Quiz 6 for Statistics 201: CORRECTION

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Question 1:

Suppose that the monthly average cost of living of a random sample from a population is $\bar{X} = \$3250$ with a standard deviation of $\sigma_{\bar{X}} = 20$.

1. Calculate a 96% [which is a $(1-\alpha)\%$] confidence interval for the mean of monthly cost of living of the whole *population*. We assume that the monthly cost of living is approximately normally distributed and the size of the sample is $n = 15$.

Answer:

Since the population standard deviation is unknown we can then use the sample standard deviation to estimate it and for that since the sample size is small (<30) we can use the t value instead of the z -value for the confidence interval.

$$CI_{96\%} = [\bar{X} - t_{\alpha/2}\frac{\sigma_{\bar{X}}}{\sqrt{n}}, \bar{X} + t_{\alpha/2}\frac{\sigma_{\bar{X}}}{\sqrt{n}}] = [3250 - t_{(0.02,14)}\frac{20}{\sqrt{15}}; 3250 + t_{(0.02,14)}\frac{20}{\sqrt{15}}]$$

$$=[3250 - 2.145\frac{20}{\sqrt{15}}; 3250 + 2.145\frac{20}{\sqrt{15}}] = [\ 3238.9\ ;\ 3261.1]$$

Question 2:

The mean of a sample of 65 customer satisfaction ratings is $\bar{x} = 41.95$. If we let μ denote the mean of all customers satisfaction ratings and if the standard deviation of all customers is $\sigma = 2.55$.

1. calculate the 95 percent and the 99 percent confidence intervals for μ.
2. using the 95 percent confidence interval, can we be 95 percent confident that μ is greater than 41 (a very satisfied customer gives a rating greater than 41).
3. using the 99 percent confidence interval , can we be 99 percent confident that μ is greater than 41? .

Answer

1. $CI_{95\%}\ [\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}] = [41.95 - z_{(0.025)}\frac{2.55}{\sqrt{65}}; 41.95 + z_{(0.025)}\frac{2.55}{\sqrt{65}}]$

$$=[41.95 - 1.96\frac{2.55}{\sqrt{65}}; 41.95 + 1.96\frac{2.55}{\sqrt{65}}] = [\,41.33,\ 42.570]$$

$$CI_{99\%}\ [\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}] = [41.95 - z_{(0.005)}\frac{2.55}{\sqrt{65}}; 41.95 + z_{(0.005)}\frac{2.55}{\sqrt{65}}]$$

$$[41.95 - 2.345\frac{2.55}{\sqrt{65}}; 41.95 + 2.345\frac{2.55}{\sqrt{65}}] = [\,41.208,\ 42.692]$$

2. Both confidence interval shows that the assumpation that μ is greater than 41 is mostly correct.

Question 3

For each of the following confidence level, find the $z_{\alpha/2}$ point needed to compute a confidence interval for μ

(a) 88 % $= (1-\alpha)\%$ (b) 92 % $= (1-\alpha)\%$ (e) 98 % $= (1-\alpha)\%$

(a) $z_{\alpha/2} = z_{6\%} = 1.55$
(b) $z_{\alpha/2} = z_{4\%} = 1.75$
(c) $z_{\alpha/2} = z_{1\%} = 2.33$