Quiz 7: Correction | November 18, 2002

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Question 1:

States the five steps of a hypothesis testing analysis.

Answer:
1. Null hypothesis and alternative
2. Level of significance α
3. Test Statistics
4. Critical value
5. Decision

Question 2:

Suppose that a random sample of 100 measurements gives has a mean $\overline{x} = 100$. If we know that the standard deviation of the original population where the sample were randomly selected from is $\sigma = 20,$ use the hypothesis testing to

1. Test $H_0 : \mu = 97$ versus $H_a : \mu \neq 97$ using level of significance $\alpha = 0.03$.
2. Test $H_0 : \mu = 97$ versus $H_a : \mu > 97$ using level of significance $\alpha = 0.05$.

answer:
1. H_0 $\mu = 97$, $H_a : \mu \neq 97$
Test statistics:

$$Test = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{100 - 97}{20/\sqrt{10}} = 0.474$$

Critical value=$z_{\alpha/2} = z_{0.03/2} = z_{0.015} \sim 1.89(almost)$
test statistics is smaller than 1.89 so it belongs to the non rejection region, so we should not reject the null hypothesis H_0

2.H_0 $\mu = 97$, $H_a : \mu > 97$
Test statistics:

$$Test = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{100 - 97}{20/\sqrt{10}} = 0.474$$

Critical value=$z_{\alpha} = z_{0.05} = \sim 1.98(almost)$

test statistics is not greater than 1.89 so it belongs to the non rejection region so we should not reject the null hypothesis H_0

Question 3:

A sample of size 20 has been selected to represent a population of measurements. The sample mean is $\overline{x} = 21.5$ and the standard error of the sample is $s = 0.5$ (we suppose that we do not know the population standard deviation).

1. Calculate the 97% confidence interval of a population mean μ.
2. Do you think that an estimated population mean $\mu = 22,$ belongs to your confidence interval at 97%?

$$1.\ CI_{97\%} = [\bar{X} - t_{\alpha/2}\frac{s}{\sqrt{n}}; \bar{X} + t_{\alpha/2}\frac{s}{\sqrt{n}}] = [21.5 - t_{(0.015,19)}\frac{0.5}{\sqrt{20}}; 21.5 + t_{(0.015,19)}\frac{0.5}{\sqrt{20}}]$$
$$= [21.5 - 2.093\frac{0.5}{\sqrt{20}}; 21.5 + 2.093\frac{0.5}{\sqrt{20}}]$$
$$= [21.266, 21.734]$$

2. No