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Passive Decomposition of Multiple Mechanical Systems under Coordination Requirements

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***Abstract*— We propose a general control framework for multiple mechanical systems interacting with environments and/or humans under coordination requirements. The key innovation is the *passive decomposition* which enables us to achieve the two requirements of such systems simultaneously: motion coordination and energetic passivity of the closed-loop system. It decomposes the system dynamics into shape system addressing the coordination aspect, locked system representing overall dynamics of the coordinated system, and dynamic couplings between the locked and shape systems. The dynamic couplings can be cancelled out without violating passivity. Thus, the coordination aspect (shape system) and the dynamics of the coordinated system (locked system) can be decoupled from each other while enforcing passivity. Also, by designing the locked and shape controls to enforce passivity of their respective systems, passivity of the closed-loop system is guaranteed. We analyze and exhibit geometry of the passive decomposition and the locked and shape systems.**

I. Introduction

In this paper, we propose a general control framework for multiple (or single) mechanical systems which interact with environments and/or humans under motion coordination requirements. We call such systems interactive mechanical system. Some examples of such systems include mechanical teleoperators, collaboratively manipulating multirobot, and human interactive robots (e.g. ROBODOC [1]).

The main contribution of this paper is the *passive decomposition* which enables us to achieve the two often conflicting requirements of such interactive mechanical systems, simultaneously: motion coordination (holonomic constraint) among the multiple mechanical systems (or coordination of internal DOF for a single one) and energetic passivity for safe and natural interaction with external environments and/or humans. With the passive decomposition, the system dynamics of the interactive mechanical system splits into the three components: shape system addressing the coordination aspect, locked system representing overall dynamics of the coordinated system, and dynamic couplings between the locked and shape systems.

The most important and powerful property of the passive decomposition is that the decoupling control (i.e. cancellation of the dynamic couplings) is energetically conservative, i.e. it does not dissipate nor generate any energy by itself. This property enables us to decouple the shape and locked dynamics from each other while enforcing energetic passivity. Thus, coordination aspect and dynamics of the coordinated system can be controlled simultaneously and separately by controlling the shape and the locked systems, respectively. Moreover, by designing the locked and shape system controls to enforce passivity of their respective systems, total closed-loop interactive mechanical system is guaranteed to be energetically passive. Abstracting a group of multiple mechanical systems by its locked system whose dynamics is similar to a single system, we can also create a hierarchy for a large number of mechanical systems. Utilizing these properties, the passive decomposition has been successfully applied to teleoperation [2], [3], formation flying [4], and multirobot cooperative grasping [5].

For mechanical systems under holonomic constraints, majority of control schemes (e.g. hybrid position/force control [6], [7] or impedance control [8], [9]) are based on the "rigid constraint" condition, i.e. the holonomic constraints are somehow satisfied all the time so that the dynamics is given by the induced dynamics on the submanifold of the constraint and the second fundamental form (i.e. constraint force normal yo the submanifold). However, those schemes would not be suitable for the interactive mechanical systems, since 1) the rigid constraint is often a control objective rather than a given condition; 2) energetic passivity is not generally ensured with the cancellation of only the second fundamental form.

The terms "locked" and "shape" were adopted from reduced lagrangian approach [10] which provides a similar decomposition for mechanical systems possessing principle bundle and symmetry structure. Although many mechanical systems posses those properties, still many systems don't (e.g. two n-DOF revolutive joint tele-robot). The passive decomposition doesn't require such properties and is applicable to general mechanical systems.

The rest of the paper is organized as follows. In section II, control problem of the interactive mechanical systems under coordination requirement is formulated. In section III, we design the passive decomposition, analyze its geometric property, and exhibit geometry of the decomposed systems. Section IV presents a control design example and section V contains some concluding remarks. Several proofs are omitted due to space limit and please refer to [11] for them.

II. Problem Formulation

A. Modelling of Multiple Mechanical Systems

Consider a group of m-mechanical systems s.t. the i-th agent's dynamics evolving on a n_i-dimensional differential configuration manifold $\mathcal{M}_i$ is given by

$$M_i(q_i)\nabla^i_{v_i} v_i = T_i + F_i, \qquad i = 1, ..., m, \tag{1}$$

where $v_i \in T_{q_i}\mathcal{M}_i$ is the tangent vector (velocity), and $T_i, F_i \in T^*_{q_i}\mathcal{M}_i$ are the control and environmental force covectors, $M_i(q_i)$ is the inertia tensor defining the kinetic energy $\kappa_i(t)$ which is a Riemannian metric on $\mathcal{M}_i$ s.t.

$$\kappa_i(t) := \frac{1}{2}\langle\langle v_i(t), v_i(t)\rangle\rangle_{q_i} = \langle M_i(q_i)v_i(t), v_i(t)\rangle, \tag{2}$$

where $\langle\langle,\rangle\rangle$ and $\langle,\rangle$ are inner product and standard pairing on $\mathcal{M}_i$, respectively. Here, ∇^i is the Levi-Civita connection on $\mathcal{M}_i$ w.r.t. the metric $M_i(q_i)$ s.t. 1) it is torsion-free: for all $X_i, Y_i \in \mathfrak{X}(\mathcal{M}_i)$ (i.e. smooth vector fields on $\mathcal{M}_i$),

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$T_i(X_i, Y_i) = 0$, where $T_i : \mathfrak{X}(\mathcal{M}_i) \times \mathfrak{X}(\mathcal{M}_i) \to \mathfrak{X}(\mathcal{M}_i)$ is torsion tensor defined by

$$T_i(X_i, Y_i) := \nabla^i_{X_i} Y_i - \nabla^i_{Y_i} X_i - [X_i, Y_i], \tag{3}$$

with $[,]$ being the Lie bracket on $\mathcal{M}$; and 2) it is compatible w.r.t. the metric $M_i(q_i)$ (2) s.t. for $X_i, Y_i, Z_i \in \mathfrak{X}(\mathcal{M})$

$$\mathcal{L}_{X_i} \langle\langle Y_i, Z_i \rangle\rangle = \langle\langle \nabla^i_{X_i} Y_i, Z_i \rangle\rangle + \langle\langle Y_i, \nabla^i_{X_i} Z_i \rangle\rangle, \tag{4}$$

where $\mathcal{L}_{X_i}$ is the Lie derivative along X_i.

Let us cast the group dynamics (1) into a $n(:= \sum_{i=1}^m n_i)$-dimensional product manifold $\mathcal{M} = \mathcal{M}_1 \times \mathcal{M}_2 \cdots \mathcal{M}_m$. Then, the group dynamics is given by:

$$M(q)\nabla_v v = T + F, \tag{5}$$

where $q := (q_1, q_2, ..., q_m) \in \mathcal{M}$, $v := (v_1, v_2, ..., v_m) \in T_q\mathcal{M}$, $T := (T_1, T_2, ..., T_m) \in T^*_q\mathcal{M}$, and $F := (F_1, F_2, ..., F_m) \in T^*_q\mathcal{M}$. Also, the product inertia metric $M(q)$ is defined by:

$$\langle\langle v, w \rangle\rangle_{\mathcal{M}} := \langle\langle v_1, w_1 \rangle\rangle_{\mathcal{M}_1} ... + \langle\langle v_m, w_m \rangle\rangle_{\mathcal{M}_m}, \tag{6}$$

$\forall v = (v_1, v_2, ..., v_m)$ and $w = (w_1, w_2, ..., w_m) \in T_q\mathcal{M}$, and the product connection ∇ on $\mathcal{M}$ is defined s.t.

$$\nabla_X Y := (\nabla^1_{X_1} Y_1, \nabla^2_{X_2} Y_2, ..., \nabla^m_{X_m} Y_m), \tag{7}$$

where $X, Y \in \mathfrak{X}(\mathcal{M})$ with X_i and Y_i being their respective projections onto $\mathfrak{X}(\mathcal{M}_i)$. Then the product connection ∇ is the Levi-Civita connection on $\mathcal{M}$ w.r.t. the product metric $M(q)$ (6) [12] .

B. Energetic Passivity

Energetically, the interactive mechanical system (5) is a m-port system. Let us define environmental supply rate for the m-port interactive mechanical system (5) s.t.:

$$s_\rho(v_i, F_i) := \langle F_1, \dot{q}_1 \rangle + ... \langle F_m, \dot{q}_m \rangle, \tag{8}$$

where $v_i \in T_{q_i}\mathcal{M}_i$ and $F_i \in T^*_{q_i}\mathcal{M}_i$ are the velocity and the environmental force of the i-th agent. Let us also define control supply rate s.t.:

$$s_c(v_i, T_i) := \langle T_1, \dot{q}_1 \rangle + ... \langle T_m, \dot{q}_m \rangle, \tag{9}$$

where $T_i \in T^*_{q_i}\mathcal{M}_i$ is the i-th agent's control in (1).

For safe and stable interaction, we would like to achieve energetic passivity condition: there exists $d \in \Re$ s.t.

$$\int_0^t s_\rho\left(v_i(\tau), F_i(\tau)\right) d\tau \geq -d^2, \quad \forall t \geq 0, \tag{10}$$

where $s_\rho(\cdot)$ is environmental supply rate (8). We also define controller passivity condition: there exists $c \in \Re$ s.t.

$$\int_0^t s_c\left(v_i(\tau), T_i(\tau)\right) d\tau \leq c^2, \quad \forall t \geq 0, \tag{11}$$

where $s_c(\cdot)$ is control supply rate (9).

As shown by the following proposition, for interactive mechanical systems (5) with the compatible connection ∇, controller passivity (11) implies energetic passivity (10).

Proposition 1 *Consider a dynamical system $\tilde{M}(\tilde{q})\tilde{D}_{\tilde{v}}\tilde{v} = \tilde{T} + \tilde{F}$ evolving on a differential manifold $\tilde{\mathcal{M}}$ with its Riemannian metric $\tilde{M}(\tilde{q})$ where $\tilde{v} \in T_{\tilde{q}}\tilde{\mathcal{M}}$, $\tilde{T}, \tilde{F} \in T^*_{\tilde{q}}\tilde{\mathcal{M}}$ are velocity, control, and environmental forcing, and $\tilde{D}$ is an affine connection compatible w.r.t. the metric $\tilde{M}(\tilde{q})$. Then, controller passivity* (11) *implies energetic passivity* (10).

Proof: Let us define $\tilde{\kappa}(t) := \frac{1}{2}\langle\langle \tilde{v}, \tilde{v} \rangle\rangle = \frac{1}{2}\langle \tilde{M}(\tilde{q})\tilde{v}, \tilde{v} \rangle$, then, from the compatibility of $\tilde{D}$, we have

$$\frac{d}{dt}\tilde{\kappa}(t) = \mathcal{L}_{\tilde{v}} \frac{1}{2}\langle\langle \tilde{v}, \tilde{v} \rangle\rangle = \langle\langle \tilde{D}_{\tilde{v}}\tilde{v}, \tilde{v} \rangle\rangle = \langle \tilde{F} + \tilde{T}, \tilde{v} \rangle, \tag{12}$$

where $\mathcal{L}_{\tilde{v}}$ is the Lie derivative. Integration of (12) with controller passivity (11) and the fact that $\tilde{\kappa}(t) \geq 0 \ \forall t$ proves energetic passivity (10), since we have, for all $t \geq 0$, $\int_0^t \langle \tilde{F}(\tau), \tilde{v}(\tau) \rangle d\tau = \tilde{\kappa}(t) - \tilde{\kappa}(0) - \int_0^t \langle \tilde{T}(\tau), \tilde{v}(\tau) \rangle d\tau \geq -\tilde{\kappa}(0) - c^2 =: -d^2$. ■

Here, we assume that suitable kinematic and power scalings are already embedded in the original dynamics (1) following procedures in [11]. Using such scalings, we would be able to match differences in size and mechanical strength of individual agents and their respective environments.

C. Holonomic Constraints as Coordination Requirements

Let us define *coordination map* h to be a smooth submersion (i.e. its push-forward map $h_{*q} : T_q\mathcal{M} \to T_{h(q)}\mathcal{N}$ is surjective $\forall q \in \mathcal{M}$) from the n-dim. manifold $\mathcal{M}$ of the product system (5) to a m-dim. differential manifold $\mathcal{N}$,

$$h : \mathcal{M}^n \to \mathcal{N}^m, \ \ n \geq m, \tag{13}$$

so that the mapped point $c := h(q)$ on the manifold $\mathcal{N}$ can represent the motion coordination aspect. We call such manifold $\mathcal{N}$ *coordination manifold* of the coordination map h. Then, from the submersion theorem [10], for every $c \in range(h) \subset \mathcal{N}$, its level set $\mathcal{H}_c$ defined by

$$\mathcal{H}_c := \{q \in \mathcal{M} \mid h(q) = c\}, \tag{14}$$

is a $(n - m)$-dimensional smooth submanifold in $\mathcal{M}$.

III. THE PASSIVE DECOMPOSITION

A. Decomposition of Tangent and Cotangent Spaces

At each configuration $q \in \mathcal{M}$, we decompose the tangent space $T_q\mathcal{M}$ into two orthogonal vector spaces s.t. $T_q\mathcal{M} = T_q^\top\mathcal{M} \oplus T_q^\perp\mathcal{M}$, where

$$T_q^\top\mathcal{M} := Ker(h_*) = span\{v \in T_q\mathcal{M} \,|\, h_*(v) = 0\},$$
$$T_q^\perp\mathcal{M} := span\{w \in T_q\mathcal{M} \,|\, \langle\langle v, w \rangle\rangle = 0, \forall\, v \in T_q^\top\mathcal{M}\},$$

where $Ker(h_*)$ is kernel of the push-forward h_*. The dimension of $T_q^\top\mathcal{M} = Ker(h_*)$ is $n - m$, $\forall q \in \mathcal{M}$, since h (13) is a smooth submersion so that every $q \in \mathcal{M}$ has a $(n - m)$-dim. submanifold $\mathcal{H}_c$ (14) with $c = h(q)$. Also, there exists a unique m-dim. orthogonal complement $T_q^\perp\mathcal{M}$ of $T_q^\top\mathcal{M}$, since $T_q\mathcal{M}$ is a n-dim. metric vector space [12]. By collecting $T_q^\top\mathcal{M}$ and $T_q^\perp\mathcal{M}$ over $\mathcal{M}$, let us construct tangential and normal distributions, $\Delta^\top$ and $\Delta^\perp$, s.t.:

$$\Delta^\top(q) := T_q^\top\mathcal{M}, \quad \Delta^\perp(q) := T_q^\perp\mathcal{M}. \tag{15}$$

Both distributions, $\Delta^\top, \Delta^\perp$ are regular, since their respective dimensions are $n - m$ and m for all $q \in \mathcal{M}$. Also, $\Delta^\top$ is integrable with the submanifold $\mathcal{H}_c$ being its integral manifold. However, $\Delta^\perp$ is generally not integrable.

The decomposition of the tangent space also splits its dual space, cotangent space $T^*_q\mathcal{M}$ of the system (5) s.t.

$\forall q \in \mathcal{M}$, $T_q^*\mathcal{M} = T_q^{*\top}\mathcal{M} \oplus T_q^{*\perp}\mathcal{M}$, where $T_q^{*\top}\mathcal{M}$ and $T_q^{*\perp}\mathcal{M}$ are the duals of $T_q^{\top}\mathcal{M}$ and $T_q^{\perp}\mathcal{M}$, s.t.

$$T_q^{*\top}\mathcal{M} := span\{\tau \in T_q^*\mathcal{M} \,|\, \langle \tau, w\rangle_{\mathcal{M}} = 0 \forall w \in T_q^{\perp}\mathcal{M}\},$$
$$T_q^{*\perp}\mathcal{M} := span\{\delta \in T_q^*\mathcal{M} \,|\, \langle \delta, v\rangle_{\mathcal{M}} = 0 \forall v \in T_q^{\top}\mathcal{M}\}.$$

We also construct tangential and normal codistributions, $\Omega^\top$ and $\Omega^\perp$, s.t.:

$$\Omega^\top(q) := T_q^{*\top}\mathcal{M}, \quad \Omega^\perp(q) := T_q^{*\perp}\mathcal{M}. \tag{16}$$

The codistributions $\Omega^\top$ and $\Omega^\perp$ are annihilating codistributions of the distributions $\Delta^\perp$ and $\Delta^\top$, respectively.

Using the decompositions (15)-(16), let us decompose the velocity $v \in T_q\mathcal{M}$ and the force $F \in T_q^*\mathcal{M}$ (or the control T) of the system (5) under the coordination map h (13) s.t.

$$v = v^\top + v^\perp, \qquad v^\top \in \Delta^\top(q), \quad v^\perp \in \Delta^\perp(q), \tag{17}$$
$$F = F^\top + F^\perp, \qquad F^\top \in \Omega^\top(q), \quad F^\perp \in \Omega^\perp(q), \tag{18}$$

where $v^\perp$ is called shape velocity defining the velocity of $h(q) \in \mathcal{N}$ on the coordination manifold $\mathcal{N}$ as given by $h_*(v) = h_*(v^\perp) = \frac{d}{dt}h(q)$, $v^\top$ is called locked velocity representing the velocity of the coordinated system (5) under the current coordination (i.e. $v^\top \in T_q\mathcal{H}_{h(q)}$ where $\mathcal{H}_{h(q)} := \{\tilde{q} \in \mathcal{M} \mid h(q) = h(\tilde{q})\}$ is the submanifold having the current configuration q), and $F^\perp$, $F^\top$ represent the portions of the environmental force F in (5) affecting the coordination aspect (shape system) and the motion of the coordinated system (locked system). Note that the decomposition (17) is metric dependent, since $v^\perp$ is orthogonal to $v^\top$ w.r.t. the metric. However, $v^\top$ is given purely by the coordination map h (13) regardless of metric structure.

B. Decomposed Dynamics and Passive Decoupling

Using the decompositions (15)-(18), let us decompose the dynamics of interactive mechanical system (5) s.t.:

$$M\nabla_v v = T^\top + T^\perp + F^\top + F^\perp \tag{19}$$
$$= M(\nabla_v v^\top)^\top + M(\nabla_v v^\top)^\perp + M(\nabla_v v^\perp)^\top + M(\nabla_v v^\perp)^\perp$$

where the term $M(\nabla_{\dot q} v^\top)^\top$ is called *locked system* dynamics describing the dynamics of the locked velocity $v^\top$ in (17) on the current submanifold $\mathcal{H}_{h(q)}$, while the term $M(\nabla_{\dot q} v^\perp)^\perp$ is called *shape system* dynamics representing the dynamics of the shape velocity $v^\perp$ in (17) on the coordination manifold $\mathcal{N}$. The remaining terms, $M(\nabla_{\dot q} v^\top)^\perp$, and, $M(\nabla_{\dot q} v^\perp)^\top$, are the dynamic couplings between the locked and shape systems.

The decomposition also decomposes kinetic energy of the interactive mechanical system (5) into the sum of those of the locked and shape systems, s.t.:

$$\kappa(t) = \frac{1}{2}\langle\langle v, v\rangle\rangle = \frac{1}{2}\langle\langle v^\top, v^\top\rangle\rangle + \frac{1}{2}\langle\langle v^\perp, v^\perp\rangle\rangle, \tag{20}$$

from the definition of the distributions $\Delta^\top, \Delta^\perp$ in (15), where $\kappa(t) := \frac{1}{2}\langle\langle v, v\rangle\rangle_{\mathcal{M}} = \sum_{i=1}^m \frac{1}{2}\langle\langle v_i, v_i\rangle\rangle_{\mathcal{M}_i}$ is defined from the product metric (6). Mechanical power of the m-port system (5) is also decomposed in the sense that

$$\langle F, v\rangle = \sum_{i=1}^m \langle F_i, v_i\rangle = \langle F^\top, v^\top\rangle + \langle F^\perp, v^\perp\rangle, \tag{21}$$

from the definition of the codistributions (16) (same for T).

The dynamic couplings $M(\nabla_{\dot q} v^\perp)^\top$, $M(\nabla_{\dot q} v^\top)^\perp$ in (19) induce a crosstalk between the locked and shape systems, which is bilinear in $\dot q$ and v. To compensate for this crosstalk, we design the decoupling control T^d s.t.

$$T = \underbrace{M(\nabla_{\dot q} v^\top)^\perp + M(\nabla_{\dot q} v^\perp)^\top}_{T^d:\ decoupling\ control} + T_\star, \tag{22}$$

where T is the total control of the interactive mechanical system (5), and $T^\star$ is an additional control action to be designed according to some task objectives. As shown by the following theorem, the decoupling control T^d (22) decouples the locked and shape systems from each other, while enforcing passivity (10) of the closed-loop system.

Theorem 1 *The decoupling control* T^d (22) *does not generate nor dissipate any energy, (energetically conservative).*

Proof: Mechanical power generated (or dissipated) by the decoupling control (22) is given by:

$$\begin{aligned}\langle T^d, v\rangle &= \langle M(\nabla_{\dot q} v^\top)^\perp, v\rangle + \langle M(\nabla_{\dot q} v^\perp)^\top, v\rangle \\ &= \langle\langle(\nabla_{\dot q} v^\top)^\perp, v^\perp\rangle\rangle + \langle\langle(\nabla_{\dot q} v^\perp)^\top, v^\top\rangle\rangle \\ &= \mathcal{L}_{\dot q}\langle\langle v^\top, v^\perp\rangle\rangle = 0.\end{aligned} \tag{23}$$

Thus, energy generated by the decoupling control T^d (22) is $\int_o^t \langle T^d(\tau), v(\tau)\rangle d\tau = 0$, $\forall t \geq 0$. ■

The decoupling control T^d (22) requires only agents' velocity and position rather than often-unavailable acceleration. Also, it ensures passivity even under model uncertainty and inaccurate position signals (robust passivity [13]). But, passivity is not generally guaranteed with incorrect velocity signal and this issue is a topic for future research.

With the passive decoupling control (22), we have the following decoupled locked and shape dynamics:

$$M\nabla_v^d v = M(\nabla_v v^\top)^\top + M(\nabla_v v^\perp)^\perp = T_\star, \tag{24}$$

where the closed-loop decoupled connection ∇^d is defined by $\nabla_X^d Y := (\nabla_X Y^\top)^\top + (\nabla_X Y^\perp)^\perp$, for $X, Y \in \mathfrak{X}(\mathcal{M})$. Then, it is easy to show that ∇^d is an affine connection compatible w.r.t. the product Riemannian metric M (6).

Proposition 2 *Suppose that the additional locked and shape system controls* $T_\star^\top$ *and* $T_\star^\perp$ *(i.e.* $T_\star = T_\star^\top + T_\star^\perp \in T_q^*\mathcal{M}$ *with* $T_\star^\top \in \Omega^\top(q)$ *and* $T_\star^\perp \in \Omega^\perp(q)$ *in* (22)*) are designed to be individually energetically passive: there exist* $c_1, c_2 \in \Re$ *s.t.* $\int_0^t \langle T_\star^\top(\tau), v^\top(\tau)\rangle d\tau \leq c_1^2$, *and* $\int_0^t \langle T_\star^\perp(\tau), v^\perp(\tau)\rangle d\tau \leq c_2^2$. *Then, the closed-loop system is energetically passive* (10).

C. Shape System Dynamics: Connection over the Map h

The coordination aspect of the system (5) is represented by a mapped point $h(q)$ on the coordination manifold $\mathcal{N}$, while its instantaneous velocity on $\mathcal{N}$ is given by:

$$\frac{d}{dt}h(q) = h_*(v) = h_*(v^\top + v^\perp) = h_*(v^\perp), \tag{25}$$

for $v \in T_q\mathcal{M}$, where $h_* : T_q\mathcal{M} \to T_{h(q)}\mathcal{N}$ is the push-forward of h s.t. $h_*(v^\top) = 0$ from the definition of the decomposition (15). Thus, if we find the acceleration of the mapped point $h(q) \in \mathcal{N}$, the coordination aspect of the system (5) can be completely described on the coordination manifold $\mathcal{N}$ with the reduced dimension m. In order for this, let us define *shape system connection* ∇^h to be a connection over the coordination map $h : \mathcal{M} \to \mathcal{N}$ as follows. See Appendix for more details.

Proposition 3 *Consider a smooth vector field over the map h, $X^h \in \mathfrak{X}^h(\mathcal{M})$, i.e. $X^h : \mathcal{M} \to T\mathcal{N}$ and $X^h(q) \in T_{h(q)}\mathcal{N}$ for every $q \in \mathcal{M}$, where we denote smooth vector fields over the map h by $\mathfrak{X}^h(\mathcal{M})$. Then, there exists a unique smooth vector field $X^\perp \in \mathfrak{X}(\mathcal{M})$ associated to $X^h \in \mathfrak{X}^h(\mathcal{M})$ s.t. $X^\perp \in \Delta^\perp$ and $h_*(X^\perp) = X^h$.*

We define shape system connection $\nabla^h : \mathfrak{X}(\mathcal{M}) \times \mathfrak{X}^h(\mathcal{M}) \to \mathfrak{X}^h(\mathcal{M})$ to be a connection over the coordination map $h : \mathcal{M} \to \mathcal{N}$: for $Y \in (\mathcal{M})$ and $X^h \in \mathfrak{X}^h(\mathcal{M})$,

$$\nabla^h_Y X^h := h_* \left(\nabla_Y X^\perp\right)^\perp, \tag{26}$$

where h_* is the push-forward map of h, $\nabla : \mathfrak{X}(\mathcal{M}) \times \mathfrak{X}(\mathcal{M}) \to \mathfrak{X}(\mathcal{M})$ is the Levi-Civita connection of the system (5), and $X^\perp \in \mathfrak{X}(\mathcal{M})$ is the smooth vector field on $\mathcal{M}$ associated to $X^h \in \mathfrak{X}^h(\mathcal{M})$ as in proposition 3.

Theorem 2 *1. the shape system connection* (26) *is an affine connection over the map h (i.e. satisfies* (39)-(42)*);*
2. Let us define the induced metric on the coordination manifold $\mathcal{N}$ s.t.: for $v^h, w^h \in T_{h(q)}\mathcal{N}$,

$$\langle\langle v^h, w^h \rangle\rangle_{\mathcal{N}} := \langle\langle v^\perp, w^\perp \rangle\rangle_{\mathcal{M}} \tag{27}$$

where $v^\perp, w^\perp \in \Delta^\perp(q)$ are the tangent vectors associated to $v^h, w^h \in T_{h(q)}\mathcal{N}$, i.e. $h_{*q}v^\perp = v^h$, $h_{*q}w^\perp = w^h$. Then, the shape system connection (26) *is compatible w.r.t. the induced metric* (27) *(i.e. satisfies* (44)*);*
3. Suppose that the evolution of the product system (5) *is confined in the normal distribution $\Delta^\perp$* (15) *in the sense that $v(t) \in \Delta^\perp(q)$ $\forall t \geq 0$. Then, the shape system connection ∇^h* (26) *becomes the unique connection over the map h w.r.t. the induced metric* (27).

Proof: Consider $X^h, Y^h \in \mathfrak{X}^h(\mathcal{M})$ with $X^\perp, Y^\perp \in \mathfrak{X}(\mathcal{M})$ being their associated vector fields on $\mathcal{M}$.
1. Here, we prove only property (41) in Appendix, since other properties are easy to prove: for every $f \in C^\infty(\mathcal{M})$, $Y \in \mathfrak{X}(\mathcal{M})$, and $X^h \in \mathfrak{X}^h(\mathcal{M})$, we have

$$\begin{aligned}\nabla^h_Y f X^h &= h_* \left(\nabla_Y f X^\perp\right)^\perp = h_* \left(\mathcal{L}_Y(f) X^\perp + f \nabla_Y X^\perp\right)^\perp \\ &= \mathcal{L}_Y(f) h_*(X^\perp) + f h_*(\nabla_Y X^\perp)^\perp = \mathcal{L}_Y(f) X^h + f \nabla^h_Y X^h,\end{aligned}$$

from the affinity of the product connection (5);
2. For $Z \in \mathfrak{X}(\mathcal{M})$ and $X^h, Y^h \in \mathfrak{X}^h(\mathcal{M})$, we have,

$$\begin{aligned}&\mathcal{L}_Z \langle\langle X^h, Y^h \rangle\rangle_{\mathcal{N}} = \mathcal{L}_Z \langle\langle X^\perp, Y^\perp \rangle\rangle_{\mathcal{M}} \\ &= \langle\langle \left(\nabla_Z X^\perp\right)^\perp, Y^\perp \rangle\rangle_{\mathcal{M}} + \langle\langle \left(\nabla_Z Y^\perp\right)^\perp, X^\perp \rangle\rangle_{\mathcal{M}} \\ &= \langle\langle h_*(\nabla_Z X^\perp)^\perp, h_*(Y^\perp) \rangle\rangle + \langle\langle h_*(\nabla_Z Y^\perp)^\perp, h_*(X^\perp) \rangle\rangle \\ &= \langle\langle \nabla^h_Z X^h, Y^h \rangle\rangle_{\mathcal{N}} + \langle\langle \nabla^h_Z Y^h, X^h \rangle\rangle_{\mathcal{N}},\end{aligned} \tag{28}$$

from compatibility of ∇ (5) and definition of ∇^h (26);
3. Torsion translation (43) of the shape connection (26) $T^h : T_q\mathcal{M} \times T_q\mathcal{M} \to T_{h(q)}\mathcal{N}$ is defined by: for $v, w \in T_q\mathcal{M}$,

$$\begin{aligned}T^h(v, w) &:= \nabla^h_X h_* Y - \nabla^h_Y h_* X - h_*[X, Y] \\ &= h_* \left(\nabla_X Y^\perp\right)^\perp - h_* \left(\nabla_Y X^\perp\right)^\perp - h_*[X, Y] \\ &= h_* \left(\nabla_X Y^\perp - \nabla_Y X^\perp - [X, Y]\right)\end{aligned} \tag{29}$$

where $X, Y \in \mathfrak{X}(\mathcal{M})$ are local extensions of $v, w \in T_q\mathcal{M}$. Then, if $v, w \in T_q\mathcal{M}$ are restricted s.t. $v, w \in \Delta^\perp(q)$, the torsion (29) vanishes, since ∇ in (5) is Levi-Civita. Also, since h_* is surjective (as h is a smooth submersion), ∇^h (26) becomes the unique torsion-free and compatible connection over h w.r.t. the induced metric (27) as given in Appendix. ∎

Using the definition (26), we have

$$\nabla^h_{\dot{q}} h_*(v^\perp) = h_*(\nabla_{\dot{q}} v^\perp)^\perp. \tag{30}$$

i.e. the shape system dynamics $M(\nabla_{\dot{q}} v^\perp)^\perp$ in (19) describes the acceleration of the mapped point $h(q) \in \mathcal{N}$ on the coordination manifold $\mathcal{N}$. Thus, we can control the coordination aspect by designing some control actions (e.g. PD-control [14]) on the coordination manifold $\mathcal{N}$ and pulling it back to the original product manifold $\mathcal{M}$.

D. Locked System Projection on a Differential Manifold

Suppose that there exists a smooth submersion l from the configuration manifold $\mathcal{M}$ of the interactive mechanical system (5) to a $(n-m)$-dim. differential manifold $\mathcal{L}$:

$$l : \mathcal{M}^n \to \mathcal{L}^{n-m}, \tag{31}$$

such that its push-forward map $l_* : T_q\mathcal{M} \to T_{l(q)}\mathcal{L}$ satisfies the following *projectability* condition similar to that for the coordination map h (25): at each $q \in \mathcal{M}$, for all $v \in T_q\mathcal{M}$,

$$\frac{d}{dt} l(q) = l_*(v) = l_*(v^\top), \tag{32}$$

i.e. $l_*(v^\perp) = 0$ with $v^\perp \in \Delta^\perp(q)$. We call such map l and manifold $\mathcal{L}$, *locked system map* and *locked system manifold*.

When there exists such a locked system map $l : \mathcal{M} \to \mathcal{L}$ (31), using the passive decoupling control (22), we can decompose the n-dimensional dynamics of the interactive mechanical system (5) completely into two decoupled dynamics while enforcing energetic passivity (theorem 1): the m-dimensional shape system dynamics on the coordination manifold $\mathcal{N}$ (representing the coordination aspect) and the $(n-m)$-dimensional locked system dynamics on the locked system manifold $\mathcal{L}$ (representing the motion of the coordinated system). Thus, we can simultaneously achieve coordination requirement, desired behavior of the coordinated system, and energetic passivity, by designing individual shape and locked system controls to be passive as in proposition 2.

As shown in the following theorem, integrability of the normal distribution $\Delta^\perp$ (15) is a necessary condition for the existence of such a locked system map $l : \mathcal{M} \to \mathcal{L}$ (31) satisfying the projectability condition (32). This condition, however, is generally not satisfied by the system (5). In contrast, the tangential distribution $\Delta^\top$ (15) is integrable.

Theorem 3 *Suppose that there exists a locked system map $l : \mathcal{M}^n \to \mathcal{L}^{n-m}$* (31) *satisfying projectability condition*

(32). *Then, the m-dim. regular normal distribution $\Delta^\perp$ (15) should be integrable, i.e. $\forall q \in \mathcal{M}$, there exists a m-dim. integral manifold $\mathcal{D} \subset \mathcal{M}$ s.t. $q \in \mathcal{D}$ and $T_q\mathcal{D} = \Delta^\perp(q)$.*

Proof: Since the locked system map l (31) is a smooth submersion, at every $q \in \mathcal{M}$, there exists a m-dimensional submanifold $\mathcal{Q}_{l(q)} \subset \mathcal{M}$ defined by

$$\mathcal{Q}_{l(q)} := \{\tilde{q} \in \mathcal{M} \ \mid \ l(\tilde{q}) = l(q)\}.$$

Also, from the projectability condition (32), the locked system map $l : \mathcal{M} \to \mathcal{L}$ (31) satisfies that, at each $q \in \mathcal{M}$, $l_{*q}\left(v^\perp\right) = 0$ for all $v^\perp \in \Delta^\perp(q)$, i.e. $\Delta^\perp(q) \subset Ker(l_{*q}) = T_q\mathcal{Q}_{l(q)}$ for every $q \in \mathcal{M}$, where l_{*q} is the push-forward of l at q and $Ker(l_{*q}) \subset T_q\mathcal{M}$ is its kernel.

Suppose that $Ker(l_{*q}) \neq \Delta^\perp(q)$ for some $q \in \mathcal{M}$. Then, there should exist a tangent vector $\tilde{v} \in \Delta^\top(q) \subset T_q\mathcal{M}$ at $q \in \mathcal{M}$ s.t. $\tilde{v} \in Ker(l_{*q})$, since $T_q\mathcal{M} = \Delta^\top(q) \oplus \Delta^\perp(q)$. This is contradictory to the projectability condition (32) and the fact that l_* is surjective. Thus, $\Delta^\perp(q) = Ker(l_{*q}) = T_q\mathcal{Q}_{l(q)}$. This condition shows that, for every $q \in \mathcal{M}$, the normal distribution $\Delta^\perp$ (15) has $\mathcal{Q}_{l(q)}$ as its m-dimensional integral manifold. Thus, $\Delta^\perp$ should be integrable. ∎

For the next theorem, we need several definitions. The curvature tensor of the system (5), $R : \mathfrak{X}(\mathcal{M}) \times \mathfrak{X}(\mathcal{M}) \times \mathfrak{X}(\mathcal{M}) \to \mathfrak{X}(\mathcal{M})$, is defined by: for $X, Y, Z \in \mathfrak{X}(\mathcal{M})$

$$R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z, \quad (33)$$

where $[,]$ is the Lie bracket on $\mathcal{M}$, and ∇ is the Levi-Civita connection in (5). When the curvature tensor R (33) vanishes, the manifold $\mathcal{M}$ is said to be flat (w.r.t. the connection ∇). When the manifold $\mathcal{M}$ is flat, the parallel transport map $\mathcal{T}_{(p,q)}$ of $p, q \in \mathcal{M}$ is independent on curves joining p and q. The manifold $\mathcal{M}$ is called (geodesically) complete if any geodesic $\gamma(t)$ starting from any point $q \in \mathcal{M}$ with $\gamma(0) = q$ and $\dot{\gamma}(0) = v_o \in T_q\mathcal{M}$ is defined for all values of $t \in \Re$ and for any $v_o \in T_q\mathcal{M}$. On a connected and complete Riemannian manifold $\mathcal{M}$, $\forall p, q \in \mathcal{M}$, there exists a minimizing geodesic γ joining p and q [12].

Theorem 4 *Suppose that the product manifold $\mathcal{M}^n$ of the interactive mechanical system* (5) *is a complete, simply connected flat manifold. Denote the parallel transport map of the flat manifold $\mathcal{M}$ by $\mathcal{T}_{(p,q)} : T_q\mathcal{M} \to T_p\mathcal{M}$ for $p, q \in \mathcal{M}$. Assume that the coordination map h (13) is designed s.t. the tangential distribution $\Delta^\top$* (15) *is invariant with respect to $\mathcal{T}$ in the sense that for every $p, q \in \mathcal{M}$, $\mathcal{T}_{(p,q)}T_q^\top\mathcal{M} = T_p^\top\mathcal{M}$ where $T_q^\top\mathcal{M} = \Delta^\top(q) = Ker(h_{*q})$.*

Choose a submanifold $\mathcal{H}_o^{(n-m)} := \{\tilde{q} \in \mathcal{M} \,|\, h(\tilde{q}) = h(q_o)\}$ of a point $q_o \in \mathcal{M}$ as the locked system manifold $\mathcal{L}$ (31)*. For every $q \in \mathcal{M}$, let us define $\pi_o(q) \in \mathcal{H}_o$ to be the intersecting point of the submanifold $\mathcal{H}_o$ by a minimizing geodesic of the ambient connection ∇ on $\mathcal{M}$ starting from q, i.e. $\pi_o(q) := \gamma(1) \in \mathcal{H}_o$ where $\gamma : [0,1] \to \mathcal{M}$ is the minimizing geodesic of $\mathcal{M}$ s.t. $\gamma(0) = q$. Then, the map $\pi_o : \mathcal{M} \to \mathcal{H}_o$ defines a locked system map satisfying the projectability condition* (32)*: for every $v = v^\top + v^\perp \in T_q\mathcal{M}$, $\frac{d}{dt}\pi_o(q) = \pi_{o*}(v^\top)$, i.e. $\pi_{o*}(v^\perp) = 0$.*

1) Constrained Dynamics under Constant Holonomic Constraints: Suppose that the motion of the interactive mechanical system (5) is confined in a submanifold $\mathcal{H}_c = \{\tilde{q} \in \mathcal{M} \,|\, h(\tilde{q}) = c\} \subset \mathcal{M}$ such that $v(t) = v^\top(t) \in T_q\mathcal{H}_c = \Delta^\top(q) \subset T_q\mathcal{M}$ (i.e. $v^\perp(t) = 0$) and $h(q(t)) = c \in \mathcal{N}$, $\forall t \geq 0$, where $c \in \mathcal{N}$ is a constant coordination. Then, the constrained dynamics of the system (5) on $\mathcal{H}_c$ is given by (from (19) with $v = v^\top$ and $v^\perp = 0$):

$$\nabla_v v = \nabla_{v^\top} v^\top = \underbrace{\nabla^L_{v^\top} v^\top}_{\substack{locked\ system\\ dynamics}} + \underbrace{B(v^\top, v^\top)}_{\substack{dynamic\\ couplings}}, \quad (34)$$

where, with $X^\top, Y^\top, Z^\top \in \mathfrak{X}(\mathcal{M})$ being respective local extensions to $\mathcal{M}$ of $X^c, Y^c, Z^c \in \mathfrak{X}(\mathcal{H}_c)$ on $\mathcal{H}_c$,
1. the induced connection $\nabla^L : \mathfrak{X}(\mathcal{H}_c) \times \mathfrak{X}(\mathcal{H}_c) \to \mathfrak{X}(\mathcal{H}_c)$ defined by: for $X^c, Y^c \in \mathfrak{X}(\mathcal{H}_c)$,

$$\nabla^L_{X^c} Y^c := \left(\nabla_{X^\top} Y^\top\right)^\top, \quad (35)$$

is the Levi-Civita connection on the submanifold $\mathcal{H}_c$ w.r.t. the induced metric [12];
2. The (bilinear and symmetric) mapping $B : \mathfrak{X}(\mathcal{H}_c) \times \mathfrak{X}(\mathcal{H}_c) \to \Delta^\perp\big|_{\mathcal{H}_c}$ is second fundamental form [12] of the submanifold $\mathcal{H}_c$ defined s.t. for $X^c, Y^c \in \mathfrak{X}(\mathcal{H}_c)$,

$$B(X^c, Y^c) := \nabla_{X^\top} Y^\top - (\nabla_{X^\top} Y^\top)^\top = (\nabla_{X^\top} Y^\top)^\perp,$$

where $\Delta^\perp\big|_{\mathcal{H}_c}$ is the restricted normal distribution $\Delta^\perp$ (15) on $\mathcal{H}_c$ (i.e. $\Delta^\perp\big|_{\mathcal{H}_c} = \{\Delta^\perp(q) | q \in \mathcal{H}_c\}$). Second fundamental form $B(v, v)$ in (34) represents the tendency of the system (5) to deviate from the submanifold $\mathcal{H}_c$.

If second fundamental form $B(v, v)$ in (34) is cancelled out (e.g. by the passive decoupling control (22)), the dynamics of the interactive mechanical system (5) constrained on the submanifold $\mathcal{H}_c$ is reduced to the locked system dynamics (34) given by the Levi-Civita connection ∇^L on $\mathcal{H}_c$ w.r.t. the induced metric. Thus, the dynamics of the system (5) is reduced to that of usual mechanical systems with the reduced dimension $n-m$ on $\mathcal{H}_c$. Thus, a variety of control algorithms developed for usual mechanical systems can be used to control this interactive mechanical system (5) constrained on the submanifold $\mathcal{H}_c$.

IV. Coordination Regulation Control

Let us consider the coordination regulation problem s.t.

$$h(q(t)) \to c_d, \quad (36)$$

where $c_d \in \mathcal{N}$ is a (constant) target coordination. To achieve this objective (36), we design the additional control $T_\star$ in (22) to be a PD (proportional-derivative)-control [14] s.t.:

$$T_\star = T^{pd} := h^*\left[d\varphi(h(q)) - K_d(h(q))\left(h_*(v^\perp)\right)\right], \quad (37)$$

where 1) $h^* : T^*_{h(q)}\mathcal{N} \to T^*_q\mathcal{M}$ is the pull-back map of h s.t. $\langle h^* w_h, v_q\rangle_\mathcal{M} = \langle w_h, h_*(v_q)\rangle_\mathcal{N}$ for any tangent vector $v_q \in T_q\mathcal{M}$ and covector $w_h \in T^*_{h(q)}\mathcal{N}$; 2) $\varphi : \mathcal{N} \to \Re$ is a smooth potential function on the coordination manifold $\mathcal{N}$ with only one critical point at c_d s.t. for any $c \in \mathcal{N}$, $\varphi(c) \geq 0$, and $\varphi(c) = 0$ and $d\varphi(c) = 0$ if and only if $c = c_d$; and 3) $K_d(c) : T_c\mathcal{N} \to T^*_c\mathcal{N}$ is smooth dissipation two-form tensor field on $\mathcal{N}$, which is positive definite in the sense that for any $v^h \in T_c\mathcal{N}$, $\langle K_d(c)(v^h), v^h\rangle_\mathcal{N} \geq 0$ with equality if and only if $v^h = 0$.

Theorem 5 *Consider the interactive mechanical system* (5) *under the control* (22) *with the PD-control* T^{pd} (37).
1. Suppose that $F^{\perp} + T'^{\perp} = 0$ *in* (19)*, and the initial coordination error is bounded in the sense that:*

$$V(h(q), h_* v^{\perp}) := \frac{1}{2}\langle\langle h_* v^{\perp}, h_* v^{\perp}\rangle\rangle + \varphi(h(q)) \quad (38)$$

is initially bounced at $t = 0$*. Then,* $(h(q(t)), v^{\perp}) \rightarrow (c_d, 0)$*.*
2. The closed-loop interactive mechanical system (5) *satisfies energetic passivity condition* (10)*.*

One application of PD-control (37) is cooperative fixtureless grasping with h (13) defined to describe grasping shape. Then, internal force can be controlled by changing the set-point c_d in (36) and the potential function φ. Also, by controlling the locked system, we can control the behavior of the grasped object. Because of the decoupling, grasping aspect and object's motion will not affect each other if the object is light. Thus, secure and tight grasping can be guaranteed. Moreover, due to the passivity, interaction safety and stability can be enhanced. For more details on this application, please refer to [5].

V. Conclusion

In this paper, we propose a general control framework for multiple (or single) mechanical systems interacting with environments and/or humans under motion coordination requirements. The key innovation is the passive decomposition which enables us to decouple the dynamics of the coordination aspect and that of the coordinated system from each other while enforcing energetic passivity. Thus, we can simultaneously achieve the motion coordination and some desired behavior of the coordinated system, while ensuring safe and natural interaction with environments and/or humans. We analyze geometric property of the passive decomposition and exhibit geometry of the decomposed systems.

We believe that the proposed passive decomposition will provide new and powerful frameworks for many traditional and emerging applications where interaction safety and motion coordination are required but often compromised or neglected, such as multirobot collaborative manipulation, multirobot telemanipulation, and multi-fingered grasping.

Appendix

Here, we introduce some mathematical concepts of *connection over a map*. For more detail, please refer to [15].

Consider a smooth map $h : \mathcal{M} \rightarrow \mathcal{N}$ of a manifold $\mathcal{M}$ to a manifold $\mathcal{N}$. Then, a smooth vector field X^h over h is a smooth map $X : \mathcal{M} \rightarrow T\mathcal{N}$ s.t. for every $q \in \mathcal{M}$, $X^h(q) \in T_{h(q)}\mathcal{N}$. We denote the set of such smooth vector fields over map h by $\mathfrak{X}^h(\mathcal{M})$.

Connection over the map h assigns to each $v \in T_q\mathcal{M}$ an operator D_v^h which maps vector fields over the map h into $T_{h(q)}\mathcal{N}$, i.e. $D^h : T_q\mathcal{M} \times \mathfrak{X}^h(\mathcal{M}) \rightarrow T_{h(q)}\mathcal{N}$. We say the connection D^h is affine, if for all $v, w \in T_q\mathcal{M}$, $a, b \in \Re$, $f \in C^{\infty}(\mathcal{M})$, $X^h, Y^h \in \mathfrak{X}^h(\mathcal{M})$, and $X \in \mathfrak{X}(\mathcal{M})$:

$$D^h_{av+bw}X^h = aD^h_v X^h + bD^h_w X^h, \quad (39)$$

$$D^h_v(aX^h + bY) = aD^h_v X^h + bD^h_v Y^h, \quad (40)$$

$$D^h_v(fX^h) = \mathcal{L}_v(f)X^h(q) + f(q)D^h_v X^h, \quad (41)$$

$$(D^h_X X^h)(q) = D_{X(q)}X^h \ is\ smooth. \quad (42)$$

Torsion translation $T^h : T_q\mathcal{M} \times T_q\mathcal{M} \rightarrow T_{h(q)}\mathcal{N}$ at each $q \in \mathcal{M}$ is defined s.t.: for $v, w \in T_q\mathcal{M}$,

$$T^h(v, w) := D^h_X(h_* Y) - D^h_Y(h_* X) - h_*[X, Y], \quad (43)$$

where $X, Y \in \mathfrak{X}(\mathcal{M})$ are local extensions of v, w to smooth vector fields on $\mathcal{M}$ with $X(q) = v, Y(q) = w$, and $[,]$ is the Lie-bracket on $\mathcal{M}$. $T^h(v, w)$ (43) is independent on the choice of extensions X, Y. A connection D^h is said to be torsion-free when $T^h = 0$, $\forall v, w \in T_q\mathcal{M}$.

Define a metric over a map h to be $G \circ h$ where G is a metric on the manifold $\mathcal{N}$. Then, a connection over the map D^h is said compatible w.r.t. the induced metric $G \circ h$, if for all $X^h, Y^h \in \mathfrak{X}^h(\mathcal{M})$ and $v \in T_q\mathcal{M}$,

$$\mathcal{L}_v\langle\langle X^h(q), Y^h(q)\rangle\rangle = \langle\langle D^h_v X^h, Y^h\rangle\rangle + \langle\langle X^h, D^h_v Y^h\rangle\rangle, \quad (44)$$

where $\langle\langle X^h(q), Y^h(q)\rangle\rangle_{\mathcal{N}} := \langle G(h(q))X^h(q), Y^h(q)\rangle_{\mathcal{N}}$.

Similar to the uniqueness of the Levi-Civita connection on a Riemannian manifold [12], there exists a connection over a map h which is torsion-free and compatible w.r.t. a metric over the map, and furthermore, it is unique in a neighborhood of any point at which h_* is onto.

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