Motivating Workers through Task Assignment: A Dynamic Model of Up-and-Down Competition for Status

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Abstract

We show how competition for status that conveys explicit benefits can motivate effort in organizations such as schools, public agencies, and unionized firms in the absence of monetary incentives or intrinsically motivated workers. We develop an indefinitely-repeated labor market tournament model in which high-status agents may be rewarded either monetarily or with favorable task assignment. If monetary incentives are unavailable and the principal relies on task assignment this entails an efficiency cost relative to the benchmark case with monetary incentives. Our model offers a new perspective on the value to an employer of flexibility over job assignments within labor contracts.

Key words: task assignment; status; dynamic tournament; non-wage compensation

JEL codes: C73; J41; L20; M51; M52

1. Introduction

A recent literature has begun to consider the motivation of workers in organizations such as schools, non-profits, and government agencies that are constrained in their ability to employ explicit monetary incentives that tie compensation to performance. Constraints on monetary incentives may also play an important role for firms during periods of stagnation or contraction (such as due to economic downturns) when bonuses cannot be funded and promotion opportunities are limited. Whether monetary rewards exist or are salient, managers are often charged with allocating among workers a set of tasks that inherently vary in their desirability. This task assignment can serve as an important motivational tool by engendering an ongoing competition over status in the organization, and rewarding those with high status with more desirable tasks. For example, school teachers in favor with their principal are likely to get desirable class assignments and be allocated scare resources such as teacher aides and new technology. In a manufacturing plant employees out of favor with their manager may be assigned undesirable weekend or night shifts, be delegated stressful tasks, or be given jobs with higher exposures to health and safety risks. Since both desirable and undesirable tasks must be performed (i.e. the set of tasks is typically fixed) and the principal has some discretion over task assignments among agents, this creates competition for agents not only to gain favor with the principal but also, once status is achieved, to avoid falling out of favor.

In this study we develop an indefinitely-repeated dynamic Markov tournament model of competition for status within the organization. Agents are required to complete contractible tasks (the satisfactory completion of which is observable) and also choose effort that increases output or performance in a non-contractible dimension. Non-contractible effort is not directly observable to the principal. An agent’s utility each period depends on his status, and in each period a fixed number of agents are moved up and moved down in the organization. We explicitly model the possibility that high status may be rewarded non-monetarily through the principal’s discretion over task assignment.

To establish a baseline for comparison, we show that when high status can be rewarded monetarily and discounting is negligible this type of dynamic tournament can function as an efficient mechanism, inducing non-contractible effort without paying rents to agents, so that first-best effort is obtained. We then show how differential assignment of contractible tasks can similarly be employed to motivate non-contractible effort. However, if the agents’ cost of the effort required to complete contractible tasks is a convex function, employing differential task assignment entails an efficiency cost. The outcome is therefore second-best as the principal faces a tradeoff between implementing incentives to induce non-contractible effort, and obtaining completion of contractible tasks at least cost. The second-best tournament using task assignment to reward in-favor workers implements levels of both contractible and non-contractible effort that are suboptimal. Nevertheless, flexibility in assignment of contractible tasks readily enables implementation of non-contractible effort and may permit organizations that are financially constrained to approach an efficient outcome.

Existing dynamic tournament models in the labor literature, beginning with Rosen's (1986) seminal work, have largely focused on elimination tournaments in which competition is "up or out" and the game has a finite number of periods (because a hierarchy is inherently finite). In contrast to elimination tournaments, in our model strong incentives arise from competition for status within an organization without a significant hierarchy.[1] Further, there are two important implications of our model that differ from results based on standard, static tournament models. First, when employing differential task assignment to motivate workers, more intense competition is desirable because it reduces the utility difference in tasks based on status required to motivate any level of effort. Therefore, an increase in the variance of the random shocks (which dulls the competitive incentive) is costly to the principal. In the standard Lazear and Rosen (1981) tournament framework a greater variance in random shocks can be offset by an increased prize spread to maintain the effort incentive at no cost to the principal (assuming agents are risk neutral). Second, in standard tournament models efficient incentives can be implemented with any number of winners. Conversely, in our dynamic mechanism the number of winners/losers (which is the number of players transitioned between status groups each period), impacts marginal incentives and the agents' binding participation constraint. Out-of-favor players won't compete if the chance of winning and gaining favored status is too low to make the expected future benefits justify the effort. As a result the optimal number of winners, which determines the persistence of status, depends in particular on the players' discount rate.

[1] The basic structure of our model follows from the indefinite Markov chain models employed in the dynamic regulatory enforcement literature (e.g. Landsberger and Meilijson, 1982; Greenberg, 1984; Harrington, 1988; Harford and Harrington, 1991), which includes recent applications involving regulation tournaments (Liu and Neilson, 2013; Gilpatric, Vossler and Liu, 2015). This labor market application is quite distinct as the objectives of labor contracts do not mirror regulations, and in the labor setting agents' participation constraints must be satisfied (unlike regulated firms who are compelled to participate).

In exploring how organizations such as public bureaucracies can motivate workers when monetary incentives are either highly constrained or entirely unavailable, our work is closest to that of Gersbach and Keil (2005), Besley and Ghatak (2005, 2008), and Prendergast (2007). Similarly to this paper, Gersbach and Keil characterize a public organization where the principal does not have direct monetary means to incentivize agents and model the use of competition over task-assignments to motivate agents. However, the contest they model does not motivate effort by agents but rather induces agents to reveal unproductive tasks in their departments. Furthermore task assignment plays a very different role with contest winners being rewarded with more tasks because managers are assumed to seek a larger role in the organization. In our framework tasks are mandated duties which require effort to complete. Thus Gersbach and Keil show how organizations can motivate managers to streamline an organization by revealing hidden opportunities for cost-cutting. Our model applies to very different settings.

Prendergast (2007) notes that employees, and particularly public bureaucrats, often have “no monetary reason for doing what they do” and that their willingness to exert effort is often attributed to intrinsic motivation. In the presence of such “motivated agents” he shows why it may nevertheless be optimal for a principal to employ agents who are biased in the sense of not sharing the preferences of the principal. Besley and Ghatak (2005) consider the importance for many organizations of employing agents who subscribe to the mission of their employer. By matching the mission preferences of the principal and agent the need for high-powered (monetary) incentives is reduced. This work on agents who are intrinsically motivated by their fit to the organizational mission suggests that agents’ utility may vary depending on their assigned tasks, which may align more or less well with their own preferences. This is one channel consistent with our model through which task assignment may be employed as an incentive mechanism.

The absence of monetary incentives does not of course rule out other mechanisms that may be used to motivate workers, and our model contributes to the nascent literature on such non-monetary mechanisms. Besley and Ghatak (2008) consider how non-monetary “status” incentives can be employed to motivate workers. Status in their context is conveyed by job title (e.g. “employee of the month” or “vice president”) which does not carry with it higher compensation nor any other explicit reward, but does convey high achievement to other agents within the organization. The authors show how competition for status can motivate workers who obtain utility from their perceived positional ranking within the organization. Our model is similar insofar as agents are motivated by competition for status. However, agents in our framework do not value status for its own sake. Rather, our emphasis is on how task-assignment can be employed to make status within the organization quite consequential, and the trade-offs present when this mechanism is used in lieu of monetary incentives. Moldovanu, Sela, and Shi (2007) also model competition for status within an organization, with a focus on the optimal number of status classes. A key aspect of their framework is that agents differ in ability, and rank achieved in competition is therefore a signal to other agents of relative ability and valued for that reason. Our framework differs from both of these prior works in considering an indefinitely repeated game where status is temporary but has direct, explicit consequences regarding task assignment, and competition arises from both the desire to achieve high status and to retain it once achieved.

By demonstrating how competition for status which determines task assignments can be employed as an incentive mechanism, our model offers a new perspective on the value to an employer of flexibility over job assignments within labor contracts. An employer is likely to value such flexibility for many reasons, such as being better able to adapt to changing technology or market conditions. But our model illustrates that contractual flexibility that gives a manager significant discretion over employees' task assignments yields an important motivational tool that can elicit greater effort. In this context, it is not surprising that unions may resist such flexibility, or demand compensation for it, in labor negotiations.[2] Workers will recognize that if they accept a contract that permits greater discretion in their assignments, this can compel them to exert more costly effort in the ongoing competition for status. Indeed, such negotiations have occurred between UPS and the Teamsters Union. A clause in one of their contracts reads: "Job reassignments will be on an as-needed basis only, due to reduction or transfer of the work. Seniority will be recognized in all job reassignments" (Teamsters Local 150, 2014). Further, the

[2] Anecdotal evidence supports this: "Unions typically direct their job-description efforts toward setting defined boundaries for positions, usually wanting to define the work that employees can perform within specific job classifications" (Joinson, 2001). It is also possible to think of task assignment in a broader sense that includes schedule flexibility; a particularly undesirable task could be one that needs to be completed at night or on a holiday (e.g. teaching an evening class). Zeytinolgu (2005) states, "… traditional union preference [includes] regularity of work and/or skepticism regarding flexible scheduling, which they tend to view as a risk for losing control to employers." Further, a recent contract negotiated by the teacher's union in New York highlights how the city hopes to achieve higher quality in education by providing flexibility to schools in terms of work rules and length of school day (The New York Times Editorial Board, 2014). This could be explained in part by the principals' ability to motivate more effort from teachers when they have more control.

Union states that one of their goals is "stronger language that strengthens the rights of… workers to bid on overtime and job assignments" (Teamsters for a Democratic Union, 2013). It is evident that the balance of control over scheduling and task assignments between the employer and union is a key issue in these labor negotiations.

The relevance of our model for government agencies is potentially quite large. Several pertinent aspects of government culture, as characterized by Wilson (1989) in his text on bureaucracy, suggests that managers have limited ability to use monetary incentives to motivate effort. He makes clear that as an alternative managers can "give people attractive or miserable job assignments" (p. 156). In certain branches of government, including the military, the goal is not only to use job assignment as a reward mechanism, but also to provide equal opportunities to all officers by rotating them, even though this may be disadvantageous from the standpoint of developing expertise in a particular area. This demonstrates that task assignments are not static in many agencies and further that organizations may have reasons (outside our model) for keeping task assignments in flux. This could suggest that the efficiency cost of using our mechanism may not be too great. Finally, government agencies "are often prepared to accept less money with greater control than more money with less control" (p. 179). This mentality favors incentive mechanisms which rely on operational control, like task assignments, instead of money.

The present work significantly complements work on public organizations (e.g. Heinrich and Marschke, 2010; Perry and Porter, 1982), which emphasizes that a manager's ability to use monetary incentives in a public organization (such as a school) is limited. One potential non-monetary incentive is public recognition. According to Heinrich and Marschke (2010), there is some evidence that employees in the public sector may be particularly motivated by public recognition relative to monetary compensation. Our model can be applied to this context in that certain tasks or projects may be more likely to result in public recognition and are therefore more desirable.

Finally, our model is applicable to sports leagues and businesses. The competition we model has a parallel in many sports leagues worldwide, such as the English football league, which employ a system of promotion and relegation in which a fixed number of the lowest performing teams in the top league are demoted at the end of a season while the highest performing teams in the second-tier league are promoted. For a business, during a transition from a period of rapid growth to being more "mature" and stable in size, it may move from having many opportunities for promotion (and thus more traditional tournament rewards) to having a much larger role played by task assignments. Similarly, an organization that has money for bonuses or salary increases during good times may institute a pay freeze or eliminate the bonus pool during a downturn, thus forcing the organization to rely on non-monetary incentives.

2. A Dynamic Markov Tournament with Task Incentives

Our model consists of a firm or other organization whose objective is to maximize the value of expected output of its workforce net of labor costs, subject to participation and incentive-compatibility constraints. There are two dimensions of labor effort, one of which is contractible and one of which is non-contractible. The completion of contractible tasks is observable by the principal, and verifiable by a third party; therefore, employees who do not fulfill the task requirements can be denied compensation. On the other hand, non-contractible effort cannot be directly observed by the principal or verified by a third party. This effort contributes to the production of a valuable output, but as in a standard tournament model, the output is subject to random shocks.

As an example, consider a school principal who must ensure all classes are taught each semester (among many required tasks). Whether a teacher is present for classes and meets certain basic requirements of teaching is readily observable, failure to do so can be demonstrated to a third party, and thus these are contractible tasks. However, the teacher's output (student learning) depends on unobservable effort which may interact in a complex manner with other inputs and random factors. Although student learning may be assessed from student evaluations, test scores, peer evaluations, etc., these assessments are subject to many factors and random shocks and thus imperfectly map to effort, making it impossible to directly contract over effort of this type. We model how the principal can use a dynamic tournament to induce this type of non-contractible effort from agents, even if all agents receive equal payment every period.[3]

While the principal seeks to maximize net benefit, he faces two constraints. First, the agents must agree to enter into the contract and thus participate in the tournament. Agents are assumed to have an outside option and the present value of their expected utility from

[3] Of course teacher effort may be motivated by monetary rewards, as with bonuses tied to "high stakes" assessments. But such incentive schemes are often meet with strong resistance from teachers, and implementation may entail very high administrative and other costs.

employment under the tournament at any point in time must be greater than their reservation utility. Second, it must be incentive compatible for the agents to provide the level of non-contractible effort the principal seeks, which in our context requires that the desired effort is the Nash equilibrium of the dynamic tournament. In this section we will demonstrate how non-contractible effort can be implemented in a dynamic tournament satisfying these participation and incentive-compatibility constraints by employing differential monetary compensation and/or differential task assignment conditional on agents' status, or group. We will show later the efficiency consequences of employing task assignment rather than money to motivate effort in this context.

Our framework consists of N risk-neutral agents who are sorted into high status (G_1) and low status (G_2) groups, where agents in the high status group are better off. [4] The game is indefinitely repeated with discount factor δ. In each period, each agent chooses non-contractible effort μ at cost $c(\mu)$, where $c' > 0$, $c'' > 0$. The agent's output from non-contractible effort is given by y, which is the sum of effort and a random component: $y = \mu + \varepsilon$ (the random component is an *i.i.d.* draw from the distribution H across agents and periods). We utilize the following additional notation:

f_1, f_2	fixed payment to an agent in a period, conditional on group assignment
n_1, n_2	number of agents in G_1 and G_2
τ	number of agents in each period transitioned from G_1 to G_2 and vice versa
A	value of output from non-contractible effort (common to both groups)
B	value of output from contractible effort
e_1, e_2	contractible effort required of each agent in G_1 and G_2 in each period
E	total contractible effort required to complete all necessary tasks in each period, such that $n_1 e_1 + n_2 e_2 = E$
$z(e)$	cost of contractible effort for the agents, where $z' > 0$, $z'' > 0$

We assume the principal can commit to group sizes, monetary compensation, and contractible effort in each group. This parallels the assumption in standard tournament models that the principal can commit to prize levels (and has no incentive to award the top prize to anyone other

[4] Both Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) discuss risk-averse contestants. As for other incentive mechanisms, risk aversion among agents implies only a second-best outcome can be achieved with a tournament because motivating effort is traded-off against agents' exposure to risk. This result applies in the present context as well.

than the agent with the highest output). The commitment may in some cases be achieved through reputation. As we have discussed, union contracts may limit employer discretion over task assignment, which can be thought of as a commitment regarding differentiation of contractible effort.

We will first model the dynamic competition which determines non-contractible effort under the assumption that the participation constraint is satisfied. In each period, a separate tournament occurs within each group, with transitions across groups resulting only from tournament outcomes. Specifically, the τ agents with the lowest output in G_1 are placed in G_2 the following period, while the other agents remain in G_1. Similarly, the τ agents with the highest output in G_2 are placed in G_1 the following period, while the other agents remain in G_2. Both tournaments are standard symmetric multiplayer rank-order tournaments with properties characterized by Nalebuff and Stiglitz (1983) on which we will draw.

The principal can motivate agents using differential task assignment between the high and low status groups by setting $e_1 < e_2$ and/or by varying the monetary payment to agents in each group $f_1 > f_2$. The prize at stake in the tournament is the utility difference an agent experiences from being in the high status group, either due to the higher monetary payment or the assignment of less burdensome contractible tasks. The current-period payoff to an agent in the high status group is $\pi_{1,it} = f_1 - c(\mu_{it}) - z(e_1)$, and analogously for an agent in the low status group. Let the probability that an agent in G_1 ranks among the bottom τ agents (and therefore gets transitioned to G_2) be represented by $Q_i(\mu_i, \mu_{-i})$ and the probability that an agent in G_2 ranks among the top τ agents (and therefore gets transitioned to G_1) be represented by $R_j(\mu_j, \mu_{-j})$.

Applying a result from Nalebuff and Stiglitz (1983), the probability that an agent in G_1 who chooses effort μ_i when the other agents in G_1 choose μ_{-i} ranks in exactly the *k*th position up from the bottom (i.e. *k*=1 denotes ranking last and so on) is the following.

$$Q_{ik}(\mu_i, \mu_{-i}) = \int \frac{(n_1 - 1)!}{(n_1 - k)!\,(k-1)!} h(\varepsilon_i)\big(H(\varepsilon_i + \mu_i - \mu_{-i})\big)^{k-1}\big(1 - H(\varepsilon_i + \mu_i - \mu_{-i})\big)^{n_1 - k} d\varepsilon_i$$

The probability that *i* ranks among the bottom τ is then $Q_i(\mu_i, \mu_{-i}) = \sum_{k=1}^{\tau} Q_{ik}(\mu_i, \mu_{-i})$. For identifying the symmetric equilibrium of the tournament, we require the marginal effect of effort on the probability of ranking among the bottom τ be evaluated when $\mu_i = \mu_{-i}$. The marginal effect on the probability of ranking in position k is given below.

$$\frac{\partial Q_{ik}(\mu_i, \mu_{-i})}{\partial \mu_i}|_{\mu_i=\mu_{-i}} = \int \frac{(n_1-1)!}{(n_1-k)!\,(k-1)!}\left(h(\varepsilon_i)\right)^2\left\{\left(1-H(\varepsilon_i)\right)^{n_1-k-1}\left(H(\varepsilon_i)\right)^{k-2}\right\} *$$
$$\left\{(k-1)\left(1-H(\varepsilon_i)\right)-(n_1-k)H(\varepsilon_i)\right\}d\varepsilon_i$$

The effect of effort in symmetric equilibrium on the probability of ranking among the bottom τ is then: $\frac{\partial Q_i(\mu_i,\mu_{-i})}{\partial \mu_i}|_{\mu_i=\mu_{-i}} = \sum_{k=1}^{\tau}\frac{\partial Q_{ik}(\mu_i,\mu_{-i})}{\partial \mu_i}|_{\mu_i=\mu_{-i}}$. The G_2 tournament is directly analogous except that effort increases the probability of an agent's ranking among the top τ in the group. In the special case that group sizes are equal (i.e. $n_1 = n_2$) it follows that $\frac{\partial Q_i(\mu_i,\mu_{-i})}{\partial \mu_i}|_{\mu_i=\mu_{-i}} = -\frac{\partial R_j(\mu_j,\mu_{-j})}{\partial \mu_j}|_{\mu_j=\mu_{-j}}$ because the random component of output is drawn from the same distribution *H* regardless of group assignment, and *H* is symmetric.

The dynamic game follows a Markov chain process where the probabilities of transitioning from group to group are $Q_i(\mu_i, \mu_{-i})$ and $R_j(\mu_j, \mu_{-j})$. For example, the probability that an agent who is in G_1 in period t will remain in G_1 in period $t+1$ is $1 - Q_i$.

Let V_{1t} be the expected present value to an agent of being in group 1 at time t (and analogously for group 2). Then we have:

$$V_{1t} = \pi_{1t} + \delta(1 - Q_{it})V_{1,t+1} + \delta Q_{it}V_{2,t+1}$$
$$V_{2t} = \pi_{2t} + \delta R_{jt}V_{1,t+1} + \delta\left(1 - R_{jt}\right)V_{2,t+1}$$

The expected present value of utility is the sum of utility in the current period and the discounted expected present value of utility starting from the next period, accounting for the probabilities associated with the two possible states the agent may experience in the following period. Applying the ergodic theorem for Markov chains, the optimal strategy for an agent is stationary, i.e. conditioned only on an agent's current state (group), not on the period in the game (Kohlas, 1982; Harrington, 1988). Stationarity allows us to drop the time subscript and we impose symmetric behavior on agents within each group. Thus, we obtain the following first-order conditions:

(1) $$G_1: \frac{\partial \pi_i}{\partial \mu_i} = -c'(\mu) = \delta(V_1 - V_2)\frac{\partial Q_i}{\partial \mu_i}|_{\mu_i=\mu_{-i}} \quad \forall i$$

(2) $$G_2: \frac{\partial \pi_j}{\partial \mu_j} = -c'(\mu) = -\delta(V_1 - V_2)\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}} \quad \forall j$$

where:

(3) $$(V_1 - V_2) = \frac{(\pi_1 - \pi_2)}{1-\delta\left(1-\left(\frac{\tau}{n_1}\right)-\left(\frac{\tau}{n_2}\right)\right)}$$

These equations implicitly define the equilibrium of the dynamic game entailing symmetric behavior by agents (all agents follow identical strategies conditional on their group).[5] Note that maximization of the stage-game payoff yields no effort in this model. The equations above show the incentive arising from the dynamic game which depends on the value of $(V_1 - V_2)$. This difference is the prize at stake in both tournaments. The magnitude of the difference depends on two things: the difference in payoffs, $\pi_1 - \pi_2$; and the equilibrium transition probabilities, which determine the "stickiness" of the states (high or low status).[6] The lower the transition probabilities, the more valuable it is to be in the high status group.

2.1 Optimizing the Dynamic Tournament Labor Contract

The principal's problem is to maximize profit from the agents' effort subject to the constraints. Because the mechanism is stable over time yielding a constant expected payoff each period, the principal maximizes her expected per-period payoff, denoted Π. Recall that the incentive-compatibility constraint is satisfied if the desired effort is incentivized by the tournament. Further, the participation constraint will be slack for agents in the high group when it holds for agents in the low group. We assume the outside option for an agent yields constant utility per period of $\underline{u}$. Then the relevant constraint is $V_2 \geq \frac{u}{1-\delta}$, that is, the present value of the equilibrium payoff stream for an agent in the low group equals the present value of the utility stream from opting out of the tournament. Total benefits to the principal are the sum of the value of contractible and expected non-contractible output, $BE + NA\mu$. (Recall that output, y, is the

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[5] The existence of a pure strategy equilibrium in any rank-order tournament requires sufficient variance of the random component of agents' output. This is required to make the equilibrium effort satisfy general incentive compatibility such that the effort identified by the marginal optimality conditions is not dominated by "opting out" of competition and choosing zero effort. Nalebuff and Stiglitz (1983) discuss this detail. As is standard, we assume this condition is met.

[6] The first-order conditions defining effort in each group (equations 1 and 2) show that effort is increasing in $(V_1 - V_2)$, which is effectively the prize spread. Also, we have $(\pi_1 - \pi_2) = [f_1 - f_2] + [z(e_2) - z(e_1)] + [c(\mu_i) - c(\mu_j)]$; therefore, by looking at equation (3), we can see $(V_1 - V_2)$ increases with $f_1 - f_2$ and with $z(e_2) - z(e_1)$. The denominator of the r.h.s. of (3) is clearly increasing with the number of players transitioned each period, τ, which therefore decreases the prize spread $(V_1 - V_2)$ and reduces equilibrium effort.

sum of non-contractible effort, μ, and an error term, ε; in expectation, y is simply μ.) Therefore, the principal's problem is as follows:

$$\max \Pi = BE + NA\mu - n_1 f_1 - n_2 f_2 \qquad s.t. \quad V_2 \geq \frac{\underline{u}}{1-\delta}$$

The first-best non-contractible effort level for agents in both groups, denoted μ^*, is defined by $c'(\mu^*) = A$. Because the effort of agents in both groups is valued equally, it is optimal to equate the effort incentive across groups which can be achieved by setting $n_1 = n_2$. Assuming group sizes are equated then, as noted earlier, $\frac{\partial Q_i(\mu_i,\mu_{-i})}{\partial \mu_i} = -\frac{\partial R_j(\mu_j,\mu_{-j})}{\partial \mu_j}$ in equilibrium and the effort of all agents can be implicitly stated as:

$$(4) \qquad c'(\mu) = \delta(V_1 - V_2)\frac{\partial R_j(\mu_j,\mu_{-j})}{\partial \mu_j}$$

When agents in both groups choose a common effort level, then $(\pi_1 - \pi_2) = [f_1 - f_2] + [\mathrm{z}(e_2) - z(e_1)]$. The prize spread required to implement a given effort level is found by substituting (3) into (4) and noting that $1 - \left(\frac{\tau}{n_1}\right) - \left(\frac{\tau}{n_2}\right) = 1 - \frac{4\tau}{N}$ when $n_1 = n_2$. This yields

$$(5) \qquad [f_1 - f_2] + [\mathrm{z}(e_2) - z(e_1)] = \frac{c'(\mu)}{\delta \frac{\partial R_j}{\partial \mu_j}|_{\mu_j = \mu_{-j}}}\left(1 - \delta\left(1 - \frac{4\tau}{N}\right)\right).$$

We now identify when the participation constraint binds for agents in G_2 (recall that the participation will be slack for agents in the high group when it holds for agents in the low group). The participation constraint binds when

$$(6) \qquad f_2 = \underline{u} + c(\mu) + z(e_2) - \frac{2\tau}{N} * \frac{c'(\mu)}{\frac{\partial R_j}{\partial \mu_j}|_{\mu_j = \mu_{-j}}}.$$

This constraint shows that ensuring agents do not opt out of the tournament when they are in low status requires that the per-period fixed payment in the low group, f_2, be sufficient to compensate them for foregoing their outside option plus the cost of contractible and non-contractible effort, less the present value of expected future rents that the agent expects to obtain from future periods of being in favor.

Finally, the total cost of compensation in each period is $TC = n_1 f_1 + n_2 f_2 = N f_2 + n(f_1 - f_2)$. We can substitute (5) and (6) into this expression for total cost to find

$$(7) \qquad TC = N\left\{\underline{u} + c(\mu) + z(e_2) - \frac{2\tau}{N} * \frac{c'(\mu)}{\frac{\partial R_j}{\partial \mu_j}|_{\mu_j = \mu_{-j}}}\right\}$$

$$+\frac{N}{2}\left\{\frac{c'(\mu)}{\delta\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}}\left(1-\delta\left(1-\frac{4\tau}{N}\right)\right)-[\mathrm{z}(e_2)-z(e_1)]\right\}.$$

This can be restated as

(7') $$TC = N\left\{\underline{u}+c(\mu)+\frac{z(e_1)+z(e_2)}{2}+\frac{c'(\mu)}{\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}}\left[\frac{(1-\delta)}{2\delta}\right]\right\}.$$

Note that the cost per agent is the opportunity cost of the foregone alternative employment plus the cost of non-contractible effort and average cost of contractible effort, plus the final term in the brackets. This final term can be viewed as the cost of frictions in this dynamic mechanism and is determined by two factors: the discount rate, and the ratio of the marginal cost of effort to the marginal effect of effort on the probability of being transitioned. As we will discuss further below, the marginal effect of effort on the probability of being transitioned may depend on the number of players transitioned each period, so although τ doesn't appear in the simplified form of the expression (7') it remains a determinant of costs. However, it is evident that in the absence of discounting the final "friction" term becomes zero. Thus we find that in the limit, as discounting becomes negligible,

(8) $$\lim_{\delta\to 1} TC = N\left\{\underline{u}+c(\mu)+\frac{z(e_1)+z(e_2)}{2}\right\}.$$

We will proceed to show that the principal can achieve a first-best outcome given negligible discounting when the difference in monetary compensation between groups is not constrained, and then examine the second-best optimum when it is constrained. The principal's objective in the absence of discounting is:

(9) $$\max \Pi = BE + NA\mu - N\left\{\underline{u}+c(\mu)+\frac{z(e_1)+z(e_2)}{2}\right\}.$$

We assume the principal can deny payment to an agent who fails to complete his assigned contractible tasks, and thus completing these tasks is incentive compatible for any agent who chooses to accept employment.

2.2 Task Assignment with Flexible Monetary Incentives

Let $S = e_2 - e_1$ denote the spread between the contractible effort required to complete the tasks assigned to each of the two groups, in which case $e_1 = \frac{E}{N} - \frac{S}{2}$ and $e_2 = \frac{E}{N} + \frac{S}{2}$. If the principal can use monetary payments to motivate agents, then any desired non-contractible effort can be implemented while equating contractible effort across agents; i.e. with $S = 0$. Note that because $z'' > 0$, for any strictly positive S then $\left[z(\frac{E}{N} - \frac{S}{2}) + z(\frac{E}{N} + \frac{S}{2})\right]/2 > z(\frac{E}{N})$. Therefore the cost of implementing any desired non-contractible effort is minimized by setting $S = 0$ and using only monetary incentives to motivate this effort. The payments required to implement the desired effort, (f_1, f_2), will be determined by equations (5) and (6) above, but this only impacts the principal's objective through the participation constraint, which is incorporated in the objective (9). In this case, the principal's optimization problem can be modeled as choosing total contractible effort, E, and the implemented non-contractible effort, μ (with f_1, f_2 adjusting as required):

(10) $$\max_{E,\mu} \Pi = BE + NA\mu - N\left\{\underline{u} + c(\mu) + z\left(\frac{E}{N}\right)\right\}.$$

The first order conditions for the principal's optimal choices of effort to implement, which we denote $\hat{E}$ and $\hat{\mu}$, are $c'(\hat{\mu}) = A$ and $z'\left(\hat{E}/N\right) = B$.

PROPOSITION 1: If the principal can use the difference in fixed monetary payoffs $(f_1 - f_2)$ to induce effort, as discounting becomes negligible (δ approaches 1) the dynamic tournament mechanism implements optimal effort, μ^*, E^*, such that $c'(\mu^*) = A$ and $z'\left(\frac{E^*}{N}\right) = B$.[7]

2.3 Task Assignment with Constrained Monetary Incentives

We next turn to the situation where the principal is constrained in her ability to use financial incentives, but has the ability to reward high-status agents with preferential task assignment that incurs a lower effort, i.e. $S > 0$. For ease of exposition, consider the case where monetary payment is constrained to be equalized across groups, $f_1 = f_2$. Employing differential task assignments such that $S > 0$ causes the total effort cost of completing any given set of contractible tasks to increase relative to the case when the tasks are spread evenly among agents because $z'' > 0$. In this case the total cost to the principal of implementing a particular non-contractible effort μ together with the total tasks E is

[7] All proofs are included in the Appendix.

(11) $$TC = N\left\{\underline{u} + c(\mu) + \left[z(\frac{E}{N} - \frac{S}{2}) + z(\frac{E}{N} + \frac{S}{2})\right]/2 + \frac{c'(\mu)}{\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}}\left[\frac{(1-\delta)}{2\delta}\right]\right\},$$

and in the absence of discounting the total cost becomes

(12) $$TC = N\left\{\underline{u} + c(\mu) + \left[z(\frac{E}{N} - \frac{S}{2}) + z(\frac{E}{N} + \frac{S}{2})\right]/2\right\}.$$

We will show that even in the absence of discounting the mechanism entails an efficiency cost when monetary incentives are unavailable. In this case we can state the optimization problem as

(13) $$\max_{E,\mu} \Pi = BE + NA\mu - N\left\{\underline{u} + c(\mu) + \left[z(\frac{E}{N} - \frac{S}{2}) + z(\frac{E}{N} + \frac{S}{2})\right]/2\right\}.$$

The first-order conditions that yield the constrained-optimal effort levels are

$c'(\hat{\mu}) = A - \frac{\partial\left[z(\frac{E}{N}-\frac{S}{2})+z(\frac{E}{N}+\frac{S}{2})\right]/2}{\partial S}\frac{\partial S}{\partial \mu}$, and

$\frac{1}{2}\left\{z'\left(\frac{E}{N} - \frac{S}{2}\right) + z'\left(\frac{E}{N} + \frac{S}{2}\right)\right\} = B.$

These conditions can be restated as

(14) $$c'(\hat{\mu}) = A + \frac{1}{4}\frac{\partial S}{\partial \mu}[z'(e_1) - z'(e_2)], \quad \text{and}$$

(15) $$\frac{1}{2}[z'(e_1) + z'(e_2)] = B.$$

Note that $[z'(e_1) - z'(e_2)] < 0$. Thus, $A > c'(\hat{\mu})$. Furthermore $\frac{1}{2}[z'(e_1) + z'(e_2)] \geq z'\left(\frac{E}{N}\right)$ with $\frac{1}{2}[z'(e_1) + z'(e_2)] > z'\left(\frac{E}{N}\right)$ if $z'''(e) > 0$. Therefore, the optimal contract when the principle is constrained to motivate non-contractible effort through task assignment yields non-contractible effort below first-best and contractible effort also below first-best if the marginal cost of contractible effort increases at an increasing rate. Intuitively, motivating non-contractible effort entails an efficiency cost related to differential task assignment, and when $z'''(e) > 0$ the magnitude of the efficiency cost grows as the total amount of contractible effort implemented, E, grows, leading to a distortion in the contractible dimension as well.

PROPOSITION 2: When monetary incentives are unavailable, the principal can use the assignment of tasks to motivate effort, but doing so entails an efficiency cost that increases with total contractible effort. The second-best contract therefore entails $\hat{\mu} < \mu^*$ and $\hat{E} \leq E^*$ with $\hat{E} < E^*$if $z'''(e) > 0$.

In the analysis above we have considered two polar cases: flexibility over the use of money and task assignment incentives; and flexibility over task assignment only. There is a continuum of intermediate cases, characterized by a situation where the firm has some ability to use a differential fixed monetary payment, but where the difference across groups is insufficient to induce optimal effort. As we have shown, inefficiencies arise when task assignment is used instead of monetary payment differentials, and as a result the firm will do best to use monetary incentives to the extent possible.

3. Additional Analysis

3.1 Error Variance

An important result of standard static labor tournament models which also applies in our framework is that the prize spread required to motivate a given effort level increases with the variance of random shocks that impact output. In the standard static framework such as Lazear and Rosen (1981), and in the dynamic tournament model presented here when monetary incentives can be flexibly employed, increasing the spread of payoffs is not costly to the principal (i.e. the principal does not need to pay extra compensation to agents as a result of increasing the prize spread because total payments to the group of contestants can be held constant while the spread is increased). However, in the task assignment model, when $f_1 = f_2$, increasing the spread between $z(e_2)$ and $z(e_1)$ in order to motivate non-contractible effort is costly due to the convexity of the contractible cost function and this causes inefficiency. O'Keeffe, Viscusi and Zeckhauser (1984) argue that the variance of output in a tournament can reflect a variety of phenomena such as uncertainty in environmental factors or the precision with which a principal monitors his agents. If it is the case that monitoring precision partially determines the variance of output, then in the task differentiation model, the principal's ability to more precisely evaluate his employees' output is important for efficiency.[8] This result is in contrast to the standard tournament framework, although other factors can have a similar effect

[8] The existence of a pure strategy equilibrium in any rank-order tournament requires sufficient variance of the random component of agents' output. This is required to make the equilibrium effort satisfy general incentive compatibility such that the effort identified by the marginal optimality conditions is not dominated by "opting out" of competition and choosing zero effort. Nalebuff and Stiglitz (1983) discuss this detail. As is standard, we assume this condition is met.

in making greater output variance costly to the principal. These include risk aversion or limited liability among agents.

PROPOSITION 3: In the task assignment model, when sufficient monetary incentives are not available, an increase in the variance of ε increases the spread of payoffs, $z(e_2) - z(e_1)$, required to motivate any effort level. This is costly for the principal and decreases the elicited non-contractible effort, $\hat{\mu}$.

3.2 Transition Probability

An unusual characteristic of the dynamic tournament studied here is that the number of winners in the tournament in G_2 (and the number of losers in G_1), τ, is also a key parameter determining the size of the prize at stake in the contest. Consequently, the number of agents transitioned between groups in each period also has an effect on the principal's cost of implementing any level of effort. From equation (3), it is straightforward to see that increasing τ decreases the spread in the present value of utility, $(V_1 - V_2)$, *ceteris paribus*, and this effect decreases equilibrium effort for any given payoff differential between groups. This leads to the effect evident in equation (5) that a larger τ requires a larger payoff spread to induce a given non-contractible effort. When group sizes are equal the payoff spread (combining monetary and task-assignment differences) required to induce a specific μ is: $[f_1 - f_2] + [\mathrm{z}(e_2) - z(e_1)] = \frac{c'(\mu)}{\delta \frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}}\left(1 - \delta + \frac{4\delta\tau}{N}\right)$, which is increasing in τ *ceteris paribus*. However, it is quite possible that the marginal effect of effort on the probability of being transitioned, $\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}$, may increase when τ increases. This will generally be the case for unimodal distributions (e.g. normal) as τ increases from 1 until τ reaches half the groups size (i.e. until it reach $\frac{N}{4} = \frac{n_1}{2} = \frac{n_2}{2}$). This effect could offset the negative effect of an increase in τ on implemented effort. On the other hand, the impact of increasing τ on the marginal effect of effort on the probability of being transitioned may be quite small for flat distributions, and in particular there is no effect if errors are uniformly distributed.[9]

If more players being transitioned increases the payoff spread required to implement a given effort level this will be costly to the principal when task incentives must be used. Recall the principal's total cost function from equation (7'):

$$(7') \qquad TC = N\left\{\underline{u} + c(\mu) + \frac{z(e_1)+z(e_2)}{2} + \frac{c'(\mu)}{\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}}\left[\frac{(1-\delta)}{2\delta}\right]\right\}.$$

If τ affects the task differential required to implement non-contractible effort it will impact the average cost of contractible effort, $\frac{z(e_1)+z(e_2)}{2}$. Furthermore, if the marginal effect of effort on the probability of being transitioned, $\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}$, increases when τ increases then the number of players transitioned has an additional impact on costs through the "friction" component when discounting is present.

Given the various factors in play, the effect of τ on the cost of implementing effort through the dynamic tournament mechanism is unambiguous only in the special case when discounting is negligible and errors are uniformly distributed, as stated in Proposition 4.

PROPOSITION 4: : In the task assignment model, when monetary incentives are not available, discounting is negligible and random shocks are uniformly distributed, it is optimal to transition one agent from G_1 to G_2, and vice versa (i.e. $\tau = 1$).

To understand the effect of τ on incentives and the optimal choice of number of players to transition when errors follow other symmetric distributions we conducted simulations under the assumption that random errors are *i.i.d.* normal. We first consider the effect in the absence of discounting. In that case the only effect of τ is through the payoff spread. The effect on the payoff spread is determined by $\frac{\left(1-\delta+\frac{4\delta\tau}{N}\right)}{\delta\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}} = \frac{4\tau}{N\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}}$. In this case, as τ increases from 1 then

[9] This is simply an application of the Nalebuff and Stiglitz's (1983) finding that when errors are uniform at a symmetric point a contestant's effort increases his probability of ranking first and reduces his probability of ranking last while leaving the probability of any interior rank unchanged.

$\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}$ increases, but by less than a factor of 4, therefore $\frac{4\tau}{N\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}}$ increases.[10] As a result, the payoff spread required to induce any effort level increases with τ, so in order to minimize the cost of implementing any effort it is optimal to set $\tau = 1$, as in the case of uniformly distributed shocks.

Unlike the case of uniform random shocks, when discounting is present and shocks are normally distributed the optimal choice of τ will deviate from 1. For this discussion we will ignore the friction component of implementation costs, which may increase the optimal τ further. Therefore the following cases indicate lower bounds for the optimal τ.[11] The variance of the normally distributed shocks impacts $\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}$ proportionally for all τ and therefore does not impact the optimal choice. Two factors do play a role in determining the optimal number of players transitioned: δ and the number of players N (as noted before we assume group sizes are equal, $n_1 = n_2 = \frac{N}{2}$). We find that the optimal τ monotonically decreases with δ. For example, when $N = 20$, the τ that minimizes the implementation cost is as follows: for $\delta \in [0,0.16]$, $\tau^* = 5$; for $\delta \in [0.17,0.47]$, $\tau^* = 4$; for $\delta \in [0.48,0.72]$, $\tau^* = 3$; for $\delta \in [0.73,0.90]$, $\tau^* = 2$; and for $\delta \in [0.91,1]$, $\tau^* = 1$. Notice that it is optimal to transition a maximum of half of each group when discounting is extremely great, and the optimal number declines to a single player in each group when discounting is minimal. The optimal number of players to transition as a *share* of the number of competitors is stable. For example, for $\delta = 0.8$: for $N = 20,\ \tau^* = 2$; and for $N = 200$, $\tau^* = 20$.

4. Conclusion

Tournaments are frequently used to model dynamic labor market settings where there is a natural employment hierarchy; typically, there is a clear potential for promotion and, sometimes, demotion. We demonstrate that a tournament can also be used in non-hierarchal situations where

[10] To obtain this result, we used simulations to calculate $\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}$. Aside from the assumed distribution, this derivative is only conditional on n_1 and τ, and we considered a wide range of values for these arguments.

[11] The effect of the friction component on the optimal τ depends on the relative values of contractible versus non-contractible effort. The more relatively valuable non-contractible effort is the greater the importance of this friction cost. Because increasing τ will increase $\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}$ and thus reduce the friction cost the optimal τ will increase with the relative value of non-contractible effort.

many employees are at a similar rank, promotion does not play a significant role in employees' motivation, and monetary incentives are not readily available (e.g. employees in a government agency, teachers in a public school or perhaps professors in a university). While the manager in such an organization may not be able to set up a monetary incentive program, he can still induce non-contractible effort by differentially assigning contractible tasks. However, this mechanism relies on spreading the burden of tasks unevenly, which is inherently inefficient under the standard assumption of convex effort costs, and therefore induced effort is suboptimal.

Our framework has important implications for understanding incentives in public organizations. Several papers have noted that monetary incentives are rarely used in the public sector. This may be partially due to the fact that there are insufficient funds for large bonuses and partially due to the unpopularity of rewarding employees in public service with cash payments (Heinrich and Marschke, 2010). Further, it is often the nature of public service, especially in the case of education, that the hierarchy of the organization is fairly flat, at least for certain groups of employees. We have shown that this does not imply that managers have no opportunities to motivate employees with competition. On the contrary, managers are frequently responsible for assigning courses, administrative tasks and other contractible responsibilities. Such assignment decisions are easily conditioned on which employees are "in favor" and which employees are "out of favor" at the time. Further, employees are likely to fall out of favor, or be "promoted" to being in favor, with the manager from time to time based on past performance. A dynamic Markov tournament is therefore a mechanism that may enable an organization without explicit performance incentives or hierarchy to achieve efficient labor outcomes.

Further, our model offers a new perspective on the value to managers of flexibility over job assignments. Of course flexibility is important for a firm or any organization in that it allows adaptation to changing technology or market conditions without having to re-negotiate labor contracts (Wright and Snell, 1998). However, we show that there may be an additional advantage of flexibility in that it can be used as an incentive mechanism. A manager who can use task assignments to reward agents in favor and punish those out of favor has the opportunity to motivate effort through the competition for status.

As a possible extension to our model, the principal may value output between the high and low status groups differently. For example, a principal may value non-contractible effort from teachers more highly in honors classes (a possible task "reward" for the in-favor group) because the parents of honors students are more demanding, or he may value non-contractible effort more highly in lower-level classes (a possible task "punishment" for the out-of-favor group) due to minimum standardized testing requirements. Similarly, in a government organization, the manager could place a high value on non-contractible effort put into a report that gets publicized even if this report is tedious to produce and is therefore a task "punishment" for the low status group. When one group has more highly-valued output it is possible to elicit differential effort from them if the relative group sizes can differ or the variance of random shocks can be calibrated (such as through the intensity of monitoring).

Another possible extension is to allow constraints on the proportion of agents in each group that are selected for their respective tournaments. In our model, all agents compete in tournaments. However, there may be settings where the manager may not have information with which to adequately compare agent performance, e.g. performance evaluations may not be undertaken for all agents in all periods. Further, who to evaluate may be subject to manager preferences, allowing him to influence the effort induced by varying the likelihood agents in each group are scrutinized.

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Appendix

Proof of Proposition 1:

The principal's optimization problem is given by equation 10:

$$\max_{E,\mu} \Pi = BE + NA\mu - N\left\{\underline{u} + c(\mu) + z\left(\frac{E}{N}\right)\right\}.$$

The first order conditions for a maximum are $B - Nz'\left(\frac{E}{N}\right)\left(\frac{1}{N}\right) = 0$ and $NA - Nc'(\mu) = 0$.

These simplify to $B = z'\left(\frac{E}{N}\right)$ and $A = c'(\mu)$, which are equivalent to the conditions for μ^*, E^*.

Proof of Proposition 2:

The principal's optimization problem is given by equation (11):

$$\max_{E,\mu} \Pi = BE + NA\mu - N\left\{\underline{u} + c(\mu) + \left[z(\frac{E}{N} - \frac{S}{2}) + z(\frac{E}{N} + \frac{S}{2})\right]/2\right\}$$

The first order conditions for a maximum are:

(A1) $$NA - Nc'(\hat{\mu}) - N\left(\frac{1}{2}\right)\frac{\partial\left[z\left(\frac{E}{N}-\frac{S}{2}\right)+z\left(\frac{E}{N}+\frac{S}{2}\right)\right]}{\partial S}\frac{\partial S}{\partial \mu} = 0$$

(A2) $$B - N\left[\frac{1}{2}\left\{z'\left(\frac{E}{N} - \frac{S}{2}\right)\left(\frac{1}{N}\right) + z'\left(\frac{E}{N} + \frac{S}{2}\right)\left(\frac{1}{N}\right)\right\}\right] = 0$$

Equation A1 can be simplified to: $A = c'(\hat{\mu}) + \left[-\frac{1}{4}\left(z'\left(\frac{E}{N} - \frac{S}{2}\right)\right) + \frac{1}{4}\left(z'\left(\frac{E}{N} + \frac{S}{2}\right)\right)\right]\frac{\partial S}{\partial \mu}$.

Note that $E = n_1 e_1 + n_2 e_2$ and $n_1 = n_2$. Thus, $e_2 = \frac{E}{N} - e_1$. Further note that $S = e_2 - e_1$ and $e_1 = \frac{E}{N} - \frac{S}{2}$. Similarly, it can be shown that $e_2 = \frac{E}{N} + \frac{S}{2}$. A1 can be further simplified to: $c'(\hat{\mu}) = A + \frac{1}{4}\frac{\partial S}{\partial \mu}[z'(e_1) - z'(e_2)]$, as shown in equation (12). Since $[z'(e_1) - z'(e_2)] < 0$, it must be the case that $c'(\hat{\mu}) < A$.

Equation (A2) can be simplified to: $\frac{1}{2}[z'(e_1) + z'(e_2)] = B$. By comparing this to the condition for the first-best level of contractible effort, $B = z'\left(\frac{E^*}{N}\right)$), and noting that by Jensen's inequality $\frac{1}{2}[z'(e_1) + z'(e_2)] \geq z'\left(\frac{E}{N}\right)$ with $\frac{1}{2}[z'(e_1) + z'(e_2)] > z'\left(\frac{E}{N}\right)$ if $z'''(e)$>0 for any E and for $S > 0$, it follows that $\hat{E} \leq E^*$ and $\hat{E} < E^*$ for $z'''(e)$>0.

Proof of Proposition 3:

Standard tournament theory results indicate that as the error variance in a contest increases the marginal effect of effort on the probability of ranking in the top (winning) positions decreases, i.e. $\frac{\partial Q_i(\mu_i,\mu_{-i})}{\partial \mu_i}|_{\mu_i=\mu_{-i}}$ and $\frac{\partial R_j(\mu_j,\mu_{-j})}{\partial \mu_j}|_{\mu_j=\mu_{-j}}$ decrease in magnitude (Nalebuff and Stiglitz, 1983). When the principal cannot use monetary incentives, $f_1 = f_2$, then the condition defining equilibrium non-contractible effort in the contest is:

$[\mathrm{z}(e_2) - z(e_1)] = \frac{c'(\mu)}{\delta \frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}}\left(1 - \delta\left(1 - \frac{4\tau}{N}\right)\right)$. Greater error variance which decreases $\frac{\partial R_j(\mu_j,\mu_{-j})}{\partial \mu_j}|_{\mu_j=\mu_{-j}}$ thus increases the spread $\mathrm{z}(e_2) - z(e_1)$ required to achieve any effort level. That is, $\frac{\partial S}{\partial \mu}$ increases with the error variance for all μ. From the condition for $\hat{\mu}$, $c'(\hat{\mu}) = A + \frac{1}{4}\frac{\partial S}{\partial \mu}[z'(e_1) - z'(e_2)]$, it is evident that as $\frac{\partial S}{\partial \mu}$ increases the optimal $\hat{\mu}$ declines.

Proof of Proposition 4:

From equation (5) the prize spread required to implement a given effort is:

$[\mathrm{z}(e_2) - z(e_1)] = \frac{c'(\mu)}{\delta \frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}}\left(1 - \delta\left(1 - \frac{4\tau}{N}\right)\right)$. This can be re-written as

$c'(\mu) = \delta\left(\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}\right)\frac{[\mathrm{z}(e_2)-z(e_1)]}{\left(1-\delta\left(1-\frac{4\tau}{N}\right)\right)}$. When errors are distributed uniformly $\frac{\partial R_j}{\partial \mu_j}|_{\mu_j=\mu_{-j}}$ is unaffected by τ. Therefore the only effect of an increase in τ is to increase the right-hand-side denominator, which requires an offsetting increase in the spread $z(e_2) - z(e_1)$ in order to implement any given non-contractible effort.

Recalling equation (8), the total cost as discounting becomes negligible is:

$\lim_{\delta\to 1} TC = N\left\{\underline{u} + c(\mu) + \frac{z(e_1)+z(e_2)}{2}\right\}$. This can be rearranged as:

$$\lim_{\delta\to 1} TC = N\left\{\underline{u} + c(\mu) + \frac{[z(e_1) + z(e_2) - z(e_1) + z(e_1)]}{2}\right\}$$

$$= N\left\{\underline{u} + c(\mu) + z(e_1) + \frac{[z(e_2) - z(e_1)]}{2}\right\}$$

Clearly, $\partial TC/\partial[z(e_2) - z(e_1)] > 0$. Thus, $\partial TC/\partial\tau > 0$ and the total cost to the organizer of implementing any effort level is minimized when $\tau = 1$.