed-form solution for $0 < r < 1$ in the appendix, which is intricate as the problem can only be reduced to a cubic equation. For $r = \frac{1}{2}$, which coincides with the experiment, the algebra is simpler. In this case, and focusing on the case of cost heterogeneity, the (pure strategy) symmetric Bayesian-Nash equilibrium is[3]

$$[10] \qquad X_A^{**} = \frac{[1+\alpha(N-1)]v}{c_A} \left(\frac{4\frac{c_A}{c_D} + \left(1 + \frac{c_A}{c_D}\right)^2}{8\left(1 + \frac{c_A}{c_D}\right)^2} \right) \text{ and } X_D^{**} = \frac{[1+\alpha(N-1)]v}{c_D} \left(\frac{4\frac{c_A}{c_D} + \left(1 + \frac{c_A}{c_D}\right)^2}{8\left(1 + \frac{c_A}{c_D}\right)^2} \right).$$

Table 2 presents the equilibria for the incomplete information condition for $r = \frac{1}{2}$, and again we focus on cases where there is inter-group heterogeneity across types with respect to effort cost, prize value, or group size. Given the expectation about opponent's strategies, individuals in

[3] When $\alpha = 0$, the equilibrium does not depend on the number of players, and the group contest equilibria are identical to those based instead on a contest among individuals. It follows that similar results can be found in Malueg and Yates (2004) and Fey (2008) for the $r = \frac{1}{2}$ case.

either an advantaged or disadvantaged group have unique effort levels that do not depend on the type of group they are actually competing against, which is of course unknown. For the special case of $r = \frac{1}{2}$, effort in the incomplete information setting is the average effort (for the same group type) across even and uneven contests with complete information.

As in the complete information case, for both cost and value heterogeneity, the advantaged team is predicted to exert more (collective) effort relative to the disadvantaged team which, in turn, increases their probability of winning an uneven contest. Further, altruism continues to increase efforts for both advantaged and disadvantaged groups. With group size heterogeneity, when players are altruistic, group-level effort is strictly increasing in the size of either group and the larger group is more likely to win. However, when players are purely self-interested, equilibrium effort is $\frac{v}{4c}$ regardless of group size or r.

Turning to the effects of information, there are interesting results, which we summarize through three propositions. In the appendix, we provide proofs for the $r = \frac{1}{2}$ case, and further use numerical calculations to illustrate that the propositions hold more generally.

Proposition 1: When players are altruistic towards members of their own team, for group contests with cost, prize value, or group size heterogeneity, incomplete information *increases* contest-level effort in *uneven* contests. This information effect also holds in the cost or prize value heterogeneity cases when players are purely self-interested.

Proposition 2: When players are altruistic towards members of their own team, for group contests with cost, prize value, or group size heterogeneity, incomplete information *decreases* contest-level effort in *even* contests. This information effect also holds in the cost or prize value heterogeneity cases when players are purely self-interested.

Proposition 3: When players are altruistic towards members of their own team, for group contests with cost, prize value, or group size heterogeneity, as compared to the benchmark complete information condition: (i) expected contest-level effort is *higher* with incomplete information for $r < \frac{1}{2}$; (ii) expected contest-level effort is *equal* under both information conditions for $r = \frac{1}{2}$; and (iii) expected contest-level effort is *lower* with incomplete information for $r > \frac{1}{2}$. These results also hold in the cost or prize value heterogeneity cases when players are purely self-interested.

For uneven contests, an advantaged team increases effort under incomplete information. This is because the team does not know for sure that the other team is disadvantaged and increases effort accordingly. The effect of incomplete information is increasing in r and the extent of the advantage. For the special case of $r = \frac{1}{2}$, the effort from a disadvantaged team also increases with incomplete information. However, in general, the effect is ambiguous and depends upon the extent of the advantage together with the probability that the other team is advantaged. When the advantage is relatively small, then the discouragement effect discussed previously is also small. Then only for very high r does incomplete information motivate lower effort. As the size of the advantage increases, however, the range of probabilities for which incomplete information discourages effort increases. Overall, the effect of incomplete information on the advantaged group unambiguously dominates its effect on the disadvantaged group. As a result, contest-level effort (i.e., the sum of efforts across the disadvantaged and advantaged team) is strictly higher under incomplete information as compared to complete information.

For even contests, that incomplete information decreases group-level effort is intuitive. Note first that both teams exert equal effort in this case, and effort is higher when competing teams are advantaged rather than disadvantaged. With incomplete information, a team does not know the opposing team's type. An advantaged team will only suspect they are playing another advantaged team with some probability less than 1, and as a result will be incentivized to put forth less effort relative to the case where the opponent is for sure advantaged. A disadvantaged team will suspect their opponent may be advantaged, and this also lowers effort relative to the case where they know for sure the opponent is disadvantaged. This is due to the discouragement effect.

When considering contest-level effort, unconditional on contest type, the differential effects of incomplete information across uneven and even contests of course will counteract. When the probability a team is advantaged is exactly 50%, there is no difference in expected effort between contests with complete and incomplete information. As the (negative) effect of incomplete effort in even contests between two disadvantaged teams is relatively small, for $r < 1/2$ it is the case that expected effort is higher with incomplete information. This is because for $r < 1/2$ the positive effect in uneven contests dominates the negative effect in even (disadvantaged) contests. The opposite is true when conditions make it more probable that the contest is between two advantaged teams; i.e., when $r > 1/2$. Although the effects on expected effort (unconditional on contest type) are in general ambiguous, differences are relatively small.

The exception to these results occurs for contests with group size heterogeneity, but only when one assumes that individuals are purely self-interested. In this case, when competing under complete information, advantaged and disadvantaged groups put forth equal effort. As a result, whether a team has incomplete information on their opponent is irrelevant.

3. Experimental Design

In an experiment session, participants are randomly placed into groups, and are then paired with a competing group. Players are randomly re-matched into groups prior to each of 20 independent decision rounds. The total number of rounds is not disclosed to minimize end-of-game effects. In a round, the task of each player is to decide how many points to contribute to a "group project". Contributing points ("effort" in the theory) comes at a constant per-unit cost, and participants can select any integer amount between 0 and 50 points (inclusive). To avoid negative earnings, in each round a participant receives a "fixed income" sufficient to cover any effort costs. After all choices are made, the points contributed are added up for both groups, and the probability a group wins is given by equation [1]. Each member of the winning group receives a prize, the value of which is the same for all, regardless of how many points they contributed.

We employ a 2 x 3 between-subjects experimental design that varies the information condition (complete or incomplete information) and the potential source of heterogeneity (cost, value, or group size). Experiment parameters are summarized in Table 3. As in the theory, we characterize a group as either advantaged or disadvantaged. Regardless of treatment, a disadvantaged group has three players (i.e., $N_D = 3$), and players therein can contribute at a cost of 1 lab dollar per point $(c_D = 1)$ in attempt to win a prize that yields a payoff of 50 lab dollars per player ($v_D = 50$). To construct advantaged groups, we varied the relevant parameter by a factor of three. For a group with a cost advantage, effort cost is 1/3 per point; i.e., $c_A = 1/3$. For a group with a prize value advantage, $v_A = 150$, and a group with a size advantage includes nine members ($N_A = 9$).

Players engage in a mix of even and uneven contests. In each round, a team has a 50% chance of being assigned the parameters for an advantaged team, and determinations are made independently for each team. For example, in a cost treatment, each group has a 50% chance of facing a 1 lab dollar effort cost and a 50% chance of a 1/3 lab dollar cost. Overall, this means that there is a 25% chance that both teams are disadvantaged, a 25% chance that both are advantaged, and a 50% chance of an uneven contest between advantaged and disadvantaged teams.[4]

In the complete information condition, players have full knowledge of the incentives (cost, value, and group size) facing members of their team as well as the competing team. In the incomplete information condition, players have full information on the parameters facing their own team only. They do know that the other team has a 50% chance (i.e., $r = 0.5$) of being advantaged, and that the status of each group is determined independently. In other words, when a team is advantaged, this provides no additional information on the state of the competing group. Consistent with the theory, under the incomplete information condition players are uncertain about a single parameter facing their opponent but know the values for the other two parameters.

A. Theoretical predictions and testable hypotheses

Tables 4 and 5 present point predictions for the complete and incomplete information conditions, respectively. Separate predictions are given for the self-interest model ($\alpha = 0$), and the within-group altruism model for the case of $\alpha = 1$. Given the three-fold difference in heterogeneous parameters (e.g., $c_A/c_D = 3$), when engaged in an uneven contest theory predicts an advantaged team exerts three times more effort than a disadvantaged team when teams differ in terms of cost-of-effort or prize value. Moreover, for any (common) value of α, effort will be the

[4] The group size treatment presents a logistical challenge. To overcome this, we included 18 participants in a session, which allowed for them to play in a mix of even and uneven contests. The experiment software is programmed in such a way that gives rise to a participant playing in an uneven contest 50% of the time, an even small group contest 25% of the time, and an even large group contest 25% of the time, as stated in the experiment instructions. Important for the incomplete information treatment, a participant's knowledge of the size of her group provides no information on the size of the competing group.

same across these treatments. When $\alpha = 1$, predictions for all three sources-of-advantage are identical for both information conditions and all contest types. Relative to other sources-of-advantage, contest-level effort in uneven contests and even contests between advantaged teams will be higher in group size treatments when $\alpha > 1$, and lower when $\alpha < 1$.

The experimental design lends itself to testing group contest theories in several ways. The main hypotheses to be tested based on group-level effort are summarized below, and are deliberately written as "null" hypotheses:

Hypothesis 1. In an uneven contest, incomplete information has no impact on group effort.

Hypothesis 2. In an even contest, incomplete information has no impact on group effort.

Hypothesis 3. Unconditional on contest type, incomplete information has no effect on effort.

Hypothesis 4. Group effort does not vary according to the source of the advantage.

Hypothesis 5. Group effort is equal for advantaged and disadvantaged groups.

The first three hypotheses parallel the three theory propositions, and tests of the first four hypotheses are unique to this study. For any level of altruism (including pure self-interest), theory predicts for the cost or value treatments that incomplete information increases effort in uneven contests, decreases effort in even contests, and with $r = 0.5$ has no effect on expected effort unconditional on contest type. While the first result only holds more generally at the contest-level, based on the chosen experiment parameters this is nevertheless predicted to hold at the group-level. Effort is expected to be the same across the value and cost treatments and be higher for advantaged groups. Parallel predictions as they related to all hypotheses except for Hypothesis 4 arise for the group size treatments, but only when players are altruistic. Hypothesis 4 is only expected to hold if $\alpha = 1$. Otherwise, the self-interest model predicts that a large group holds no advantage over a small group. Moreover, in this case, incomplete information has no effect, and disadvantaged and advantaged groups exert equal effort. While Hypothesis 5 has been previously tested under conditions of complete information, our experimental design provides an additional test under incomplete information as well as across multiple sources of advantage.

B. Pilot Experiment and Power Analysis

To help inform the experimental design, a pilot experiment was conducted using the cost treatment with incomplete information. Participants were drawn from the same population and experimental procedures closely followed the final protocols described later.[5] Based on the estimated group-level variances from the pilot (within and across periods), and an assumed $\alpha = 1$ motivated by prior research, we settled on a plan to run three sessions of 18 participants for each of the cost and value treatments, and four sessions of 18 participants each for the two group size treatments.[6] The additional group size sessions help adjust for the fact that fewer group-level observations are generated from these treatments.

Based on the econometric methods employed and the planned sample sizes, power calculations suggest that we can detect a minimum treatment effect size of 9.4 units of effort based on 80% power and a 5% significance level (two-sided test) when testing Hypothesis 1 for the group size treatments, and an effect size of 8.4 when testing the same hypothesis based on either the cost or value treatments. For tests of Hypothesis 2, these figures are 8.9 and 8.4, respectively. Tests of Hypothesis 3 are powered to detect somewhat smaller differences, given that data from all even and uneven contests are pooled (7.7 for group size treatments, 7.0 for value and cost treatments).

[5] For this reason, we include data from the pilot in the analysis.
[6] In the Abbink et al. (2010) experiment, the four-person teams in no-punishment treatments expend 1,035 points on average, which is 4.1 times the prediction of standard theory (250). This implies an altruism parameter of slightly above 1. Bhattacharya (2016), with three-person groups, finds that group-level effort averages 689 in even contests, and effort for advantaged and disadvantaged teams in uneven contests average 951 and 555, respectively. The self-interest model, in contrast, predicts 125, 222, and 111 in these three cases. The data can be reconciled with predictions of the within-group altruism model with $\alpha \approx 1.8$.

For Hypothesis 4 and Hypothesis 5, minimum detectable effect sizes range from 7.0 to 10.5, and from 9.5 to 11.0, respectively.

Power calculations are only approximations as the true underlying outcome distributions are unknown. For comparisons where theory predicts treatment effects to arise, the calculations suggest we can detect differences predicted by the within-group altruism model with sufficient power. We expect lower variation in group-level effort for the complete information cases, and to the extent this is true, the minimum detectable effect sizes are smaller than those provided above. Controlling for other factors in the econometric models, such as participant characteristics, is expected to increase power as these factors should be uncorrelated with treatment assignment.

C. Experimental procedures

A typical experimental session proceeds as follows. Participants are randomly assigned an ID number and a computer station in the laboratory. The same moderator reads instructions aloud and follows several protocols that are clearly mentioned in the consent form as well as in written instructions provided to participants. All decisions are made on the computer. The experiment was programmed and conducted using the software z-Tree (Fischbacher 2007).

Prior to the group contest experiment participants complete a (paid) risk elicitation task of the sort popularized by Holt and Laury (2002). Following standard procedures, the outcome of this task is not revealed until the end of the session. After reading instructions for the group contest experiment, participants take a quiz designed to test and educate participants on earnings calculations and the incentives they face. Participants are paid for correct answers and provided detailed answers to the questions posed. Participants then proceed through one unpaid training round and any questions are answered by the moderator prior to the 20 paid rounds.

For complete information treatments, the computer decision screen displays all three parameters (cost, value, group size) in effect for the participant's group as well as the opponent group. For incomplete information treatments, identical information is displayed with the exception of the one parameter for the opponent group for which there is uncertainty. In this case, the two possible values are displayed. After all decisions are entered, a result screen reveals which team won, total effort for the participant's group, and earnings for the round. Participants do not see the individual efforts of their team members nor do they receive any direct information on the choices of their opponent. Participants earn money based on the outcome in each of the 20 paid decision rounds. The experiment concludes with a demographic questionnaire. Representative instructions and the questionnaire are provided in the appendix.

D. Participants

Eighteen experiment sessions were conducted during the summer and fall of 2019 as well as fall of 2020. We have data from 360 participants.[7] All sessions were conducted in a designated experimental economics laboratory at a major public research university. Participants were recruited from a large existing database of undergraduate students that had previously registered to receive invitations for economics experiments. Participants were not allowed to attend more than one session of the experiment. Earnings were dominated in "lab dollars" and exchanged for U.S. dollars at an announced exchange rate. As theory predicted earnings in the value treatments to be considerably higher, we used an exchange rate of 120-to-1 for the value treatments, and 90-to-1 for the remaining treatments. The experiment lasted approximately 50 minutes and on average

[7] Due to variations in participant show-up rates, there are 42 participants in the complete information cost treatment and 66 participants in the incomplete information cost treatment. Revising our power calculations based on these realized sample sizes has only a negligible effect. We met exactly our sample size targets for the other treatments.

participants earned $18 for the session.

Overall, 42% of participants are female, 56% had participated in a prior economics experiment, and 47% can be characterized as risk averse based on the incentivized risk elicitation task. The average score on our instructions quiz is about 86%. Sixty percent of participants answered all quiz questions correctly, 27.5% answered three correctly, and the remainder answered 2 or fewer questions correctly. Responses from the post-experiment questionnaire suggest that the vast majority (88%) felt they were sufficiently compensated. In response to a Likert-scale question that ranged from "1" ("poorly understood") to "5" ("well understood"), the vast majority (89%) selected a 4 or 5, indicating a strong self-assessment of how well instructions were understood.

4. Results

Table 4 and Table 5 allow for basic comparisons between observed group-level effort and theoretical predictions. Overall, the most prominent observation is that group efforts are much closer to point predictions based on the within-group altruism model with $\alpha = 1$. In fact, assuming a common altruism parameter for all settings, we find that $\alpha = 0.99$ minimizes the squared deviations between predicted and actual values.[8] Accordingly, observed averages are approximately three times those predicted by the self-interest model for the cost and value treatments. Also consistent with the within-group altruism, but not the standard self-interest model, there are stark differences in effort between advantaged and disadvantaged groups in the group size treatments. The tables do suggest possible differences between the group size treatments and the value and cost treatments. For the cost and value treatments, in uneven contests, the observed

[8] To be clear, this value of α is based on the 18 observed group-level averages presented in Tables 4 and 5. If we instead let α vary by treatment, we obtain the following estimates: cost-of-effort (complete) = 0.72; prize value (complete) = 0.66; group size (complete) = 1.29; cost-of-effort (incomplete) = 0.92; prize value (incomplete) = 0.90; group size (incomplete) = 1.26.

effort for disadvantaged groups is much higher (about double) under incomplete relative to complete information; in contrast, the opposite is true in group size treatments. For group size treatments relative to the cost and value treatments, effort for advantaged groups is about 60% and 40% higher, respectively, in complete and incomplete information contests.

We begin the analysis by analyzing group-level outcomes: effort and the probability of winning. We cluster standard errors by period within a session, which corrects for heteroskedasticity and allows contemporaneous correlation across groups. Recalling that participants are randomly re-sorted into groups every period, nearly all groups will be unique. For models that include participant characteristics, we use group-specific averages. Then, we explore individual behavior by analyzing variation in free-riding and a measure of within-group variation. In these regressions, we cluster standard errors by individuals, which allows for heteroskedasticity as well as within-subject serial correlation.

A. Group-level Effort

Tables 7 and 8 present regressions that allow for tests of information effects for uneven and even contests, respectively. It is important to keep in mind that with incomplete information participants do not know whether they are engaged in an even or uneven contest, and theory predicts differences across information conditions. Further, when testing for information effects we can include all data from incomplete information treatments regardless of the contest type.[9] Specification (1) estimates the effect of incomplete information, averaged across all potential sources of heterogeneity. Specification (2) includes interactions to allow for tests of information

[9] Tables A1 and A2 in the appendix present results that restrict the data used from the incomplete information treatments to only include observations from what are in actuality uneven and even contests, respectively. As participants in the incomplete information treatments do not know whether they are playing in an even or uneven contest, restricting the data has the expected effects – treatment effects are very similar but less precisely estimated.

by advantage source. Specification (3) adds control variables to the interactions model.

In uneven contests, when averaged across heterogeneity sources, incomplete information significantly increases group-level effort by 11 points on average. This positive and significant effect is largely driven by the cost and value treatments, where the effects are 20 and 19 points, respectively. For group size treatments, the estimate of the information effect is unexpectedly negative in specification (2), although this result is not robust to the inclusion of control variables.[10] Thus, we reject Hypothesis 1 for the value and cost treatments and in turn find support for Proposition 1. The self-interest model does not predict an information effect for the group size treatments, although based on other evidence this model overall inadequately explains behavior.

For even contests, we find no effect of information when averaging across all sources of heterogeneity. Thus, we fail to reject Hypothesis 2. This is contrary to Proposition 2 where we expect incomplete information to lower effort. In fact, based just on the value treatments, there is a *positive* and marginally significant effect of incomplete information.[11]

Table 9 pools data across the two contest types. Theoretically, effort for an advantaged (disadvantaged) group under incomplete information is equal to the effort, averaged across uneven and even contests, for an advantaged (disadvantaged) group under complete information. Thus, theory predicts the expected effort in this special case to be the same regardless of the information condition; however, empirically there are significant differences. We find, pooling over sources of heterogeneity, incomplete information increases group-level effort by approximately 6 points. By allowing the information effects to vary with the source of advantage, incomplete information

[10] We randomized treatments across sessions prior to implementation, but by chance have a slight imbalance of characteristics across the two group size treatments, with the complete information treatment utilizing a higher proportion of females, and a lower proportion of experienced and risk averse participants.
[11] We present regressions restricted to advantaged groups in Table A3 in the appendix, which reveal large and positive effects of incomplete information for the cost and value treatments only statistically significant for the latter.

increases effort in the value and cost treatments. In contests with potential group size heterogeneity, this effect is insignificant when we include control variables. Thus, overall Proposition 3 is supported by the group size treatments but not for the value and cost treatments.

The effects of control variables are very similar regardless of contest type. Effort decreases as the experiment progresses. Groups with a higher proportion of players with prior experience in economics experiments put forth less effort on average. Groups with more risk averse players also contribute lower effort, and groups with more players that identify as female have higher effort. However, specific to the proportion of females or risk averse players, when testing for information effects, we note that the coefficient magnitudes are large and statistically significant.[12]

Result 1. In uneven contests, incomplete information increases group-level effort when one team has either a cost or value advantage, but has no effect when one team has a group size advantage.

Result 2. In even contests, there is marginal evidence that incomplete information increases effort in the value treatment. Incomplete information has no effect in the cost and group size treatments.

Result 3. When the data is pooled across the two contest types, incomplete information increases group-level effort for the value and cost treatments, but has no effect for the group size treatment.

The experimental design parameters were selected so that predicted effort does not vary according to the source of heterogeneity for the special case $\alpha = 1$, or across cost-of-effort and

[12] As the overall risk tends to be higher in the incomplete information treatments, this may partially counteract the information effects on effort possibly shrinking the size of treatment effects.

prize value treatments for any degree of altruism. Table 9 allows us to test Hypothesis 4 across a few dimensions. From specification (2) and (3), we deduce that effort for the group size treatment with complete information is statistically different and higher when compared with either the value or cost treatment. With incomplete information, we find no difference based on any pairwise comparison of treatments.[13] Comparisons based on the results in Tables 7 and 8 demonstrate that effort is equivalent across the value and cost treatments regardless of contest type. Further, these results reveal that the overall differences observed between the group size and other treatments under complete information are driven by behavior in both uneven and even contests.[14, 15]

Table 10 presents regressions that allow for differences between disadvantaged and advantaged groups for each treatment, which allows for tests of Hypothesis 4, as well as additional tests relevant to Hypothesis 3. Specifications (1) and (2) use data from complete information treatments and specifications (3) and (4) use data from incomplete information treatments. It is clear from these regressions that there are very large differences in effort between disadvantaged and advantaged groups for all treatments. These differences range from 33 points (value treatment with incomplete information) to 78 points (group size treatment, incomplete information). These regressions also reveal that differences between the group size and other treatments arise on additional dimensions. In particular, with complete information, effort is higher for both the disadvantaged (by about 11 points) and advantaged groups (by about 46 points). With incomplete information, effort in the group size treatment remains higher when comparing advantaged groups,

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[13] Based on specification (3) in Table 9, additional test results are as follows: value versus group size, complete information (F=26.85, p=0.00); value versus cost, incomplete information (F=0.04, p=0.84); value versus group size, incomplete information (F=0.06, p=0.80); cost versus group size, incomplete information (F=0.01, p=0.92).
[14] Based on specification (3) in Table 7, additional test results are as follows: value versus group size, complete information (F=42.43, p=0.00); value versus cost, incomplete information (F=0.05, p=0.82); value versus group size, incomplete information (F=0.09, p=0.76); cost versus group size, incomplete information (F=0.02, p=0.88).
[15] Based on specification (3) in Table 8, additional test results are as follows: value versus group size, complete information (F=3.60, p=0.05); value versus cost, incomplete information (F=0.08, p=0.78); value versus group size, incomplete information (F=0.21, p=0.64); cost versus group size, incomplete information (F=0.08, p=0.78).

with a disparity of about 34 points on average. On the other hand, aside from the case of incomplete information and disadvantaged groups, there are no statistical differences when comparing the cost and value treatments. The above results are robust to the inclusion or exclusion of control variables.

Result 4. Based on a large set of comparisons, group effort is similar across cost and value treatments. On the other hand, many differences arise when comparing group size treatments with either the cost or value treatments.

Result 5. Group effort is higher for advantaged groups, regardless of the source of advantage or information condition.

The above results provide support for (either) theory in the case of cost and value treatments, and in turn support Hypothesis 4 and reject Hypothesis 5. However, the magnitudes of the estimated advantage effects are considerably higher than those predicted by standard theory. Continuing the theme from the tests of information effects, we have uncovered additional differences between the group size treatments and the other treatments. The self-interest theory predicts that larger groups have no advantage, and so we should expect stark differences when comparing efforts from larger groups with groups with either a cost or value advantage. While these differences arise, they are in the opposite direction. There is some qualified support for the within-group altruism theory, which accurately predicts that group effort increases with group size. The magnitudes of the predicted advantage effects from this theory closely coincides with the estimated effects, assuming that players are sufficiently altruistic.

B. Probability of Winning

Table 11 presents regressions that estimate differences in the chances of winning between advantaged and disadvantaged groups in uneven contests for each source-of-advantage.[16] Separate regressions are run for the two information conditions. The probability of winning is endogenous and determined by the relative effort of the competing groups. The theory predicts that, in uneven contests where one team has either a cost or prize value advantage, that the advantaged team has a higher chance of winning. In particular, the three-fold advantage we implement gives rise to the advantaged group being three times more likely to win; in other words, theory predicts the advantaged team has a 75% chance of winning. Of course, only the within-group altruism model predicts any differences for group size treatments, as there is no advantage according to the self-interest theory. The extent of the advantage for the group size treatments does depend on the altruism parameter. The advantage is three-fold only if $\alpha = 1$, and is otherwise higher when $\alpha > 1$ or lower when $\alpha < 1$.

With complete information, win probabilities are consistent with a roughly three-fold advantage for all treatments. For the group size treatment, from specification (1), the advantaged and disadvantaged groups have 75% and 25% chances to win, respectively. The advantaged team has an 80% and 74% chance of winning, respectively, in the cost and value treatments. Under incomplete information, the win percentages remain very close to the 75%/25% split for the group size treatment. However, advantaged groups in the value and cost treatments have a significantly lower chance of winning than what theory predicts: 62% and 67%, respectively, based on specification (3). This is largely driven by the fact that, as illustrated in Table 5, the actual ratio of

[16] A parallel analysis for even contests is uninformative given that both advantaged and disadvantaged teams will have a 50% win probability by construction. It is also for this reason that the data from the incomplete information treatments are restricted to uneven contests, regardless of the fact that participants did not know the contest type.

advantaged to disadvantaged group effort is noticeably less than 3-to-1. There are significant differences in the chances of winning (about 5 percentage points) for a particular group type across both information conditions when comparing cost and value treatments.

C. Within-group Heterogeneity

Last, we briefly investigate heterogeneous behavior within groups by estimating regressions based on individual-level effort choices. As in other social dilemma games, the possibility arises for players to free-ride off the effort expenditures of other players. About 21% of individual-level effort expenditures (1506 of 7200 observations) are zero, so it makes sense to take a closer look at the extent of free-riding. Model (1) in Table 12 presents a linear regression where the dependent variable is an indicator that equals 1 in cases where the participant contributed 0 effort.[17] Being in an uneven contest increases free-riding by 8 percentage points. Competing on an advantaged team decreases free-riding by 13 percentage points. Incomplete information has no effect. There is significantly more free-riding in the group size treatments relative to either the cost or value treatments, and the estimated difference is approximately 12 percentage points. When effects are considered in tandem, the highest rate of free-riding comes from players on a small team that are, with or without their full knowledge, competing against a larger team. Of course, regardless of what may be true in theory, the optics for those on a small team are bleak. Players with prior participation in economics experiments and those classified as risk averse are more likely to free ride, whereas females are less likely to free ride. On average, free-riding is 14 percentage points more likely in the last round of the experiment relative to the first round.

[17] While we continue to use linear regression because of its robustness properties, similar results arise if we instead estimate a probit model.

We analyze as a second measure of within-group heterogeneity the squared deviation of a player's effort from the group mean; i.e., $(x_{ig} - \overline{x}_g)^2$. Given random re-sorting into groups, $\overline{x}_g$ is specific to the particular group one is in for a specific decision round. In the extreme case where each group member makes the same effort choice, the measure equals zero. Analysis of this outcome variable is presented as Model (2) in Table 12. Participant characteristics are strongly correlated with this variance measure. Within-group variation decreases with risk aversion, as well as experience in prior economics experiments. The latter is suggestive of a learning effect. The contribution variance, however, does not vary as the experiment progress. Overall, most of the variation in the experimental design does not appear to impact within-group variation. The main exception is that there are larger disparities among members of an advantaged team. This is somewhat unexpected, given that free-riding is less likely for advantaged team members. As a possible explanation, some players may feel a stronger frustration of losing when on an advantaged team and, in turn, over-expend. If players hold expectations that their team members will behave this way, a logical response is to then contribute significantly less.

5. Discussion

The primary goal of this paper is to use theory and experiments to study the impact of incomplete information on group-level effort in a heterogeneous inter-group competition. In theory, incomplete information over the opponent's type (advantaged or disadvantaged) causes contest-level effort to be higher under "uneven" contests between an advantaged and disadvantaged group, and lower under "even" contests where both teams are advantaged, or both are disadvantaged. We find some support for the theory, but only for uneven contests and only when the source of advantage is a lower effort cost or a higher prize value for the winning team.

When data from both uneven and even contests are pooled, we find that incomplete information increases group-level effort for contests with potential value or cost heterogeneity, even though theory predicts there to be no difference.

As secondary objectives, we test across three source-of-advantage – due to cost-of-effort, prize value, and group size –, and further consider in the theory the possibility that players are altruistic towards other team members. Whereas effort is quite similar between cost and value treatments, several differences emerge in comparisons involving the group size treatments. The within-group altruism model, with $\alpha = 1$, does a reasonable job of predicting group-level effort across many but not all cases, whereas the standard model based solely on self-interest severely underpredicts efforts. Below we elaborate on what behavioral drivers might explain unanticipated results. We then discuss the broader implications of our results.

Why doesn't incomplete information lower effort in even contests? In the incomplete information treatments, it is common knowledge that the probability an opponent is advantaged is 50%. It is possible that players form subjective beliefs over this probability that deviate from actuality.[18] In particular, players on an advantaged team may act as if this percentage is greater than 50%, which then serves to counteract the effects of incomplete information when comparing even contests. One reason for this could be pessimism – that although you are on an advantaged team as (bad) luck would have it, the other team is also probably advantaged (Baharad and Nitzan 2008). Another reason to increase effort is to minimize regret from losing (Hart et al. 2015). If you lose while on an advantaged team, it could have been that you underestimated your opponent – by thinking they were not advantaged when they actually were.

[18] Bhattacharya (2016) provides evidence that subjective beliefs are important in explaining behavior in heterogenous group contests.

Why does incomplete information increase effort in heterogenous cost and value contests, but not contests between groups of different size? As with other social dilemma games, there is the potential in group contests to free ride off the efforts of others. Free-riding is presumably more likely to occur with larger groups, and groups with a size advantage in our experiment have nine players rather than three. The predictions of the within-group altruism model with $\alpha = 1$ are similar to the observed means for the group size treatments with incomplete information (see Table 5) (effort in the advantaged groups is a little high which could be explained by beliefs as described above). However, with complete information, effort from the advantaged groups in uneven contests is very high. Clearly, an advantaged group should win much of the time, but could lose the opportunity to do so if there is significant free-riding. As the number of free-riders on a team are unknown (especially with a "strangers" experimental design), those not inclined to free ride may over-expend to help insure the win. With incomplete information, players on an advantaged team never know when they "should" win, and incentives to counteract the effects of free-riding are likewise diminished. This differential behavior, to the extent that it exists, counteracts the effect of incomplete information predicted by theory. Such behavior may explain why, with complete information, there is little difference in the effort of advantaged groups across uneven and even contests in contrast to what theory predicts; in particular, there is not the same pressure to over-expend in an even contest, as free-riding is a potential concern among both teams.

Free-riding incentives provide one reason why behavior in the group size treatments is differentiated from the cost and value treatments, which in turn could explain the empirical differences observed across these sources-of advantage. Our analysis does find that those in group size treatments are more likely to free-ride. While effort from advantaged groups is unexpectedly high in uneven contests under complete information, effort from disadvantaged groups is as well.

This could be the result of players in a disadvantaged team learning to best-respond in our repeated game setting, or perhaps those prone to free-riding on large teams “make up” for this socially undesirable behavior by over-expending when placed on a smaller team.

Some of our results serve to reinforce and extend prior findings. In an uneven group contest with cost-of-effort heterogeneity, Bhattacharya (2016) finds that advantaged teams contribute significantly more effort than disadvantaged teams. This is consistent with our results, and we further demonstrate that this advantage effect holds for different sources of advantage as well as with incomplete information. In individual lottery contests involving four players, incomplete information over marginal cost-of-effort causes players with a low (high) marginal cost to submit higher (lower) bids over complete information (Brookins and Ryvkin, 2014). In our group contest setting, we find that incomplete information also alters effort both in theory and in practice. The theory also suggests that, while incomplete information increases effort for an advantaged team in an uneven contest, its effect on a disadvantaged team is ambiguous in general. (However, for the discrete case that we study using experiments, incomplete information increases group-level effort for both types.) Last, consistent with the group contest literature but inconsistent with a standard theory model of self-interest, increasing group size does lead to higher group-level effort. Prior group contest experiments on use groups with five or fewer players and thus we demonstrate that the stylized fact continues to hold with nine-member groups. We find that being in a larger group is in practice a more significant advantage relative to a cost-of-effort or prize value advantage.

Motivated by prior experimental evidence that players over-expend effort relative to a self-interest theory that assumes players maximize individual-level utility, we instead consider a theory that assumes players care about the payoffs of their group members. As this alternative model shows some promise in explaining observed behavior, it warrants investigation in other settings, especially those where the standard model predicts that a group size paradox should arise in the sense that smaller teams are more likely to win. An interesting extension to our experimental design would be to provide feedback on own (and possibly opponent) group effort and explore how individuals and groups update beliefs in response. We conjectured above that some of the effects of incomplete information may be attributable to subjective beliefs, and providing such information could lead to informative results.

On a final note, our findings have relevance for contest design. When competing groups are asymmetric (uneven contests), according to theory incomplete information causes only a wasteful increase in efforts (lower efficiency) as there are no significant changes in the probability of winning for advantaged and disadvantaged groups. Thus, if the organizer cares about efficiency, they can engender this by promoting transparency (e.g., by providing details on the competitors). On the other hand, our experimental results suggest that if the goal is to reduce free-riding behavior, then the contest designer may prefer less transparency as we observe less free-riding with incomplete information.

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Table 1. Group effort: *complete* information

Source of heterogeneity	Contest type	Equilibrium effort
Cost-of-effort	Uneven	$(X_A^*, X_D^*) = \left(\frac{c_D v[1+\alpha(N-1)]}{(c_A+c_D)^2}, \frac{c_A v[1+\alpha(N-1)]}{(c_A+c_D)^2}\right)$
	Even	$(X_A^*, X_A^*) = \left(\frac{v[1+\alpha(N-1)]}{4c_A}, \frac{v[1+\alpha(N-1)]}{4c_A}\right)$ $(X_D^*, X_D^*) = \left(\frac{v[1+\alpha(N-1)]}{4c_D}, \frac{v[1+\alpha(N-1)]}{4c_D}\right)$
Prize Value	Uneven	$(X_A^*, X_D^*) = \left(\frac{v_A^2 v_D[1+\alpha(N-1)]}{c(v_A+v_D)^2}, \frac{v_A v_D^2[1+\alpha(N-1)]}{c(v_A+v_D)^2}\right)$
	Even	$(X_A^*, X_A^*) = \left(\frac{v_A[1+\alpha(N-1)]}{4c}, \frac{v_A[1+\alpha(N-1)]}{4c}\right)$ $(X_D^*, X_D^*) = \left(\frac{v_D[1+\alpha(N-1)]}{4c}, \frac{v_D[1+\alpha(N-1)]}{4c}\right)$
Group Size	Uneven	$(X_A^*, X_D^*) = \left(\frac{v[1+\alpha(N_A-1)]^2[1+\alpha(N_D-1)]}{c(2+\alpha[N_A+N_D-2])^2}, \frac{v[1+\alpha(N_A-1)][1+\alpha(N_D-1)]^2}{c(2+\alpha[N_A+N_D-2])^2}\right)$
	Even	$(X_A^*, X_A^*) = \left(\frac{v[1+\alpha(N_A-1)]}{4c}, \frac{v[1+\alpha(N_A-1)]}{4c}\right)$ $(X_D^*, X_D^*) = \left(\frac{v[1+\alpha(N_D-1)]}{4c}, \frac{v[1+\alpha(N_D-1)]}{4c}\right)$

Notes: An "uneven" contest refers to a case where an advantaged (A) group plays a disadvantaged (D) group. The advantaged team has either a lower cost of effort (i.e., $c_A < c_D$), higher prize value ($v_A > v_D$), or larger group size (i.e., $N_A > N_D$) relative to the disadvantaged team. In an "even" contest, both groups are advantaged or disadvantaged. For clarity, we drop the subscripts on parameters that are held fixed across groups within a comparison set.

Table 2. Group effort: *incomplete* information

Source of heterogeneity	Equilibrium effort
Cost-of-effort	$X_A^{**} = \frac{[1+\alpha(N-1)]v}{c_A}\left(\frac{4\frac{c_A}{c_D} + \left(1+\frac{c_A}{c_D}\right)^2}{8\left(1+\frac{c_A}{c_D}\right)^2}\right)$ $X_D^{**} = \frac{[1+\alpha(N-1)]v}{c_D}\left(\frac{4\frac{c_A}{c_D} + \left(1+\frac{c_A}{c_D}\right)^2}{8\left(1+\frac{c_A}{c_D}\right)^2}\right)$
Prize Value	$X_A^{**} = \frac{[1+\alpha(N-1)]v_A}{c}\left(\frac{4\frac{v_D}{v_A} + \left(1+\frac{v_D}{v_A}\right)^2}{8\left(1+\frac{v_D}{v_A}\right)^2}\right)$ $X_D^{**} = \frac{[1+\alpha(N-1)]v_D}{c}\left(\frac{4\frac{v_D}{v_A} + \left(1+\frac{v_D}{v_A}\right)^2}{8\left(1+\frac{v_D}{v_A}\right)^2}\right)$
Group Size	$X_A^{**} = \frac{[1+\alpha(N_A-1)]v}{c}\left(\frac{4\frac{1+\alpha(N_D-1)}{1+\alpha(N_A-1)} + \left(1+\frac{1+\alpha(N_D-1)}{1+\alpha(N_A-1)}\right)^2}{8\left(1+\frac{1+\alpha(N_D-1)}{1+\alpha(N_A-1)}\right)^2}\right)$ $X_D^{**} = \frac{[1+\alpha(N_D-1)]v}{c}\left(\frac{4\frac{1+\alpha(N_D-1)}{1+\alpha(N_A-1)} + \left(1+\frac{1+\alpha(N_D-1)}{1+\alpha(N_A-1)}\right)^2}{8\left(1+\frac{1+\alpha(N_D-1)}{1+\alpha(N_A-1)}\right)^2}\right)$

Notes: The equilibrium effort of advantaged and disadvantaged teams are denoted by X_A^{**} and X_D^{**}, respectively. An advantaged team has either a lower cost of effort (i.e., $c_A < c_D$), higher prize value ($v_A > v_D$), or larger group size (i.e., $N_A > N_D$) relative to the disadvantaged team. For clarity, we drop the subscripts on parameters that are held fixed across groups within a comparison set. Equilibria are for the special case of $r = \frac{1}{2}$.

Table 3. Experiment parameters

Source of heterogeneity	Group type	Cost	Value	Group size
Cost-of-effort	Advantaged	$c_A = 1/3$	$v = 50$	$N = 3$
	Disadvantaged	$c_D = 1$	$v = 50$	$N = 3$
Prize Value	Advantaged	$c = 1$	$v_A = 150$	$N = 3$
	Disadvantaged	$c = 1$	$v_D = 50$	$N = 3$
Group Size	Advantaged	$c = 1$	$v = 50$	$N_A = 9$
	Disadvantaged	$c = 1$	$v = 50$	$N_D = 3$

Table 4. Theoretical predictions and observed group-level effort: *complete* information

		Self-Interest model ($\alpha = 0$)		**Within-group altruism model ($\alpha = 1$)**		**Observed**	
Source of heterogeneity	Contest Type	X_A^*	X_D^*	X_A^*	X_D^*	X_A	X_D
Cost-of-effort	Uneven	28.13	9.38	84.38	28.13	75.04	20.10
Cost-of-effort	Even	37.50	12.50	112.50	37.50	80.79	49.43
Prize Value	Uneven	28.13	9.38	84.38	28.13	70.69	27.04
Prize Value	Even	37.50	12.50	112.50	37.50	76.84	43.72
Group Size	Uneven	12.50	12.50	84.38	28.13	119.57	41.14
Group Size	Even	12.50	12.50	112.50	37.50	128.50	51.91

Notes: X_A and X_D refer to effort for advantaged and disadvantaged groups, respectively.

Table 5. Theoretical predictions and observed group-level effort: *incomplete* information

	Self-Interest model ($\alpha = 0$)		**Within-group altruism model ($\alpha = 1$)**		**Observed**	
Source of heterogeneity	X_A^{**}	X_D^{**}	X_A^{**}	X_D^{**}	X_A	X_D
Cost-of-effort	32.81	10.94	98.44	32.81	89.48	42.70
Prize Value	32.81	10.94	98.44	32.81	84.85	51.66
Group Size	32.81	12.50	98.44	32.81	119.20	39.80

Notes: X_A and X_D refer to effort for advantaged and disadvantaged groups, respectively.

Table 6. Description of data

Variable name	Description	Mean	S.D.
Dependent Variables			
Group Effort	Total points contributed by all group members	74.90	45.77
Probability of Winning	Calculated as a function of own and opponent group effort, using equation [1]	52.42	22.84
Individual Effort	Points contributed by the participant, 0 to 50 points	18.03	16.06
Effort Variance	Squared deviation of a participant's contribution relative to the mean contribution within the group	145.81	211.13
Zero Effort	= 1 if participant contributed zero points; 0 otherwise	0.21	0.41
Experimental treatment Variables			
Advantaged	= 1 for advantaged groups; 0 otherwise	0.56	0.50
Incomplete	= 1 for incomplete information treatments; 0 otherwise	0.53	0.50
Uneven	= 1 for uneven contests; 0 otherwise	0.47	0.50
Cost	= 1 for cost treatments; 0 otherwise	0.33	0.46
Value	= 1 for value treatments; 0 otherwise	0.33	0.46
Group	= 1 for group size treatments; 0 otherwise	0.40	0.49
Additional Control Variables			
Risk Averse	= 1 if participant selected safe option at least six times in Risk Elicitation task; 0 otherwise	0.47	0.50
Experience	=1 if the participant had partaken in a prior economics experiment; 0 otherwise	0.56	0.50
Female	= 1 if participant is female; 0 otherwise	0.42	0.49
Round	Decision round in the experiment, 1 to 20	10.50	5.76

Table 7. Analysis of information effects: *uneven* contests

	Dependent variable: Group Effort		
	(1)	(2)	(3)
Value		1.30	0.95
		(3.22)	(3.04)
Group		32.79***	26.87***
		(4.37)	(4.28)
Cost × Incomplete		20.01***	20.36***
		(3.01)	(2.86)
Value × Incomplete		19.12***	18.76***
		(3.37)	(3.02)
Group × Incomplete		-11.19**	-6.05
		(4.83)	(4.42)
Incomplete	11.33***		
	(2.34)		
Experience			-18.31***
			(3.23)
Risk Averse			-7.18**
			(3.41)
Female			10.00***
			(3.65)
Round			-1.30***
			(0.17)
Constant	56.80***	47.57***	71.88***
	(1.88)	(2.33)	(4.14)
Observations	1,498	1,498	1,498
R-squared	0.016	0.050	0.106

Notes: Cluster-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 8. Analysis of information effects: *even* contests

	Dependent variable: Group Effort		
	(1)	(2)	(3)
Value		-2.27	-3.73
		(4.73)	(4.65)
Group		11.12*	6.51
		(5.92)	(5.95)
Cost × Incomplete		3.81	3.74
		(4.24)	(4.05)
Value × Incomplete		6.49*	6.67*
		(3.80)	(3.66)
Group × Incomplete		-5.72	-1.88
		(5.51)	(5.38)
Incomplete	1.60		
	(2.61)		
Experience			-16.85***
			(3.02)
Risk Averse			-10.34***
			(3.14)
Female			10.19***
			(3.39)
Round			-1.20***
			(0.19)
Constant	66.52***	63.77***	87.96***
	(2.21)	(3.79)	(5.06)
Observations	1,566	1,566	1,566
R-squared	0.000	0.008	0.062

Notes: Cluster-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 9. Analysis of information effects: pooled over contest types

	Dependent variable: Group Effort		
	(1)	(2)	(3)
Value		-0.13	-1.02
		(3.21)	(3.02)
Group		21.47***	16.54***
		(3.86)	(3.85)
Cost × Incomplete		11.91***	12.06***
		(3.15)	(2.92)
Value × Incomplete		12.45***	12.49***
		(3.23)	(2.93)
Group × Incomplete		-7.97*	-4.16
		(4.27)	(3.98)
Incomplete	6.10***		
	(2.07)		
Experience			-18.45***
			(2.96)
Risk Averse			-6.73**
			(3.01)
Female			8.24***
			(3.09)
Round			-1.15***
			(0.16)
Constant	62.02***	55.67***	79.25***
	(1.53)	(2.51)	(3.91)
Observations	1,988	1,988	1,988
R-squared	0.006	0.033	0.081

Notes: Cluster-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 10. Analysis of advantage effects

	(1) Complete info.	(2) Complete info.	(3) Incomplete info.	(4) Incomplete info.
Value	0.16	-0.91	9.20**	6.68**
	(3.07)	(2.80)	(3.58)	(3.17)
Group	12.95***	8.20***	-2.23	-3.56
	(2.91)	(2.98)	(3.14)	(2.72)
Cost × Advantaged	42.42***	41.70***	46.78***	45.20***
	(4.43)	(4.30)	(2.31)	(2.36)
Value × Advantaged	38.52***	38.70***	33.16***	33.46***
	(2.61)	(2.48)	(3.33)	(3.21)
Group × Advantaged	75.37***	76.29***	78.23***	78.62***
	(4.61)	(4.56)	(4.42)	(4.25)
Experience		-20.82***		-17.56***
		(3.79)		(2.87)
Risk Averse		-1.45		-14.62***
		(3.72)		(2.92)
Female		0.80		12.94***
		(3.44)		(2.81)
Round		-0.91***		-1.24***
		(0.16)		(0.17)
Constant	35.37***	58.53***	42.49***	69.28***
	(2.28)	(3.92)	(2.07)	(3.85)
Observations	912	912	1,076	1,076
R-squared	0.458	0.494	0.442	0.508

Notes: Cluster-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 11. Probability of winning in *uneven* contests

	(1) Complete Info.	(2) Complete Info.	(2) Incomplete Info.	(4) Incomplete Info.
Value	5.86**	5.06**	4.77**	4.71**
	(2.32)	(2.31)	(2.33)	(2.29)
Group	4.80*	2.06	-8.43***	-7.85***
	(2.71)	(3.00)	(2.39)	(2.47)
Cost × Advantaged	60.48***	59.98***	34.28***	34.56***
	(3.53)	(3.55)	(3.02)	(3.03)
Value × Advantaged	48.77***	48.75***	24.73***	24.47***
	(3.02)	(2.89)	(3.55)	(3.41)
Group × Advantaged	50.89***	51.22***	51.14***	51.27***
	(4.10)	(4.12)	(3.70)	(3.79)
Experience		-4.90		-1.96
		(3.17)		(2.10)
Risk Averse		-4.99		-7.44***
		(3.05)		(2.59)
Female		6.64*		0.23
		(3.98)		(3.04)
Round		0.005		0.003
		(0.022)		(0.018)
Constant	19.76***	22.97***	32.86***	37.38***
	(1.76)	(3.22)	(1.51)	(2.54)
Observations	422	422	482	482
R-squared	0.735	0.742	0.602	0.610

Notes: Cluster-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

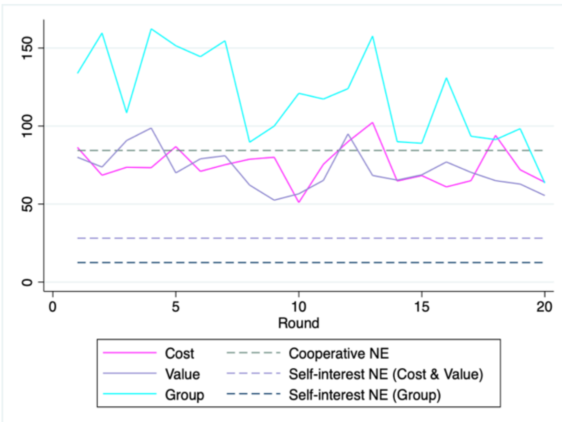
Table 12. Free-riding behavior and intra-group variation in effort

	(1) Dep. Var.: Zero Effort	(2) Dep. Var.: Contribution Variance
Value	0.005	8.75
	(0.030)	(11.27)
Group	0.118***	20.25
	(0.032)	(13.25)
Incomplete	-0.039	12.53
	(0.026)	(10.93)
Advantaged	-0.129***	17.96**
	(0.017)	(7.64)
Uneven	0.083***	-1.05
	(0.013)	(6.72)
Experience	0.082***	-33.89***
	(0.027)	(11.75)
Risk Averse	0.071***	-22.17**
	(0.027)	(11.05)
Female	-0.062**	6.97
	(0.026)	(11.51)
Round	0.007***	0.034
	(0.001)	(0.49)
Constant	0.088***	145.1***
	(0.033)	(15.92)
Observations	7,200	7,200
R-squared	0.079	0.015

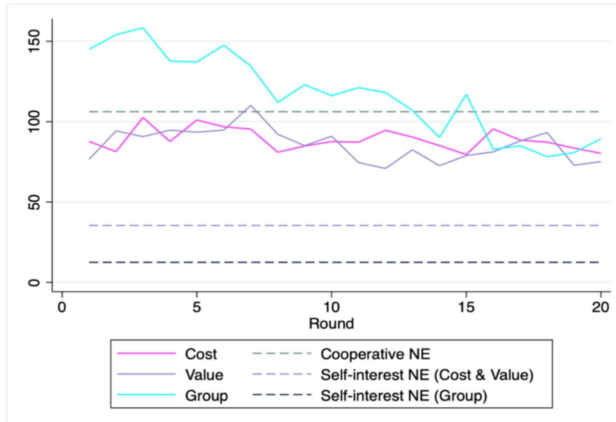
Notes: Standard errors clustered at the individual-level in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Figure 1. Group effort, *uneven* contests

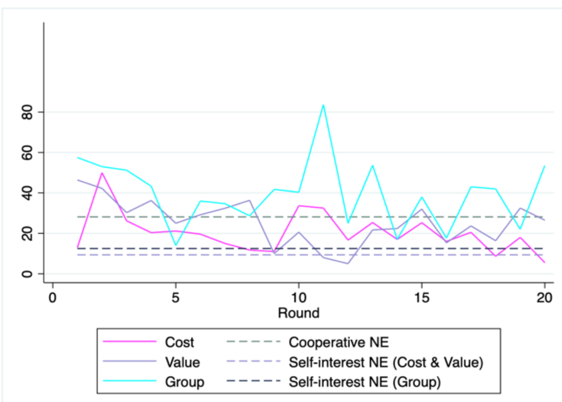
(a) Complete Information, Advantaged



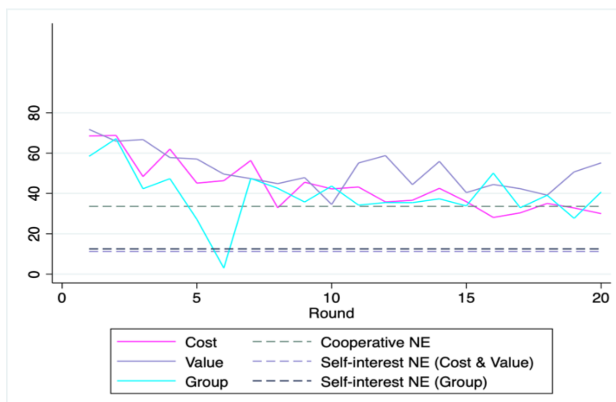
(b) Incomplete Information, Advantaged



(c) Complete Information, Disadvantaged



(d) Incomplete Information, Disadvantaged



Appendix

A. Support of propositions

We first prove the three propositions for the special case where $r = \frac{1}{2}$. For brevity we focus on the case of cost heterogeneity. Parallel proofs for other sources of heterogeneity follow in a straightforward way. We then present the general solution for $0 < r < 1$, and the results of numerical calculations to provide further support of the propositions. For convenience, throughout this appendix we define $\tilde{N} \equiv 1 + \alpha(N-1)$.

Proof of Proposition 1: We claim that contest-level effort in an uneven contest is higher with incomplete information. Using the solutions provided in Tables 1 and 2, we then need to show

[A1] $$\frac{\tilde{N}v}{c_A}\left(\frac{4\frac{c_A}{c_D}+\left(1+\frac{c_A}{c_D}\right)^2}{8\left(1+\frac{c_A}{c_D}\right)^2}\right)+\frac{\tilde{N}v}{c_D}\left(\frac{4\frac{c_A}{c_D}+\left(1+\frac{c_A}{c_D}\right)^2}{8\left(1+\frac{c_A}{c_D}\right)^2}\right) > \frac{c_D v\tilde{N}}{(c_A+c_D)^2}+\frac{c_A v\tilde{N}}{(c_A+c_D)^2}$$

Combining terms, and dividing both sides by $v\tilde{N}$ yields:

[A2] $$\left(\frac{4\frac{c_A}{c_D}+\left(1+\frac{c_A}{c_D}\right)^2}{8\frac{c_A}{c_D}(c_A+c_D)^2}\right)(c_A+c_D) > \frac{c_A+c_D}{(c_A+c_D)^2}.$$

Dividing both sides by $\frac{c_A+c_D}{(c_A+c_D)^2}$ yields:

[A3] $$\frac{4\frac{c_A}{c_D}+\left(1+\frac{c_A}{c_D}\right)^2}{8\frac{c_A}{c_D}} > 1,$$

which simplifies to:

[A4] $$\frac{1}{2}+\frac{\left(\frac{c_A+c_D}{c_D}\right)^2}{8\frac{c_A}{c_D}} > 1.$$

Subtracting ½ from both sides, and then multiplying both sides by $8c_Ac_D$ we obtain

[A5] $$(c_A+c_D)^2 > 4c_Ac_D.$$

Finally, this inequality simplifies to

[A6] $(c_A - c_D)^2 > 0,$

which holds true for any $c_A < c_D$.

Proof of Proposition 2**:** We claim that incomplete information decreases effort in an even contest. Using the solutions provided in Tables 1 and 2, we then need to show that:

[A7] $$\frac{v\tilde{N}}{4c_g} > \frac{v\tilde{N}}{c_g}\left(\frac{4\frac{c_A}{c_D}+\left(1+\frac{c_A}{c_D}\right)^2}{8\left(1+\frac{c_A}{c_D}\right)^2}\right) \text{ for } g = A, D$$

Cancelling terms on both sides, we are left with the following condition:

[A8] $$\frac{1}{4} > \left(\frac{4\frac{c_A}{c_D}+\left(1+\frac{c_A}{c_D}\right)^2}{8\left(1+\frac{c_A}{c_D}\right)^2}\right).$$

Expanding the r.h.s. of [A8], and simplifying, we obtain

[A9] $$\frac{1}{4} > \frac{c_A c_D}{2(c_A+c_D)^2} + \frac{1}{8}\ .$$

Subtracting 1/8 from both sides, and then multiplying both sides by $8(c_A + c_D)^2$ yields:

[A10] $(c_A + c_D)^2 > 4c_A c_D.$

As in the prior proof, this reduces to

[A11] $(c_A - c_D)^2 > 0,$

which holds true for any $c_A < c_D$.

Proof of Proposition 3: We claim that expected contest-level effort is the same under both information conditions when $r = \frac{1}{2}$. When $r = \frac{1}{2}$, there is a 50% chance of an uneven contest, a 25% chance of an even contest among disadvantaged teams, and a 25% chance of an even contest between advantaged teams. Using the equilibria presented in Table 1, expected contest-level effort under the complete information condition is:

$$[\text{A12}]\quad \frac{1}{2}\left[\frac{c_D v\tilde{N}}{(c_A+c_D)^2}+\frac{c_A v\tilde{N}}{(c_A+c_D)^2}\right]+\frac{1}{4}\left(\frac{v\tilde{N}}{4c_A}+\frac{v\tilde{N}}{4c_A}\right)+\frac{1}{4}\left(\frac{v\tilde{N}}{4c_D}+\frac{v\tilde{N}}{4c_D}\right).$$

Rearranging terms,

$$[\text{A13}]\quad \frac{1}{2}\left[\left(\frac{c_D v\tilde{N}}{(c_A+c_D)^2}\right)+\left(\frac{v\tilde{N}}{4c_A}\right)\right]+\frac{1}{2}\left[\left(\frac{c_A v\tilde{N}}{(c_A+c_D)^2}\right)+\left(\frac{v\tilde{N}}{4c_D}\right)\right].$$

Simplifying further and combining terms,

$$[\text{A14}]\quad \frac{v\tilde{N}}{2}\left[\frac{4c_Ac_D+(c_A+c_D)^2}{4c_A(c_A+c_D)^2}\right]+\frac{v\tilde{N}}{2}\left[\frac{4c_Ac_D+(c_A+c_D)^2}{4c_D(c_A+c_D)^2}\right].$$

Last, multiplying the numerator and denominator of both bracketed terms by $1/c_D^2$, and simplifying, yields:

$$[\text{A15}]\quad \frac{v\tilde{N}}{c_A}\left[\frac{4\frac{c_A}{c_D}+\left(1+\frac{c_A}{c_D}\right)^2}{8\left(1+\frac{c_A}{c_D}\right)^2}\right]+\frac{v\tilde{N}}{c_D}\left[\frac{4\frac{c_A}{c_D}+\left(1+\frac{c_A}{c_D}\right)^2}{8\left(1+\frac{c_A}{c_D}\right)^2}\right].$$

Under incomplete information, expected contest-level effort is simply $X_A^{**}+X_D^{**}$ as in expectation the contest includes one advantaged and one disadvantaged team. From Table 2, one can easily verify that the first and second terms in [A15] are the expected effort for advantaged and disadvantaged teams, respectively, for an incomplete information contest.

General solution for group contest with incomplete information

Below we derive the closed-form solution for the case of cost-of-effort heterogeneity based on the within-group altruism model. Other cases follow in a similar fashion. First, beginning with the first order condition defined by equation [9], if $g = D$, $N_A = N_D = N$, and $v_A = v_D = v$, then

$$[\text{A16}]\quad \{(1-r)(X_D+X_A)^2+4rX_AX_D\}v\tilde{N} = 4X_Dc_D(X_D+X_A)^2.$$

In a similar vein, if $g = A$, $N_A = N_D = N$, and $v_A = v_D = v$ it follows that

[A17] $\{4(1-r)X_AX_D + r(X_D+X_A)^2\}v\tilde{N} = 4X_Ac_A(X_D+X_A)^2$.

This gives us two equations and two unknowns. Dividing [A12] by [A13], and rearranging yields:

[A18] $X_A = \frac{(1-r)}{r}\frac{c_D}{c_A}X_D - \frac{(1-2r)}{r}\frac{v\tilde{N}}{4c_A}$.

In the special case of $r = \frac{1}{2}$, the second term equals 0 and this yields the simple relationship $X_A = \frac{c_D}{c_A}X_D$. For convenience, let $\beta = \frac{(1-r)}{r}\frac{c_D}{c_A}$ and $\alpha = \frac{(2r-1)}{r}\frac{v\tilde{N}}{4c_A}$, in which case [A18] can be written as

[A19] $X_A = \beta X_D + \alpha$.

Now, substitute [A19] into [A17] to eliminate X_A:

[A20] $\{(1-r)(X_D(\beta+1) + \alpha)^2 + 4r(\beta X_D + \alpha)X_D\}v\tilde{N} = 4X_Dc_D(X_D(\beta+1) + \alpha)^2$.

Rearranging and combining terms in [A20], we obtain the following cubic equation

[A21] $aX_D^3 + bX_D^2 + cX_D + d = 0$,

where, $a = c_D(\beta+1)^2$, $b = \left(2c_D(\beta+1)\alpha - v\tilde{N}r\beta - \frac{1}{4}v\tilde{N}(1-r)(\beta+1)^2\right)$, $c = \left(c_D\alpha^2 - v\tilde{N}r\alpha - \frac{1}{2}v\tilde{N}(1-r)(\beta+1)\alpha\right)$ and $d = -\frac{1}{4}v\tilde{N}(1-r)\alpha^2$. Last, dividing through by the coefficient a yields

[A22] $X_D^3 + a_1X_D^2 + a_2X_D + a_3 = 0$,

where $a_1 = \frac{b}{a}$, $a_2 = \frac{c}{a}$ and $a_3 = \frac{d}{a}$. Applying established methods for solving a cubic equation (i.e., using a variant of Cardano's formula), the equation [A22] has three real roots when $r \neq \frac{1}{2}$. The one root that satisfies the first-order condition of the maximization problem is:

[A23] $X_D = 2\sqrt{-Q}\cos\left(\frac{1}{3}\theta\right) - \frac{1}{3}a_1$ and $X_A = \beta\left(2\sqrt{-Q}\cos\frac{1}{3}\theta - \frac{1}{3}a_1\right) + \alpha$

where, $Q = \frac{3a_2 - a_1^2}{9}$, $R = \frac{9a_1a_2 - 27a_3 - 2a_1^3}{54}$, and $\theta = \arccos\left(\frac{R}{\sqrt{-Q^3}}\right)$. In the case of $r = \frac{1}{2}$, there are two real roots, but only one of them is non-zero. The solution in this case is:

[A24] $X_D = 2R^{1/3} - \frac{1}{3}a_1$ and $X_A = \beta\left(2R^{1/3} - \frac{1}{3}a_1\right) + \alpha$.

Here, $R^{1/3} = -a_1/3$, and it follows that $X_D = -a_1$ which simplifies to the formulas presented in Table 2.

Support of Propositions 1, 2 and 3 for $0 < r < 1$

As illustrated in Tables 1 and 2, under cost heterogeneity, the solutions for both the complete and incomplete information settings can be written as $X_g^{**} = v\widetilde{N} \cdot f_g$, where the function f_g is not a function of the altruism, group size and prize value parameters. As a result, these parameters do not independently determine differences in effort across the information conditions. This remains true in the general case.[20] As such, any differences based on information condition depend on the extent of the cost advantage and r. Without loss of generality, we can normalize $c_D \equiv 1$ in which case $0 < c_A < 1$ and the size of the advantage is decreasing in c_A. It then suffices to show that the propositions hold for all possible combinations of c_A and r.

Presented as Figures A1 to A3 are surface plots, for specific contest types, of contest-level effort in the incomplete information contest minus the contest-level effort in the complete information contest. These are based on $\widetilde{N} = 3$ and $v = 50$. Figure A1 corresponds to uneven contests, and is thus relevant for Proposition 1. The effort difference is always positive, and is

[20] To see this, note that we can write $Q = (v\widetilde{N})^2 \cdot f_1$, $a_1 = v\widetilde{N} \cdot f_2$, and $R = \left(v\widetilde{N}\right)^3 \cdot f_3$, where f_1, f_2, and f_3 are functions that do not contain v or $\widetilde{N}$. Then, [A23] becomes $X_D = v\widetilde{N}\left(2\sqrt{-f_1}\cos\left(\frac{1}{3}\arccos\left(\frac{f_3}{\sqrt{-f_1^3}}\right)\right) - \frac{1}{3}f_2\right)$.

strictly increasing in both the size of the cost advantage and the probability the opponent is an advantaged team. Figures A2 and A3 correspond to even contests between advantaged and disadvantaged teams, respectively. Confirming Proposition 2, contest-level effort is strictly higher under complete information. For a contest between advantaged teams, this difference goes to zero as $r \to 1$, as expected, as in this limit the contest is a complete information contest between advantaged teams. The effect of information is maximal when both r and c_A approach zero. For a contest between disadvantaged teams, this difference goes to zero as $r \to 0$, as this converges to a certain contest between two disadvantaged teams. The effect of information is maximal when both r approaches 1 and c_A approaches zero. Effort differences are relatively larger for even contests that involve two advantaged teams.

Figure A1. Differences in contest-level efforts based on information condition: *uneven* contests

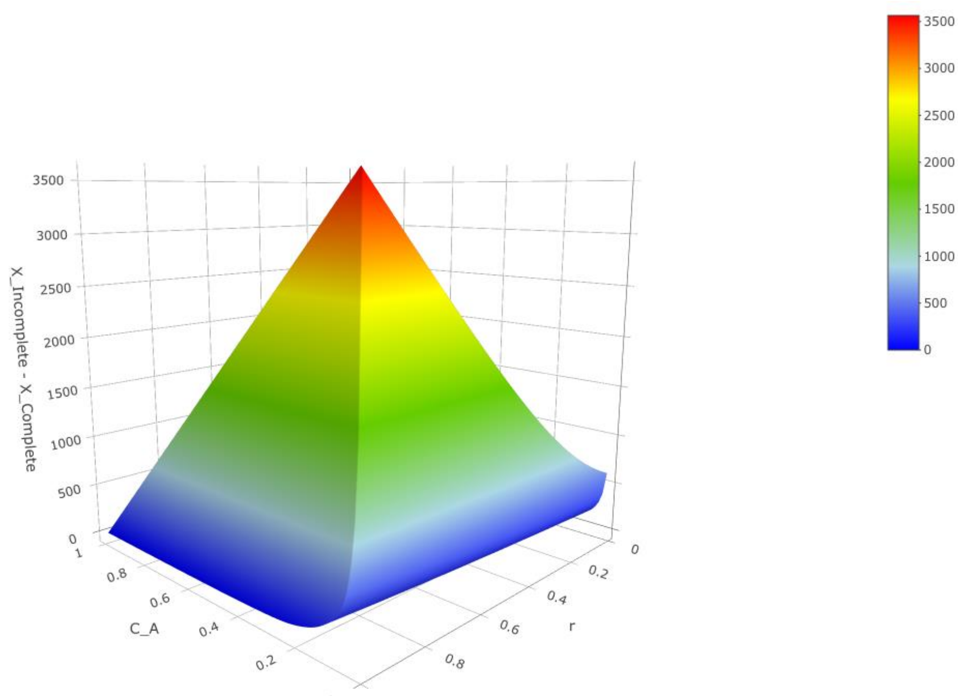


Figure A2. Differences in contest-level efforts based on information condition: *even* contests between advantaged teams

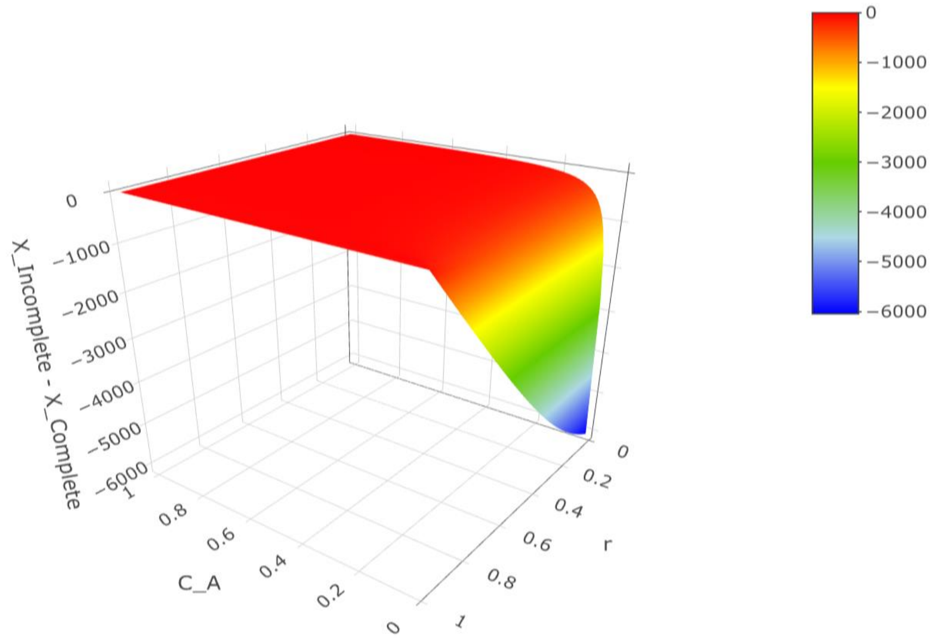


Figure A3. Differences in contest-level efforts based on information condition: *even* contests between disadvantaged teams

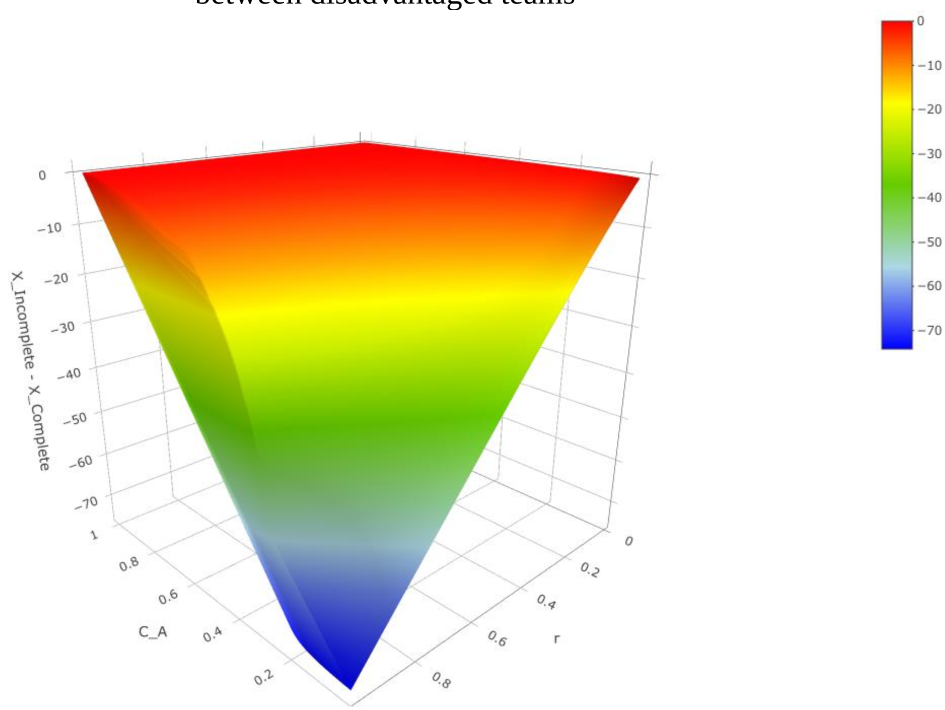
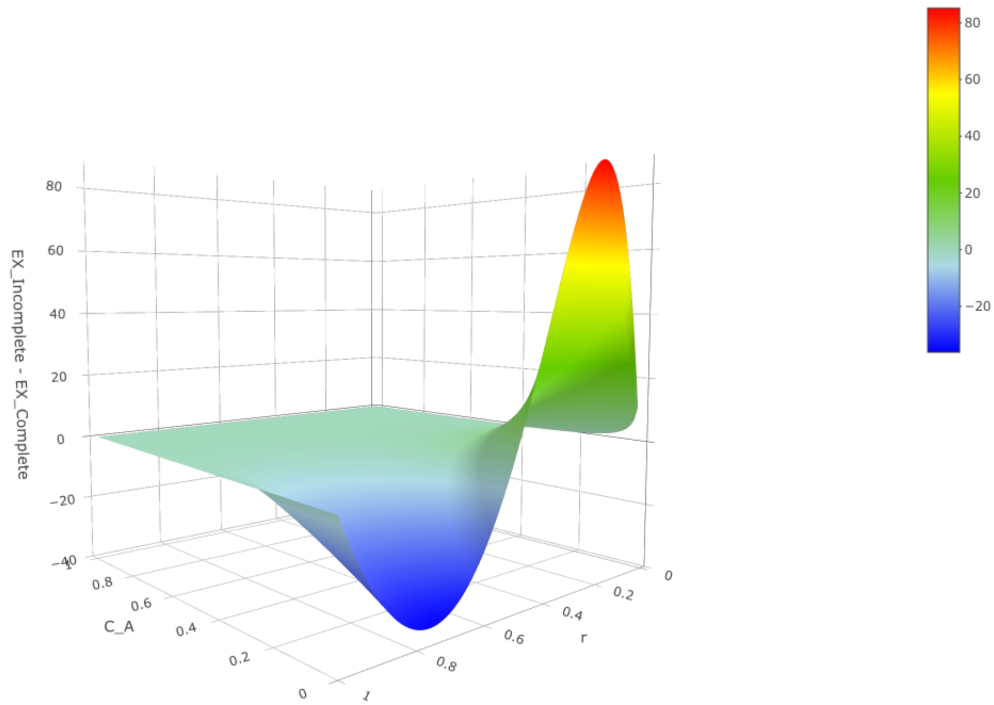


Figure A4 depicts differences in expected contest-level effort between the two information conditions. To be clear, this differs from the information provided in Figures A1 to A3 as effort is *un*conditional on contest type (e.g., even or uneven). When $r = 1/2$, there is no difference in contest-level effort as proven analytically. As r deviates from this value, differences in expected effort arise due to information conditions but in general these differences are small when compared with the differences that arise from uneven contests and even contests between advantaged teams. The largest differences occur when $c_A \rightarrow 0$.

Deviating from $r = 1/2$ in either direction increases the probability of an even contest, and from the prior results specific to contest types this would suggest expected contest-level effort would be higher with complete information. However, there turns out to be an asymmetry which is largely due to the fact that effort in an even contest between advantaged teams is considerably higher with complete information (see Figure A2), but the information effect is relatively small for an even contest between disadvantaged teams (see Figure A3). As a result, when $r > 1/2$ and it becomes more likely that an even contest between advantaged teams will occur, overall effort is higher with complete information. On the other hand, when $r < 1/2$ and it becomes more likely that an even contest between two disadvantaged teams will occur, expected effort is higher with incomplete information. Holding c_A fixed, the largest differences do not necessarily occur as r approaches 1 or 0 as there are competing effects. For instance, with $r > 1/2$, while increasing r does increase the chance of an even contest between advantaged teams, as a countervailing effect the difference in effort for an uneven contest under incomplete versus complete information is also increasing with r.

Figure A4. Differences in expected contest-level efforts between incomplete and complete information conditions



B. Additional econometric analysis

Table A1. Analysis of information effects: *uneven* contests, restricted sample

Dependent variable: Group Effort			
	(1)	(2)	(3)
Value		1.30	1.115
		(3.22)	(3.05)
Group		32.79***	27.10***
		(4.38)	(4.36)
Cost × Incomplete		20.99***	20.32***
		(3.24)	(3.07)
Value × Incomplete		19.35***	19.20***
		(3.87)	(3.51)
Group × Incomplete		-4.17	0.96
		(5.22)	(4.55)
Incomplete	13.48***		
	(2.55)		
Experience			-18.35***
			(4.53)
Risk Averse			-5.26
			(4.56)
Female			9.19*
			(4.87)
Round			-1.325***
			(0.19)
Constant	56.80***	47.57***	71.55***
	(1.88)	(2.34)	(5.05)
Observations	904	904	904
R-squared	0.025	0.081	0.133

Notes: Cluster-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Estimation sample excludes observations from incomplete information treatments associated with even contests.

Table A2. Analysis of information effects: *even* contests, restricted sample

Dependent variable: Group Effort			
	(1)	(2)	(3)
Value		-2.27	-3.54
		(4.74)	(4.570)
Group		11.12*	6.84
		(5.93)	(5.94)
Cost × Incomplete		3.05	3.80
		(4.755)	(4.46)
Value × Incomplete		6.27	6.38
		(4.07)	(3.91)
Group × Incomplete		-10.81	-7.86
		(6.57)	(6.54)
Incomplete	-0.14		
	(2.96)		
Experience			-17.86***
			(3.78)
Risk Averse			-7.79**
			(3.92)
Female			7.40*
			(3.80)
Round			-1.02***
			(0.24)
Constant	66.52***	63.77***	86.55***
	(2.21)	(3.79)	(5.67)
Observations	1,084	1,084	1,084
R-squared	0.000	0.011	0.055

Notes: Cluster-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Estimation sample excludes observations from incomplete information treatments associated with uneven contests.

Table A3. Analysis of information effects: *even* contests, advantaged groups only

	Dependent variable: Group Effort		
	(1)	(2)	(3)
Value		-3.95	-5.16
		(6.52)	(6.21)
Group		47.70***	43.26***
		(8.27)	(8.17)
Cost × Incomplete		8.47	7.62
		(6.09)	(5.81)
Value × Incomplete		8.00**	8.12**
		(4.04)	(3.645)
Group × Incomplete		-10.01	-6.84
		(7.05)	(6.74)
Incomplete	3.93		
	(3.76)		
Experience			-23.80***
			(4.015)
Risk Averse			-7.05
			(4.44)
Female			13.47***
			(4.56)
Round			-1.093***
			(0.22)
Constant	89.61***	80.80***	105.3***
	(3.35)	(5.88)	(6.95)
Observations	729	729	729
R-squared	0.002	0.187	0.259

Notes: Cluster-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Estimation sample excludes observations from incomplete information treatments associated with uneven contests.

C. Experiment Instructions: Cost treatment with incomplete information

Thank you for participating in today's study. Please follow the instructions carefully. At any time, please feel free to raise your hand if you have a question.

You have been randomly assigned an ID number for this session. You will make decisions using a computer. You will never be asked to reveal your identity to anyone. Your name will never be associated with any of your decisions. In order to keep your decisions private, please do not reveal your choices or otherwise communicate with any other participant. Importantly, please refrain from verbally reacting to events that occur.

Today's session has three parts: Experiment 1, Experiment 2, and a short questionnaire. You will have the opportunity to earn money in both experiments based on your decisions. You will be paid your earnings privately, and in cash, at the end of the experiment session. We will proceed through the written materials together. Please do not enter any decisions on the computer until instructed to do so.

Are there any questions before we begin?

Please go ahead and click "Continue" to enter the experiment.

Experiment 1

Please click "Continue" and refer to your computer screen while we read the instructions.

We would like you to make a decision for each of 10 scenarios. Each scenario involves a choice between playing a lottery that pays $4 or $0 according to specified chances (Option A) or receiving $2 for sure (Option B).

You will notice that the only differences across scenarios are the chances of receiving the high or low prize for the lottery. At the end of the today's session, **ONE** of the 10 scenarios will be selected at random and you will be paid according to your decision for this selected scenario **ONLY**. Each scenario has an equal chance of being selected.

Please consider your choice for each scenario carefully. Since you do not know which scenario will be played out, it is in your best interest to treat each scenario as if it will be the one used to determine your earnings.

Before making decisions, are there any questions?

Once you are ready to submit your decisions, please click the "Submit" button.

Experiment 2

In this experiment, all money amounts are denominated in lab dollars, and will be exchanged at a rate of **90** lab dollars to 1 US dollar at the end of the experiment.

There will be many decision rounds in the experiment. You will not know the number of rounds until the experiment has been completed. Each decision round is separate from the other rounds, in the sense that the decisions you make in one round will not affect the outcome or earnings of any other round.

In each round, participants will be randomly placed into three-person groups.

In each decision round, your group will compete with one other group to determine which group wins a **prize** of 150 lab dollars. This prize will be evenly divided among all group members. If your group wins the prize, you will personally receive 150/3 or 50 lab dollars.

Your task in each decision round is to **decide how many points to contribute towards a group project.** Which group wins the prize depends upon the total contributions from your group *relative* to the total contributions of the opponent group. The chance your group wins the prize is determined by the following formula:

$$\text{Chance of winning} = \frac{\text{Total contributions (Your group)}}{\text{Total contributions (Your group)} + \text{Total contributions (Opponent)}} \times 100\%$$

Using this formula:

- If the total contributions from both groups are *equal*, then both groups have an *equal* chance of winning the prize; i.e., the chance each group wins the prize is 50%.
- If your group contributes *more* than your opponent, then your group has a higher chance of winning the prize. For example, if your group contributes twice as much, the chance your group wins the prize is 2 in 3 or 66.7%.
- If your group contributes *less* than your opponent, then your group has a lower chance of winning the prize. For example, if the opponent group contributes four times as much as your group, your group has a 1 in 5 or 20% chance to win.

You can contribute anywhere from **0 to 50 points** (only in integer amounts) towards the group project.

While increasing contributions will increase the chance your team wins the prize, contributing points costs money. In particular, each point you contribute is associated with a per-point **contribution cost**.

The per-point **contribution cost** can have two values: either 1/3 of a lab dollar or 1 lab dollar. You will know the contribution cost when deciding.

In each round, you will receive 50 lab dollars in **fixed income.** This amount does not depend on your decision or whether your group wins this prize. Your earnings for the decision round will be calculated as follows:

IF your group wins…	Your Earnings = 100 – (points YOU contributed * contribution cost)
IF your group does not win…	Your Earnings = 50 – (points YOU contributed * contribution cost)

Before we continue, are there any questions?

<u>Instructions quiz</u>

At this time, we would like you to answer a few questions to help you understand how the experiment works. The good news is that you will be paid for correct answers. You may wish to first answer these using pen and paper. When you are ready, please read the instructions on your computer carefully, and click "I understand, Continue to Quiz" to submit your answers on the computer. If you have a question when working through the quiz, please raise your hand and your question will be answered privately.

1. Suppose the contribution cost is 1/3 of a lab dollar per point. You contribute 18 points. Your group <u>wins</u> the prize. How much money would you earn for this decision round (in lab dollars)?
   a. 27 b. 44 c. 70 d. 94

2. If your group contributes a total of 60 points and the opponent group contributes a total of 100 points, what is the chance <u>your group</u> wins the prize?
   a. 62.5% b. 37.5% c. 0% d. 50%

3. Suppose the contribution cost is 1 lab dollar per point. You contribute 40 points, and the total contributions from your group (including your own) are 50 points. Your group <u>does not win</u> the prize. How much money would <u>you</u> earn for this decision round (in lab dollars)?
   a. 30 b. –40 c. 10 d. 0

4. Suppose the other two members of your group contribute a total of 20 points. The opponent group contributes 20 points. Therefore, if you contribute nothing your group has a 50% chance of winning. By how much would you <u>increase</u> the chance your group wins if you contribute 10 points instead of contribute nothing?
   a. 0% b. 5% c. 60% d. 10%

Proceeding through the experiment

At the start of each round, you will be randomly matched into a group of three players. Your group will then be randomly matched with another group. This means that both the members of your own group as well as the members of the opponent group will vary from one round to the next.

At the start of each round, the computer will randomly determine the contribution cost for each group. Both groups will each have a 50% chance of facing the low or high contribution cost. This random determination is done independently for each group, which means that in some rounds your contribution cost will be the same as your opponent, and in other rounds it will be different. In particular:

- There will be a 25% chance that both your group and the group you are competing with have a low contribution cost (1/3 of a lab dollar);
- There will be a 25% chance that both groups have a high contribution cost (1 lab dollar); and,
- There will be a 50% chance that one group will have a low cost while the other has a high cost.

You will always know the contribution cost for your group. Throughout the experiment, however, you will not know the contribution cost for the opponent group.

Note: In the corresponding complete information treatment, the above two sentences are replaced with: "You will always know the contribution cost for your group and the opponent group."

Your **decision screen** will include relevant information for both your own group and the opponent group. Know that the prize value and group size will never change during the experiment.

At the end of each decision round you will be shown a **result screen** with the contest result, the total points contributed by all your group members, and your earnings.

We will begin with a training round to help you understand the procedures.

Aside from decisions in this training round, you will be paid based on the outcome of each decision round. This means that it is very important to consider each decision prior to making it.

Before we continue, do you have any questions?

D. Post-experiment questionnaire (computerized)

Part 1: About the Experiment

We would now like for you to complete a short questionnaire. Please know that all responses will be treated as strictly confidential and will be used for statistical purposes only. The first questions relate to your experience in today's experiment.

1. Have you previously participated in a paid study that took place in an experimental economics laboratory?
a. Yes b. No

2. Please indicate your level of agreement with the following statement: “I understood well the instructions for Experiment 2.”
1 - Strongly Disagree; 2 – Disagree; 3 – Neutral; 4 – Agree; 5 - Strongly Agree

3. Please indicate your level of agreement with the following statement: "I was well compensated for my participation in this study.”
1 - Strongly Disagree; 2 – Disagree; 3 – Neutral; 4 – Agree; 5 - Strongly Agree

4. In the past twelve months, approximately how much money (cash, check, credit card, etc.) did you donate to a charity or non-profit organization?

5. In the past twelve months, what is the approximate fair market value of non-cash property (clothing, appliances, etc.) you donated to a charity or non-profit organization?

6. In the past twelve months, approximately how many hours did you spend doing volunteer work for a charity or non-profit organization?

7. Many classes at the University of Tennessee require students to work on assignments in groups. In these settings, do you usually contribute less, about the same, or more than other people in your group?
a. Less b. About the same c. More

Please use the following space to write any comments (positive or negative) you may have about the experiment.

Part 2: Demographics

The next questions tell us something about you.
1. What is your age?
2. How do you describe yourself?
a. Male b. Female c. Transgender d. Do not identify myself as female, male, or transgender
3. What is your academic major?
4. What is your current student classification?
a. Freshman b. Sophomore c. Junior d. Senior e. Master’s Student f. Law Student g. Doctoral Student h. Other
5. What was your student status for the Spring 2019 semester?

a. Full-time student b. Part-time student c. Not a student
6. In what range is your cumulative GPA?
a. 0 to 2.0 b. 2.1 to 2. c. 2.6 to 3.0 d. 3.1 to 3.5 e. 3.6 to 4.0
7. How many economics courses have you completed at the university level?

8. How would you best describe your current employment status?
a. Employed Full-Time b. Employed Part-Time c. Self-Employed Full-Time d. Self-Employed Part-Time e. Unemployed

Part 3: Personality

Here are a number of personality traits that may or may not apply to you. Please write a number next to each statement to indicate the extent to which you agree or disagree with that statement. You should rate the extent to which the pair of traits applies to you, even if one characteristic applies more strongly than the other. All questions below are to be rated from 1-7. 1 represents strongly disagree and 7 represents strongly agree.

I see myself as:
a. Extroverted, enthusiastic
b. Critical, quarrelsome
c. Dependable, self-disciplined
d. Anxious, easily upset
e. Open to new experiences, complex
f. Reserved, quiet
g. Sympathetic, warm
h. Disorganized, careless
i. Calm, emotionally stable
j. Conventional, uncreative