

## Stat 320

### Key answer to Home assignment 2

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**(A) 30 points, 10 point for each**

A random sample ( $X$ ) of size  $n=400$ , average=35,  $S_x=5$ ,  $Z_{94\%}=1.56$  or 1.55

If we call  $Y_{94th}$  =94<sup>th</sup> percentile of the average, then we have:

$$P(\text{average} < Y_{94th})=0.94$$

$$(1) Y_{94th} = \text{average} + (S_x/\sqrt{n}) * Z_{94\%} = 35 + (5/\sqrt{400}) * 1.56 = \mathbf{35.38}$$

(2) The 97% confidence interval of the average (since the std of the population is unknown and  $n$  is large, we use normal distribution) is:

$$1-\alpha=0.97$$

$$\alpha/2=0.015$$

$$97\% \text{ CI} = \text{Mean} \pm z_{\alpha/2} * (S_x/\sqrt{n}) = 35 \pm 2.17 * (5/\sqrt{400}) = \mathbf{[34.45 ; 35.54]}$$

(3) When  $n=20$ , average=32,  $S_x=3$ ,  $t_{0.025}=2.093$

The 95% confidence interval of the average (since the std of the population is unknown and  $n$  is small, we use t distribution) is:

$$1-\alpha=0.95$$

$$\alpha/2=0.025$$

$$df=n-1=19$$

$$95\% \text{ CI} = \text{Mean} \pm t_{(0.025, 19)} * (S_x/\sqrt{n}) = 32 \pm 2.093 * (3/\sqrt{20}) = \mathbf{[33.59 ; 36.40]}$$

**(B) 20 points, 10 each**

(a)

Let  $X$  be the random for the weight of the candy bar

$n=16$ ,  $\text{mean}=4.85$ ,  $\text{sigma}$  is unknown,  $S_x=0.1$ ,  $df=n-1=15$ ,  $\alpha=0.05$

(i) We test  $H_0: \mu > 5$  versus  $H_a: \mu < 5$

$$\text{T-ratio} = (\text{mean} - \mu) / (S_x / \sqrt{n}) = (4.85 - 5) / 0.025 = -6$$

$$\text{P-value} = P(t < \text{t-ratio}) = P(t < -6) \sim \text{almost } 0$$

Since  $\text{P-value} < 0.05$ , then **we reject the null hypothesis  $H_0: \mu > 5$**

(b)

If the population is normally distributed, then:

$$\text{Z-ratio} = (\text{mean} - \mu) / (S_x / \sqrt{n}) = (4.85 - 5) / 0.025 = -6$$

$$\text{P-value} = P(Z < -6) = \text{almost } 0 < 0.05, \text{ so we reject the null hypothesis } : \mu > 5$$

**Conclusion: FTC have grounds to proceed against the manufacturer for the unfair practice of short-weight selling**

**(C) 30 points, 10 point for each**

X	1	2	3	4	5	6
Y	3	5	7	9	11	13

(a) and (b) are easy

(c) Here we may use the formula of a line which is:

If a line is summarized by  $y = a + bx$ ,

We choose two point that belongs to the line for example  $A = (x_1, y_1) = (1, 3)$  and  $B = (x_2, y_2) = (2, 5)$

We estimate the slope  $b = \text{rise/run} = (y_2 - y_1) / (x_2 - x_1) = (5 - 3) / (2 - 1) = 2$

We estimate the intercept by looking at the point of intersection of the line with the  $y$ -coordinate which is 1

So

$$\mathbf{Y = 1 + 2x}$$

**(D) 20**

**is looking at the 5 histogram, we should have some some kind of normal distribution for the last case (relatively largest sample)**