Proceedings of the 2006 IEEE International Conference on Robotics and Automation
Orlando, Florida - May 2006

Passive Bilateral Teleoperation with Constant Time Delays

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***Abstract*— We propose a novel control framework for bilateral teleoperation of a pair of multi-degree-of-freedom (DOF) nonlinear robotic systems under constant communication delays. The proposed framework utilizes the simple proportional-derivative (PD) control, i.e. the master and slave robots are directly connected via spring and damper over the delayed communication channels. Using the controller passivity concept, the Lyapunov-Krasovskii technique, and Parseval's identity, we can passify the combination of the delayed communication and control blocks altogether robustly, as long as the delays are finite constants and an upper-bound for the round-trip delay is known. Having explicit position feedback through the delayed P-action, the proposed framework enforces master-slave position coordination which is often compromised in the velocity-based schemes (e.g. conventional scattering-based teleoperation). The proposed control framework provides humans with extended physiological proprioception so that s/he can affect and sense the remote slave environments mainly relying on her/his musculoskeletal systems. Experiments are performed to validate the proposed control framework.**

I. INTRODUCTION

Energetically, as illustrated in Fig. 1, a closed-loop teleoperator is a two-port system with the master and slave ports being coupled with the human operators and slave environments, respectively. Therefore, the foremost and primary goal of the control (and communication) design for the teleoperation should be to ensure interaction safety and/or coupled stability [1] when mechanically coupled with a broad class of slave environments and humans.

To ensure such interaction safety and stability, energetic passivity (i.e. mechanical power as the supply rate [2]) of the closed-loop teleoperator has been widely utilized as the control objective [3], [4], [5]. In [6], energetic passivity of the delayed teleoperation is achieved by passifying the communication block with (possibly unknown) finite constant time-delays. This passification was made possible by applying scattering theory. In [7], this scattering-based result is further extended and the notion of the wave variables was introduced. Since these two seminal works, scattering-based (or wave-based) teleoperation has been virtually the only way to enforce energetic passivity of the time-delayed bilateral teleoperation (e.g. [3], [8], [9]).

In this paper, we propose a novel control framework for bilateral teleoperation of a pair of multi-DOF nonlinear robotic systems with finite constant communication delays. The proposed framework is based on the simple proportional-derivative (PD) control, i.e. directly connecting the master and slave robots via spring and damper over the delayed communication channels. Then, utilizing the controller passivity concept [5], [10], the Lyapunov-Krasovskii technique for delayed systems [11], and Parseval's identity [12], we can enforce energetic passivity of the closed-loop teleoperator as long as the communication delays are finite constants and an upper-bound of the round-trip delay (i.e. sum of the forward and backward delays) can be known, even if the delays are asymmetric (i.e. $\tau_1 \neq \tau_2$ in Fig. 1) or their exact estimates are not available. Relying on the controller passivity concept, this passivity is ensured even in the presence of model parametric uncertainty (i.e. robust passivity [5], [10]).

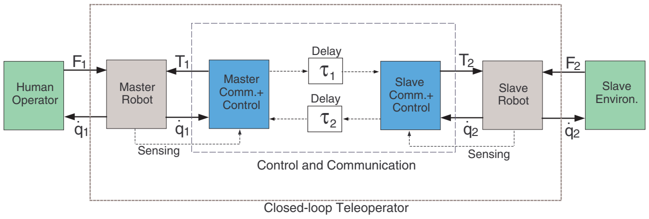

Fig. 1. Closed-loop teleoperator as a two-port system.

In [13], we proposed a similar PD-based teleoperation scheme, which also enforces passivity. However, it requires that the communication delays be exactly known and symmetric (i.e. $\tau_1 = \tau_2$). In this paper, we completely remove these two requirements, which are difficult to satisfy in practice.

Compared to the velocity-based schemes (e.g. conventional scattering-based teleoperation), the main advantage of the proposed framework is the explicit position feedback through the delayed P-control action (i.e. spring term with delayed set position). The lack of such explicit position feedback has been known as the major drawback of the conventional scattering-based teleoperation, in which, roughly speaking, the velocity information is extracted from the communicated scattering variables, and then, integrated to recover the set position information. Therefore, if this integration becomes inaccurate (e.g. slave robot makes a hard contact with a rigid wall or communication is blacked out shortly), the master and slave positions may start drifting away from each other (see [14] for example). Having the explicit position feedback, our proposed framework prevents such a position drifting. This explicit position feedback also enables us to guarantee (i.e. theoretically prove) asymptotic master-slave position coordination.

In contrast to the scattering-teleoperation where the delayed communication block is passified so that the closed-

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loop teleoperator becomes an interconnection of passive submodules (i.e. passified communication, passive control, and passive master/slave robots), the proposed framework passifies the combination of the communication and control blocks altogether. While the scattering-based teleoperation can be used without any knowledge on the (finite constant) delays, our proposed framework requires to know an upper-bound of the round-trip delay. However, we think that this is a mild restriction, since, in many applications, such a round-trip delay is relatively easy to measure/estimate [15].

Although it achieves at least a level of ideal transparency [3], the main goal of the proposed control framework is to provide humans with extended physiological proprioception (EPP) [16], i.e. the closed-loop teleoperator as a tool, by which the human operator can affect and sense the remote slave environments mainly relying on her/his musculoskeletal systems. In this sense, the proposed framework is along the line of such research works as "common passive mechanical tool" [4] and "virtual tool for wave-based teleoperation" [9].

The rest of this paper is organized as follows. The control problem is formulated in section II, and the control law is designed and its properties are detailed in section III. Experimental results are presented in section IV and section V contains some concluding remarks. This paper is a short version of our recent journal paper [17], to which we refer readers for more details.

II. PROBLEM FORMULATION

A. Modeling of Teleoperators under Constant Time Delay

Let us consider a teleoperator consisting of a pair of n-degree-of-freedom (DOF) nonlinear robotic systems:

$$M_1(q_1)\ddot{q}_1(t) + C_1(q_1,\dot{q}_1)\dot{q}_1 = T_1(t) + F_1(t), \tag{1}$$

$$M_2(q_2)\ddot{q}_2(t) + C_2(q_2,\dot{q}_2)\dot{q}_2 = T_2(t) + F_2(t), \tag{2}$$

where $q_i(t), F_i(t), T_i(t) \in \Re^n$ are the configurations, human/environmental force, and controls, and $M_i(q_i)$, $C_i(q_i,\dot{q}_i) \in \Re^{n\times n}$ are symmetric and positive-definite inertia matrices and Coriolis matrices, respectively, s.t. $\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)$ are skew-symmetric ($i = 1, 2$). Here, we assume that the gravity effects are either included in $F_1(t), F_2(t)$ or pre-compensated by the local controls.

In this work, we assume the communication structure as shown in Fig. 1, where the forward and backward communications are delayed by finite constant time-delays $\tau_1 \geq 0$ and $\tau_2 \geq 0$, respectively. Then, the controls $T_1(t), T_2(t)$ in (1)-(2) can be defined as functions of the current local information and the delayed remote information (directly received from the communication line), i.e.

$$T_1(t) := T_1(q_1(t), \dot{q}_1(t), q_2(t-\tau_2), \dot{q}_2(t-\tau_2)) \in \Re^n, \tag{3}$$

$$T_2(t) := T_2(q_2(t), \dot{q}_2(t), q_1(t-\tau_1), \dot{q}_1(t-\tau_1)) \in \Re^n. \tag{4}$$

B. Control Objectives

We would like to design $T_1(t), T_2(t)$ in (3)-(4) to achieve *master-slave position coordination*: if $(F_1(t), F_2(t)) = 0$,

$$q_E(t) := q_1(t) - q_2(t) \to 0, \qquad t \to \infty; \tag{5}$$

and *static force reflection*: with $(\ddot{q}_1(t), \ddot{q}_2(t), \dot{q}_1(t), \dot{q}_1(t)) \to 0$,

$$F_1(t) \to -F_2(t). \tag{6}$$

For safe interaction and coupled stability, we would also like to enforce the following *energetic passivity* of the closed-loop teleoperator (1)-(2): $\exists$ a finite constant $d \in \Re$ s.t.

$$\int_0^t \left[F_1^T(\theta)\dot{q}_1(\theta) + F_2^T(\theta)\dot{q}_2(\theta)\right] d\theta \geq -d^2, \qquad \forall t \geq 0, \tag{7}$$

i.e. maximum extractable energy from the two-port closed-loop teleoperator is bounded (see Fig. 1). Let us also define *controller passivity* [4], [5], [10]: $\exists$ a finite constant $c \in \Re$ s.t.

$$\int_0^t \left[T_1^T(\theta)\dot{q}_1(\theta) + T_2^T(\theta)\dot{q}_2(\theta)\right] d\theta \leq c^2, \qquad \forall t \geq 0, \tag{8}$$

i.e. the two-port controller in Fig. 1 generates only bounded amount of energy.

Lemma 1 *[5], [10] For the mechanical teleoperator* (1)-(2)*, controller passivity* (8) *implies energetic passivity* (7)*.*

Proof: Let us define the total kinetic energy

$$\kappa_f(t) := \frac{1}{2}\dot{q}_1^T(t)M_1(q_1)\dot{q}_1(t) + \frac{1}{2}\dot{q}_2^T(t)M_2(q_2)\dot{q}_2(t), \tag{9}$$

then, using (1)-(2) with its skew-symmetric property, we have

$$\frac{d}{dt}\kappa_f(t) = [T_1(t) + F_1(t)]^T\dot{q}_1(t) + [T_2(t) + F_2(t)]^T\dot{q}_2(t). \tag{10}$$

Thus, by integrating (10) with the controller passivity condition (8) and the fact that $\kappa_f(t) \geq 0$, we have, $\forall t \geq 0$, $\int_0^t [F_1^T(\theta)\dot{q}_1(\theta) + F_2^T(\theta)\dot{q}_2(\theta)]d\theta \geq -\kappa_f(0) - c^2 =: -d^2$. ■

Lemma 1 is simple but powerful in the sense that it enables us to analyze energetic passivity (7) of the closed-loop teleoperator by examining only the controller structure which is often much simpler than that of the closed-loop dynamics. Furthermore, by enforcing controller passivity (8), energetic passivity (7) will be guaranteed robustly (robust passivity [5], [10]), because controller passivity (8) does not depend on the possibly uncertain open-loop dynamics (1)-(2).

III. CONTROL DESIGN

To achieve the master-slave coordination (5), bilateral force reflection (6), and energetic passivity (7), we design the master and slave controls $T_1(t), T_2(t)$ in (3)-(4) to be

$$\begin{aligned} T_1(t) := &- K_v(\dot{q}_1(t) - \dot{q}_2(t-\tau_2)) \\ &- (K_d + P_\epsilon)\dot{q}_1(t) - K_p(q_1(t) - q_2(t-\tau_2)), \end{aligned} \tag{11}$$

$$\begin{aligned} T_2(t) := &- K_v(\dot{q}_2(t) - \dot{q}_1(t-\tau_1)) \\ &- (K_d + P_\epsilon)\dot{q}_2(t) - K_p(q_2(t) - q_1(t-\tau_1)), \end{aligned} \tag{12}$$

where $\tau_1, \tau_2 \geq 0$ are the forward and backward finite constant delays, $K_v, K_p \in \Re^{n\times n}$ are the symmetric and positive-definite proportional (P) and derivative (D) control gains, $K_d \in \Re^{n\times n}$ is the dissipation to passify the delayed P-action (i.e. with K_p) in (11)-(12) (to be designed below), and $P_\epsilon \in \Re^{n\times n}$ is an additional damping ensuring master-slave coordination

(5). Inherent device viscous damping can substitute this P_ϵ. Note that the control laws (11)-(12) contain the explicit position feedback (via the P-action), which is generally absent in the conventional scattering-based teleoperation.

To enforce energetic passivity (7), we design dissipation K_d and P-gain K_p in (11)-(12) to satisfy the following condition:

$$K_d \succcurlyeq \left[\frac{\sin\frac{w(\tau_1+\tau_2)}{2}}{w}\right]^2 K_p K_d^{-1} K_p, \qquad \forall w \in \Re, \tag{13}$$

where, for square matrices A, B, $A \succcurlyeq B$ implies that $A - B$ is positive-semidefinite. As to be shown below, with this condition (13), the delayed P-control action (i.e. with K_p) is passified by the dissipation K_d. From the fact that $\tau - \frac{\sin w\tau}{w} \geq 0$, $\forall w \in \Re$, one possible solution for the condition (13) is

$$K_d = \frac{\bar{\tau}_{rt}}{2} K_p, \tag{14}$$

where $\bar{\tau}_{rt} \geq 0$ is an upper-bound of the round-trip delay $\tau_{rt} := \tau_1 + \tau_2$ s.t. $\bar{\tau}_{rt} \geq \tau_{rt}$.

Theorem 1 *Consider the mechanical teleoperator* (1)-(2) *with the controls* (11)-(12) *under the condition* (13).
1) (Robust Passivity) The closed-loop teleoperator is energetically passive (i.e. satisfies (7)*) regardless of parametric uncertainty in the open-loop dynamics* (1)-(2)*;*
2) (Coupled Stability) Suppose that the human operator and slave environment in Fig. 1 are energetically passive: $\exists$ finite constants $d_1, d_2 \in \Re$ s.t. $\forall t \geq 0$,

$$\int_0^t \underbrace{-F_i^T(\theta)\dot{q}_i(\theta)}_{\text{power inflow to human/environ.}} d\theta \geq -d_i^2, \; i = \{1, 2\}, \tag{15}$$

i.e. the maximum extractable energy from them are bounded. Then, $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_\infty$. Thus, if the human and slave environment are $\mathcal{L}_\infty$-stable input-output impedance maps, we will also have $F_1(t), F_2(t) \in \mathcal{L}_\infty$;
3) (Position Coordination) Suppose that the human operator and slave environment are passive in the sense of (15)*. Suppose further that $M_i^{jk}(q_i)$, $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$ and $\frac{\partial^2 M_i^{jk}(q_i)}{\partial q_i^m \partial q_i^l}$ are all bounded w.r.t. q_i, where $M_i^{jk}(q_i)$ and q_i^m are the jk-th and the m-th components of $M_i(q_i)$ and q_i, respectively. Then, $q_E(t) = q_1(t) - q_2(t)$ is bounded $\forall t \geq 0$. Moreover, if $(F_1(t), F_2(t)) = 0$ $\forall t \geq 0$, $(q_E(t), \dot{q}_E(t)) \to 0$;*
4) (Static Force Reflection) If $(\dot{q}_1(t), \dot{q}_2(t), \ddot{q}_1(t), \ddot{q}_2(t)) \to 0$, then, $F_1(t) \to -F_2(t) \to -K_p(q_1 - q_2)$.

Proof: 1) Let us denote the mechanical power generated by the controls (11)-(12) by

$$\begin{aligned} s_c(t) :&= T_1^T(t)\dot{q}_1(t) + T_2^T(t)\dot{q}_2(t) \\ &= s_d(t) + s_p(t) - P(t), \end{aligned} \tag{16}$$

where $s_d(t)$ and $s_p(t)$ are the supply rates associated to the delayed D-action, and delayed P-action (+ dissipation K_d), respectively defined by

$$\begin{aligned} s_d(t) :=& -\dot{q}_1^T(t)K_v\dot{q}_1(t) + \dot{q}_1^T(t)K_v\dot{q}_2(t-\tau_2) \\ &- \dot{q}_2^T(t)K_v\dot{q}_2(t) + \dot{q}_2^T(t)K_v\dot{q}_1(t-\tau_1), \end{aligned} \tag{17}$$

$$\begin{aligned} s_p(t) :=& -\dot{q}_1^T(t)K_d\dot{q}_1(t) - \dot{q}_1^T(t)K_p(q_1(t) - q_2(t-\tau_2)) \\ &- \dot{q}_2^T(t)K_d\dot{q}_2(t) - \dot{q}_2^T(t)K_p(q_2(t) - q_1(t-\tau_1)), \end{aligned} \tag{18}$$

and $P(t)$ is the following quadratic form:

$$P(t) := \begin{pmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{pmatrix}^T \begin{bmatrix} P_\epsilon & 0 \\ 0 & P_\epsilon \end{bmatrix} \begin{pmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{pmatrix}. \tag{19}$$

We want the total controller supply rate $s_c(t)$ in (16) to satisfy the controller passivity (8).

Let us first consider the delayed D-action supply rate $s_d(t)$ in (17). Then, using the fact that, for $(i,k) = \{(1,2),(2,1)\}$,

$$\begin{aligned} &2\dot{q}_i^T(t)K_v\dot{q}_k(t-\tau_k) \\ &\qquad \leq \dot{q}_i^T(t)K_v\dot{q}_i(t) + \dot{q}_k^T(t-\tau_k)K_v\dot{q}_k(t-\tau_k), \end{aligned} \tag{20}$$

we can show that

$$\begin{aligned} s_d(t) &\leq -\sum_{i=1}^{2} \frac{1}{2}\left[\dot{q}_i^T(t)K_v\dot{q}_i(t) - \dot{q}_i^T(t-\tau_i)K_v\dot{q}_i(t-\tau_i)\right] \\ &= -\frac{d}{dt}V_v(t), \end{aligned} \tag{21}$$

where $V_v(t)$ is a Lyapunov-Krasovskii functional for delayed systems [11] defined by

$$V_v(t) := \sum_{i=1}^{2} \frac{1}{2}\int_{-\tau_i}^{0} \dot{q}_i^T(t+\theta)K_v\dot{q}_i(t+\theta)d\theta \geq 0. \tag{22}$$

Then, by integrating the inequality (21), we have

$$\int_0^t s_d(\theta)d\theta \leq -V_v(t) + V_v(0), \tag{23}$$

i.e. energy generation by the delayed D-action is always bounded by the energy stored in $V_v(t)$ (i.e. $V_v(t)$ is the storage function for the supply rate $s_d(t)$).

Now, let us consider the supply rate $s_p(t)$ in (18). Then, we can rewrite $s_p(t)$ in (18) s.t.: with $\mathcal{N} := \{(1,2),(2,1)\}$,

$$\begin{aligned} s_p(t) =& -\dot{q}_1^T(t)K_d\dot{q}_1(t) - \dot{q}_2^T(t)K_d\dot{q}_2(t) \\ &- \dot{q}_1^T(t)K_p(q_1(t) - q_2(t)) - \dot{q}_1^T(t)K_p(q_2(t) - q_2(t-\tau_2)) \\ &- \dot{q}_2^T(t)K_p(q_2(t) - q_1(t)) - \dot{q}_2^T(t)K_p(q_1(t) - q_1(t-\tau_1)) \\ =& -\dot{q}_1^T(t)K_d\dot{q}_1(t) - \dot{q}_2^T(t)K_d\dot{q}_2(t) - \frac{d}{dt}V_p(t) \\ &- \sum_{(i,k)\in\mathcal{N}} \dot{q}_i^T(t)K_p(q_k(t) - q_k(t-\tau_k)) \end{aligned} \tag{24}$$

where $V_p(t)$ is the P-action spring energy defined as

$$V_p(t) := \frac{1}{2}q_E^T(t)K_p q_E(t), \tag{25}$$

with $q_E(t) = q_1(t) - q_2(t)$ given in (5).

Let us define the truncated signal $\tilde{\dot{q}}_i^t(\theta)$ of $\dot{q}_i(t)$ s.t.

$$\tilde{\dot{q}}_i^t(\theta) := \begin{cases} \dot{q}_i(\theta) & \text{if } \theta \in [0,t] \\ 0 & \text{otherwise} \end{cases}, \tag{26}$$

$i = 1, 2$. Then, the energy generation by the supply rate $s_p(t)$ in (24) can be written as:

$$\int_0^t s_p(\theta)d\theta = -V_p(t) + V_p(0) - \sum_{i=1}^{2} \int_{-\infty}^{+\infty} \tilde{\dot{q}}_i^{tT}(\theta) K_d \tilde{\dot{q}}_i^t(\theta)d\theta$$
$$- \int_{-\infty}^{+\infty} \tilde{\dot{q}}_1^{tT}(\theta) K_p \underbrace{\left[\int_{-\infty}^{\theta} \tilde{\dot{q}}_2^t(\xi)d\xi - \int_{-\infty}^{\theta-\tau_2} \tilde{\dot{q}}_2^t(\xi)d\xi\right]}_{:=g_2^t(\theta)} d\theta$$
$$- \int_{-\infty}^{+\infty} \tilde{\dot{q}}_2^{tT}(\theta) K_p \underbrace{\left[\int_{-\infty}^{\theta} \tilde{\dot{q}}_1^t(\xi)d\xi - \int_{-\infty}^{\theta-\tau_1} \tilde{\dot{q}}_1^t(\xi)d\xi\right]}_{:=g_1^t(\theta)} d\theta. \tag{27}$$

Let us denote the Fourier transform of $\tilde{\dot{q}}_i^t$ ($i = 1, 2$) by $V_i^t(w) := \int_{-\infty}^{+\infty} \tilde{\dot{q}}_i^t(\theta) e^{-jw\theta} d\theta = \int_0^t \dot{q}_i(\theta) e^{-jw\theta} d\theta$, where $j = \sqrt{-1}$. Then, the Fourier transform of $g_i^t(\theta)$ in (27) (i.e. $G_i^t(w) := \int_{-\infty}^{+\infty} g_i^t(\theta) e^{-jw\theta} d\theta$) is given by $G_i^t(w) = \frac{1-e^{-jw\tau_i}}{jw} V_i^t(w)$. Let us denote the complex conjugate transpose of a complex vector $\star \in \mathbb{C}^n$ by $\star^*$ (i.e. $\star^* = \bar{\star}^T$). Then using the Parseval's identity [12], (27) can be rewritten as

$$\int_0^t s_p(\theta)d\theta = -V_p(t) + V_p(0) \tag{28}$$
$$- \sum_{i=1}^{2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} V_i^{t*}(w) K_d V_i^t(w) dw$$
$$- \sum_{(i,k)\in\mathcal{N}} \frac{1}{2\pi} \int_{-\infty}^{+\infty} V_i^{t*}(w) K_p \frac{1-e^{-jw\tau_k}}{jw} V_k^t(w) dw$$
$$= -V_p(t) + V_p(0) - \frac{1}{2\pi} \int_{-\infty}^{+\infty} \begin{pmatrix} \bar{V}_1^t(w) \\ \bar{V}_2^t(w) \end{pmatrix}^T H(w) \begin{pmatrix} V_1^t(w) \\ V_2^t(w) \end{pmatrix} dw,$$

where $\mathcal{N} = \{(1, 2), (2, 1)\}$ and

$$H(w) = \begin{bmatrix} K_d & \frac{K_p}{2}\frac{e^{jw\tau_1} - e^{-jw\tau_2}}{jw} \\ \frac{K_p}{2}\frac{e^{jw\tau_2} - e^{-jw\tau_1}}{jw} & K_d \end{bmatrix} \in \mathbb{C}^{2n\times 2n}.$$

For more details on these derivations, see [17]. Since $H(w)$ is Hermitian (i.e. $H^*(w) = \bar{H}^T(w) = H(w)$) with positive-definite block diagonal matrices K_d, following [18, pp.473], $H(w) \succcurlyeq 0$ (i.e. $H(w)$ is positive-semidefinite) if and only if

$$K_d \succcurlyeq \frac{e^{jw\tau_1} - e^{-jw\tau_2}}{2jw} \frac{e^{jw\tau_2} - e^{-jw\tau_1}}{2jw} K_p K_d^{-1} K_p, \tag{29}$$

which is nothing but the condition (13). Thus, with the condition (13) s.t. $H(w) \succcurlyeq 0$, we can show from (28) that

$$\int_0^t s_p(\theta)d\theta \leq -V_p(t) + V_p(0), \tag{30}$$

i.e. energy generation by the supply rate $s_p(t)$ is always bounded by the P-action spring energy $V_p(t)$ in (25).

Thus, by summing up (23) and (30) with (16) and the fact that $V_v(t) \geq 0$ and $V_p(t) \geq 0$ $\forall t \geq 0$, we can prove controller passivity (8) s.t. for all $\forall t \geq 0$,

$$\int_0^t [T_1^T(\theta)\dot{q}_1(\theta) + T_2^T(\theta)\dot{q}_2(\theta)]d\theta$$
$$\leq -V_v(t) + V_v(0) - V_p(t) + V_p(0) - \int_0^t P(\theta)d\theta$$
$$\leq V_v(0) + V_p(0) =: c^2, \tag{31}$$

where the term $V_v(0)$ will be zero if we start from zero velocities (i.e. $(\dot{q}_1(t), \dot{q}_2(t)) = 0$ $\forall t \in (-\infty, 0]$), while the term $V_p(0)$ would be small, if the initial coordination error $q_E(0) = q_1(0) - q_2(0)$ is small. Finally, from Lemma 1, energetic passivity (7) of the closed-loop teleoperator follows.

2) By integrating (10) with (31) and (15), we have, $\forall t \geq 0$,

$$\kappa_f(t) + V_v(t) + V_p(t) \tag{32}$$
$$\leq \kappa_f(0) + V_v(0) + V_p(0) - \int_0^t P(\theta)d\theta + d_1^2 + d_2^2,$$

where $P(t) \geq 0$ in (19). Here, since $V_v(0), V_p(0), d_1^2, d_2^2$ and $\kappa_f(0)$ are all bounded, $\kappa_f(t)$ is bounded. Thus, $\dot{q}_1(t), \dot{q}_2(t)$ are also bounded $\forall t \geq 0$ (i.e. $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_\infty$). Therefore, if the human operator and the slave environment are $\mathcal{L}_\infty$-stable impedance maps, $F_1(t), F_2(t) \in \mathcal{L}_\infty$.

3) Boundedness of $q_E(t) = q_1(t) - q_2(t)$ is a direct consequence of (32) with the definition of $V_p(t)$ in (25).

First step of the position-coordination proof is to show that $(\dot{q}_1(t), \dot{q}_2(t)) \to 0$. Suppose that $F_1(t), F_2(t) = 0$, $\forall t \geq 0$. Then, from (32) with $d_1 = d_2 = 0$ and the boundedness of $P_\epsilon, M_1(q_1), M_2(q_2)$, we have: $\forall t \geq 0$,

$$\kappa_f(t) \leq \kappa_f(0) + c^2 - \int_0^t P(\theta)d\theta$$
$$\leq \kappa_f(0) + c^2 - \gamma \int_0^t \kappa_f(\theta)d\theta, \tag{33}$$

where $c^2 = V_v(0) + V_p(0)$ as given in (31) and $\gamma > 0$ is a constant scalar. Here, since $\kappa_f(t) \geq 0$, the term $\int_0^t \kappa_f(\theta)d\theta$ is monotonically increasing and upper bounded, thus, it converges to a limit. Therefore, following Barbalat's lemma, if $\kappa_f(t)$ is uniformly continuous, $\kappa_f(t)$ will also converge to 0 (i.e. $(\dot{q}_1(t), \dot{q}_2(t)) \to 0$). To show this, let us consider $\frac{d}{dt}\kappa_f(t)$. Then, from (10) with $F_1(t) = F_2(t) = 0$, we have $\frac{d}{dt}\kappa_f(t) = T_1^T(t)\dot{q}_1(t) + T_2^T(t)\dot{q}_2(t)$, where $\dot{q}_1(t), \dot{q}_2(t)$ are bounded from item 2 of this theorem. Also, $T_1(t), T_2(t)$ are bounded, since, in their definitions (11)-(12), for $(i, k) = \{(1, 2), (2, 1)\}$, 1) $\dot{q}_i(t) - \dot{q}_k(t - \tau_k)$ is bounded (with bounded $\dot{q}_i(t)$); and 2) $q_i(t) - q_k(t - \tau_k) = q_E(t) + \int_{-\tau_k}^{0} \dot{q}_k(t + \theta)d\theta$ is also bounded (with bounded $\dot{q}_i(t)$, $q_E(t)$ and τ_k). Thus, $\frac{d}{dt}\kappa_f(t)$ is bounded, and $\kappa_f(t)$ is uniformly continuous. Therefore, $\kappa_f(t) \to 0$ and $(\dot{q}_1(t), \dot{q}_2(t)) \to 0$.

The next step of the proof is to show that $(\ddot{q}_1(t), \ddot{q}_2(t)) \to 0$. Let us consider the dynamics (1)-(2) with $F_1(t) = F_2(t) = 0$, where, as shown in the above paragraph, the controls $T_1(t), T_2(t)$ in (11)-(12) are bounded. Also, from the boundedness assumption of $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$, the Coriolis terms $C_i(q_i, \dot{q}_i)\dot{q}_i$ ($i = 1, 2$) in (1)-(2) are bounded. Thus, the accelerations

$\ddot{q}_1(t), \ddot{q}_2(t)$ are also bounded $\forall t \geq 0$. Now, let us consider the acceleration $\ddot{q}_i(t)$ in (1)-(2) (with $F_i(t) = 0$):

$$\ddot{q}_i = -M_i^{-1}(q_i)C_i(q_i, \dot{q}_i)\dot{q}_i + M_i^{-1}(q_i)T_i(t), \ i = 1, 2, \quad (34)$$

where the time-derivatives of the terms in the RHS are all bounded, due to the boundedness of $\ddot{q}_i(t)$, $\dot{q}_i(t)$, $q_E(t)$, $\frac{d}{dt}M_i^{-1}(q_i) = -M_i^{-1}(q_i)\frac{d}{dt}M_i(q_i)M_i^{-1}(q_i)$ (from the boundedness of $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$), and $\frac{d}{dt}[C_i(q_i, \dot{q}_i)\dot{q}_i]$ (from the boundedness of $\frac{\partial^2 M_i^{jk}(q_i)}{\partial q_i^m \partial q_i^l}$, $\dot{q}_i$, and $\ddot{q}_i$). This implies that the RHS of (34) is uniformly continuous. Thus, $\ddot{q}_1(t), \ddot{q}_2(t)$ are also uniformly continuous. Therefore, following Barbalat's lemma, $(\ddot{q}_1(t),\ \ddot{q}_2(t)) \rightarrow 0$ as $(\dot{q}_1(t), \dot{q}_2(t)) \rightarrow 0$.

Now, let us consider the dynamics (1)-(2) with the controls $T_1(t), T_2(t)$ in (11)-(12) and $F_1(t) = F_2(t) = 0$. Then, since $(\ddot{q}_1(t), \ddot{q}_2(t), \dot{q}_1(t), \dot{q}_2(t)) \rightarrow 0$, we have $K_p(q_i(t) - q_k(t - \tau_k)) \rightarrow 0$, $(i, k) = \{(1, 2), (2, 1)\}$. This condition can be rewritten as $K_p(q_E(t) + \int_{-\tau_k}^{0} \dot{q}_k(t + \theta)d\theta) \rightarrow 0$, where the second term in the parenthesis goes to zero, because $\dot{q}_k(t) \rightarrow 0$ and τ_k is finite. Therefore, since K_p is positive-definite, $q_E(t) \rightarrow 0$ (i.e. $q_1(t) \rightarrow q_2(t)$).

4) Suppose that $(\dot{q}_1(t), \dot{q}_2(t), \ddot{q}_1(t), \ddot{q}_2(t)) \rightarrow 0$. Then, from the dynamics (1)-(2) and their controls (11)-(12), we have:

$$F_1(t) \rightarrow -K_p(q_1 - q_2),\ F_2(t) \rightarrow -K_p(q_2 - q_1), \quad (35)$$

with $q_i(t - \tau) \rightarrow q_i(t) \rightarrow q_i$ as $\dot{q}_i(t), \ddot{q}_i(t) \rightarrow 0$. ■

In Theorem 1, the negative signs in the passivity condition for the human and slave environment (15) come from the fact that the power inflows to those systems are given by $-F_i(t)\dot{q}_i(t)$, i.e. the product of the reaction force $-F_i(t)$ and the interaction velocity $\dot{q}_i(t)$. The boundedness assumption on $M_i^{jk}(q_i)$, $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$ and $\frac{\partial^2 M_i^{jk}(q_i)}{\partial q_i^m \partial q_i^l}$ in Theorem 1 is guaranteed, if the robot's configuration space is compact and its inertia matrix is smooth. Such compact configuration space and smooth inertia are possessed by many practical robotic systems (e.g. revolute joint robots).

The condition (14) (or (13)) enables us to passify the delayed P-action in (11)-(12) by the dissipation K_d. This delayed P-action contains an explicit position feedback information, the lack of which is recognized as the main cause of the master-slave position drift in the conventional scattering-based teleoperation. In contrast, the delayed D-action in (11)-(12) is itself passive with the Lyapunov-Krasovskii function $V_v(t)$ in (22) as its storage function (see (23)). Since the condition (14) can be achieved as long as the delays τ_1, τ_2 are finite constants and their round-trip delay (i.e. $\tau_1 + \tau_2$) is upper-bounded, energetic passivity (7) can also be ensured with such finite constant delays, even if they are not exactly known or asymmetric (i.e. $\tau_1 \neq \tau_2$). In [13], we proposed a similar PD-based scheme, which also enforces passivity. However, it requires the delays to be symmetric and exactly known.

The condition (14) imposes the following implications on the system performance: 1) with the same static force reflection performance (i.e. same K_p, see item 4 of Theorem 1), the motion agility (i.e. less K_d) would be compromised as the delays becomes longer, since, in the condition (14), the required dissipation K_d is proportional to the round-trip delay τ_{rt}; and 2) with the delays fixed, there is a trade-off between the static force reflection performance (i.e. K_p) and motion agility (i.e. K_d), since, under the condition (14), a large K_p (i.e. sharp force reflection) requires a large K_d (i.e. poor motion agility), or a small K_d (i.e. agile free-motion) permits only a small K_p (i.e. poor force reflection).

The key step in the proof of Theorem 1 is the use of the Parseval's identity in (28) which we assume to be true. A sufficient condition for the Parseval's identity to hold is that $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_2$ [12]. As the following Lemma shows, this sufficient condition is guaranteed in many practical situations where the human and slave environment are passive in the sense of (15), the inertia matrices of the master and slave robots (1)-(2) have bounded partial-derivative w.r.t. q_i (i.e. $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$ are bounded), and the master and slave velocities and the coordination error are initially bounded. Due to the space limitation, we omit the proof here and refer readers to [17]. The Parseval's identity was also used in [19] to ensure the energetic passivity of haptic-interfaces under zero-order-hold.

Lemma 2 *Suppose that the human and slave environment are passive in the sense of* (15) *and define* $\mathcal{L}_\infty$*-stable impedance maps (i.e. if* $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_\infty,\ F_1(t), F_2(t) \in \mathcal{L}_\infty$*). Suppose further that* $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$ *are bounded. Then, if* $\dot{q}_1(0), \dot{q}_2(0), q_E(0)$ *are bounded,* $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_2$*, thus, the Parseval's identity in* (28) *holds and the items* 1-3 *of Theorem 1 are ensured.*

IV. EXPERIMENT

For the experiment, we use a pair of direct-drive planar 2-DOF serial-links revolute-joint robots. More details on the hardware/software construction of this system can be found in [17] along with more experiment and simulation results.

The control laws (11)-(12) are derived for the Cartesian-space dynamics so that, following item 4 of Theorem 1, the Cartesian (static) force reflection can be achieved. We install an aluminum wall in the slave environment, and $2.5[ms]$ sampling rate is obtained. We set the delays s.t. $(\tau_1, \tau_2) = (1.2, 1.8)[sec]$ (i.e. $\tau_{rt} = 3[sec]$). The P-gain K_p and dissipation K_d in (11)-(12) are designed according to the condition (14). Additional damping P_ϵ is omitted in the control implementation, as we leave the device friction uncompensated.

We consider the following scenario: 1) initially, a human stabilizes the teleoperator; 2) then, without seeing, moves the slave close to the aluminum wall and keeps pushing the master until s/he perceives the wall; 3) makes a hard contact; and 4) finally, retracts the slave from the wall.

Experimental results are given in Fig. 2. As shown by the force profile in Fig. 2, the human operator can perceive the aluminum wall through the force reflection. Also, when the contact is removed (i.e. free-motion), the master and slave positions become coordinated with each other. These free-motion and contact behavior are all stable, as we enforce passivity (7) through the condition (14). After the contact, the human operates the master so that the slave returns to its starting position. This leads into bumps in the human force in Fig. 2 after around $60[sec]$.

In Fig. 2, there are some errors in both the force reflection and position coordination (e.g. around $80[sec]$). These errors are due to the substantial (bearing) Coulomb friction of the robots, which we found can go up to $12[N]$. For instance, suppose in (1)-(2) that $F_1 = F_h - \mu_h$ and $F_2 = F_e - \mu_e$, where F_h, F_e are the human/contact force, and μ_h, μ_e are the frictions. During the contact, the frictions always oppose the human/contact forces. Then, from item 4 of Theorem 1, $F_h \to -F_e - (\mu_e + \mu_h)$, i.e. $(\mu_e + \mu_h)$ causes force reflection error (e.g. around $40[sec]$). These errors were not observed when we performed a simulation without such frictions [17].

Although we did not observe any substantial problems during the experiment, the required dissipation K_d (from (14)) may cause sluggish system behavior and limit the framework's usability, especially when the task requires agile operation but the delays are large. How to further minimize this required K_d while enforcing passivity is a topic for future work.

V. CONCLUSIONS

The main advantage of the proposed framework is in that we can exploit the benefits of explicit position feedback while still retaining passivity. Also, due to its simple PD-based structure which are pervasive in many industrial control systems, we think that the proposed framework would make a good impact in the real practice of teleoperation applications.

We believe that the proposed framework is promising for the Internet teleoperation, where its explicit position feedback would do an important role to recover the position coordination in the presence of packet-loss and time-varying delays.

ACKNOWLEDGMENT

Research partially supported by the Office of Naval Research under grants N00014-02-1-0011 and N00014-05-1-0186, the National Science Foundation under grants IIS 02-33314, CCR 02-09202 and ECS-01-22412, and the College of Engineering at the University of Illinois.

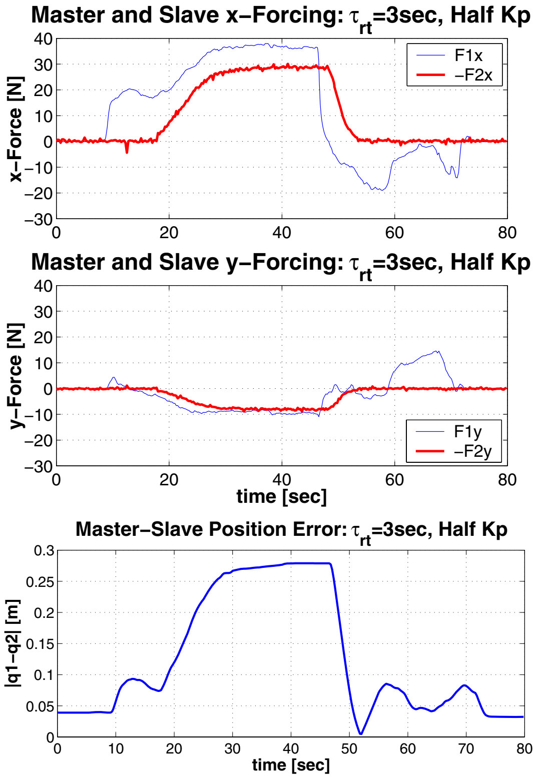

Fig. 2. Experimental results with the round-trip delay $\tau_{rt} = 3[sec]$.

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