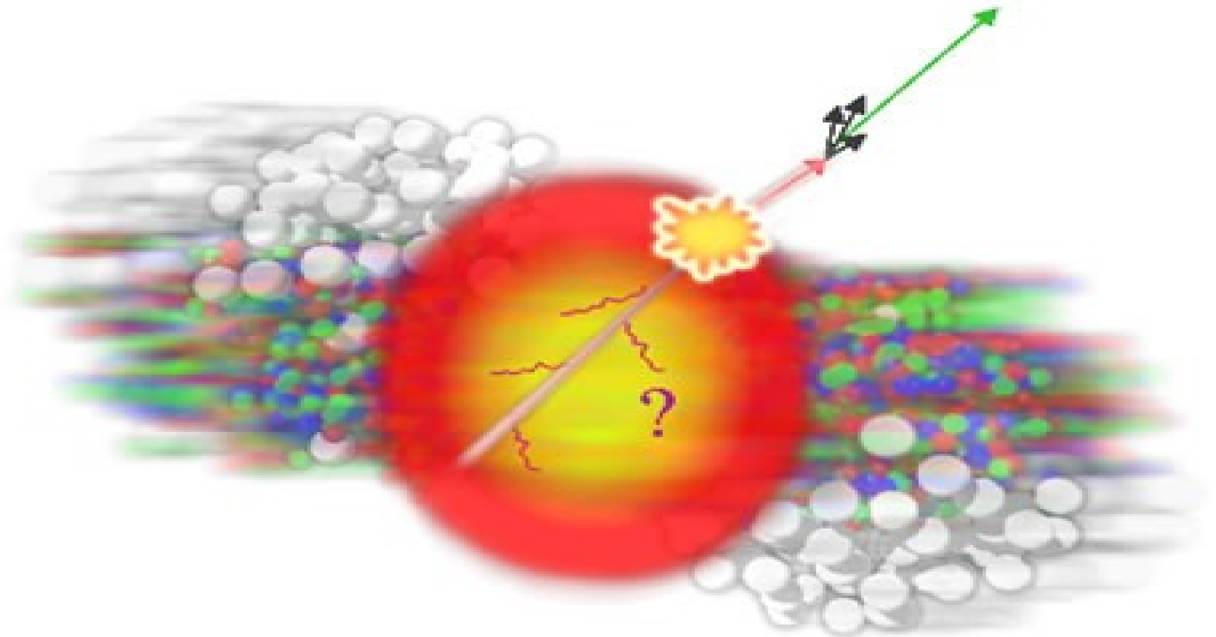
Jet spectra



Christine Nattrass
University of Tennessee, Knoxville

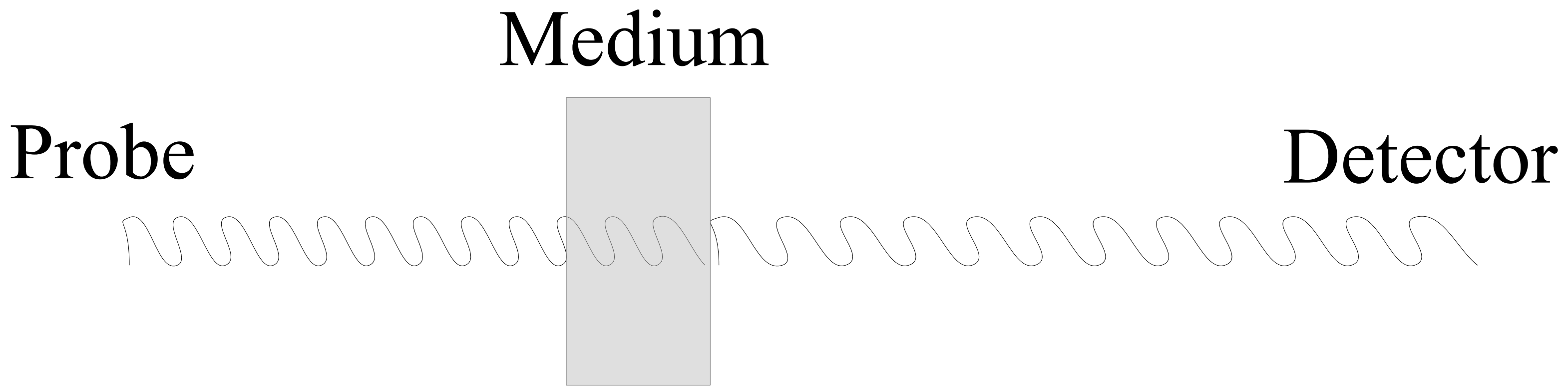
Questions an experimentalist should ask

- What do I want to learn?
- What am I measuring?
- What assumptions am I making?
- What are the dominant uncertainties?
- How do I compare to models?

The answers for jets are highly non-trivial!

What do I want to learn?
The cartoon picture

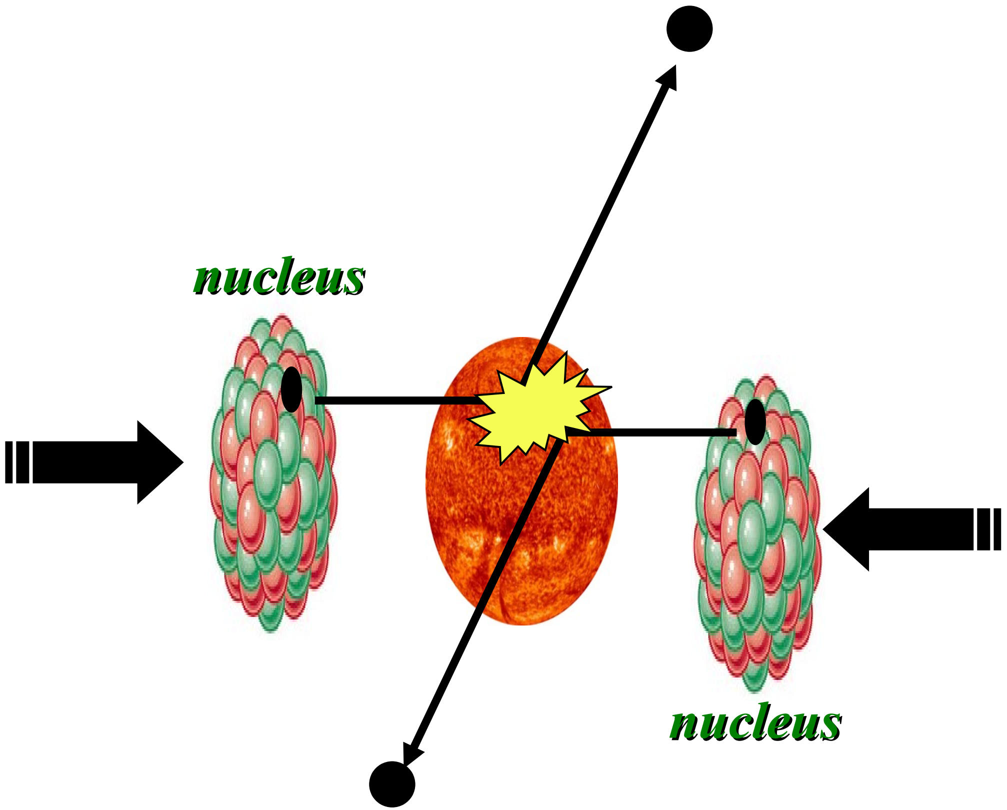
Probing the Quark Gluon Plasma



Want a probe which traveled through the collision
QGP is very short-lived (~1-10 fm/c) →
cannot use an external probe

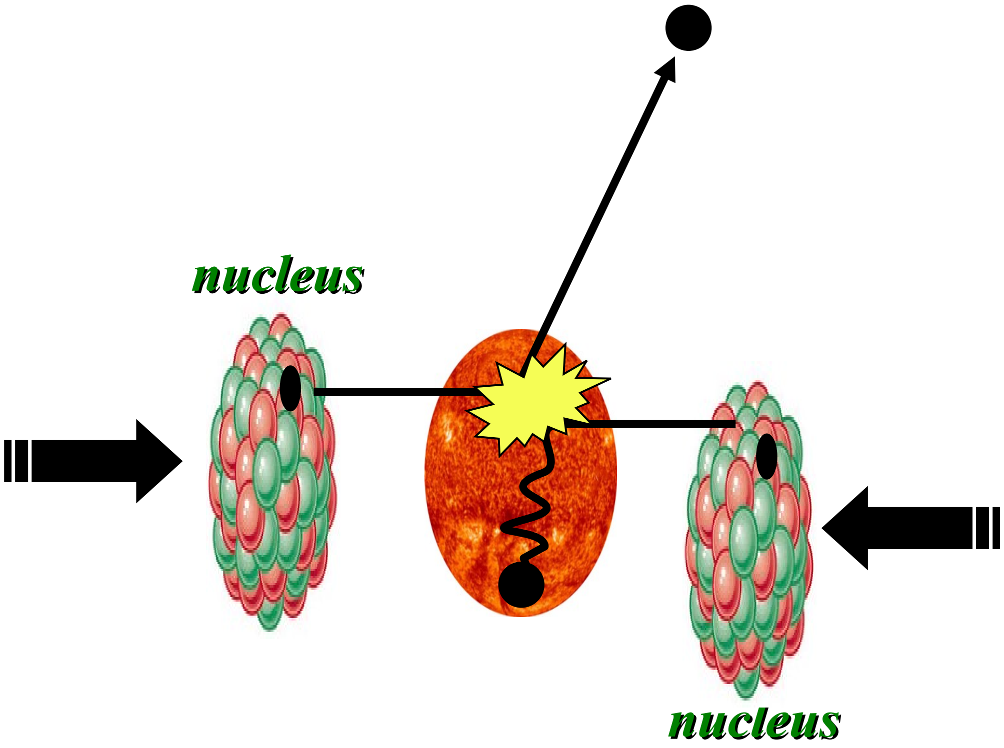
Probes of the Quark Gluon Plasma

Want a probe which traveled through the medium
QGP is short lived → need a probe created in the collision



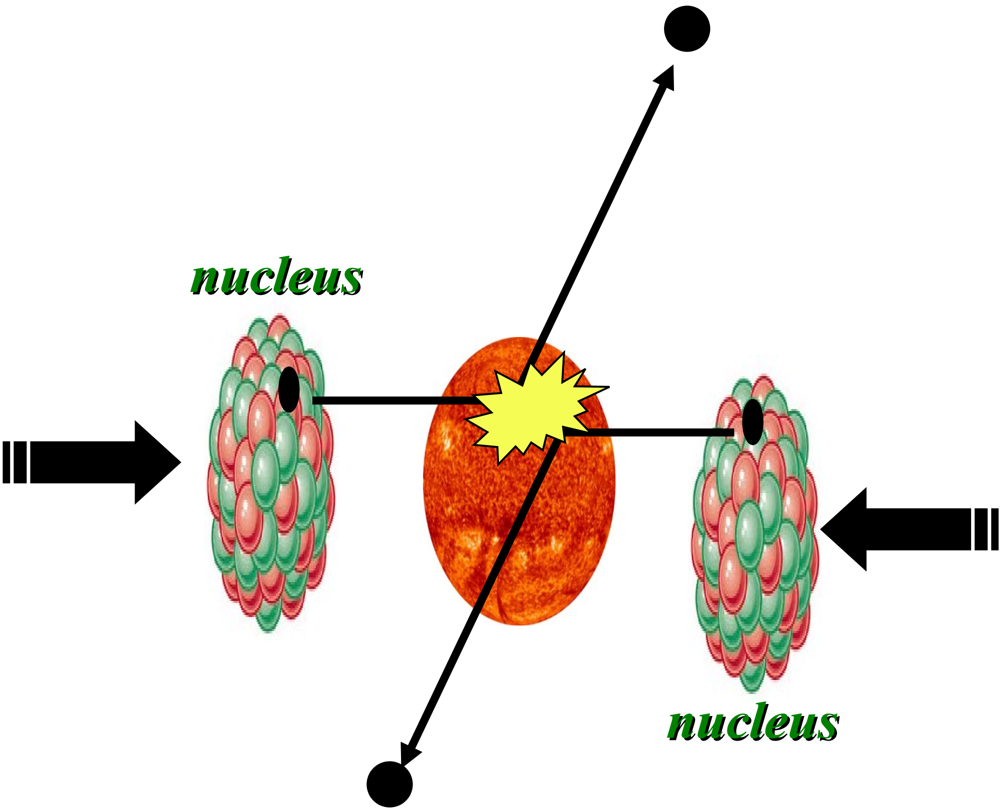
Probes of the Quark Gluon Plasma

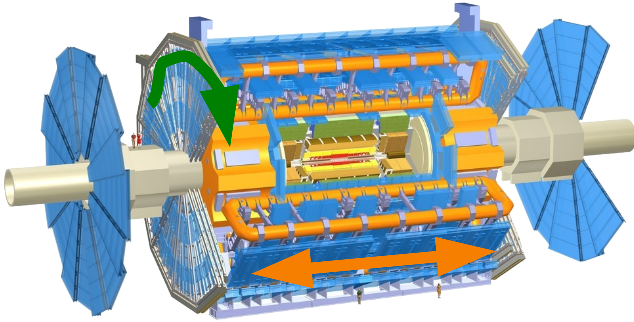
Want a probe which traveled through the medium
QGP is short lived → need a probe created in the collision
We expect the medium to be dense → absorb/modify probe

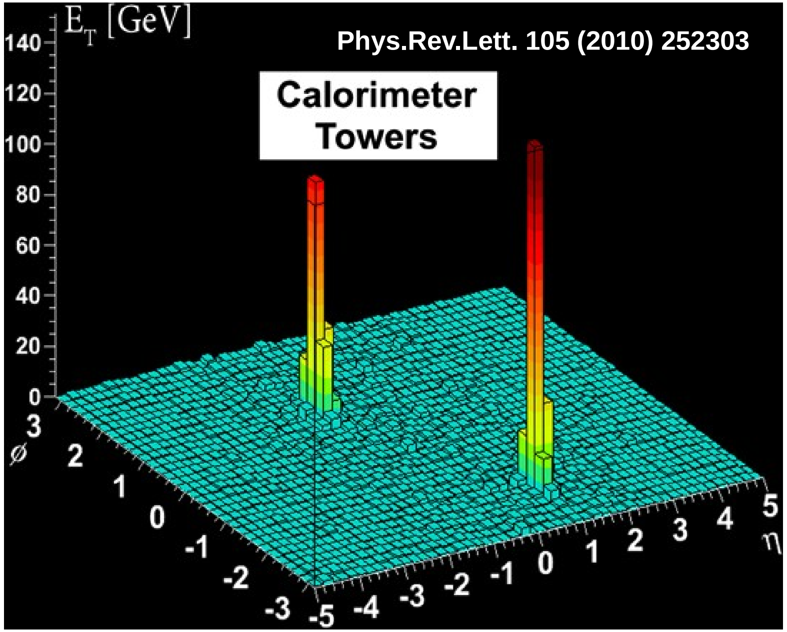


Probes of the Quark Gluon Plasma

ATLAS

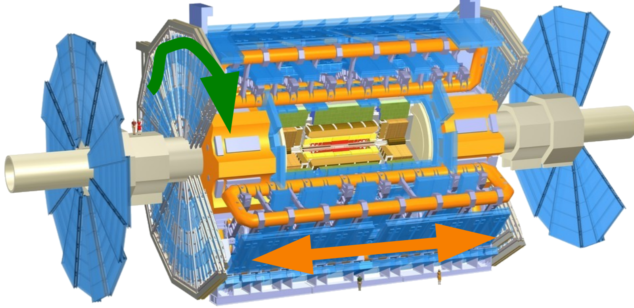


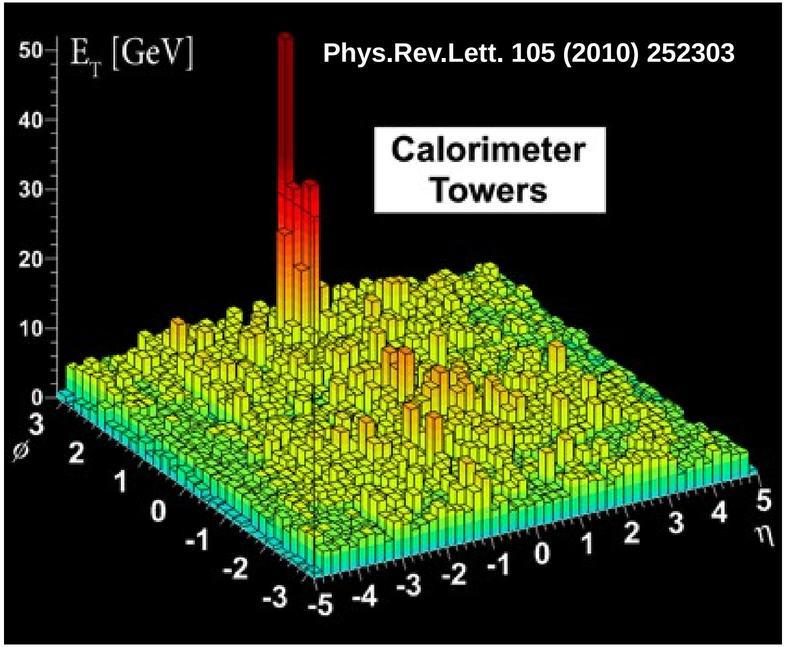


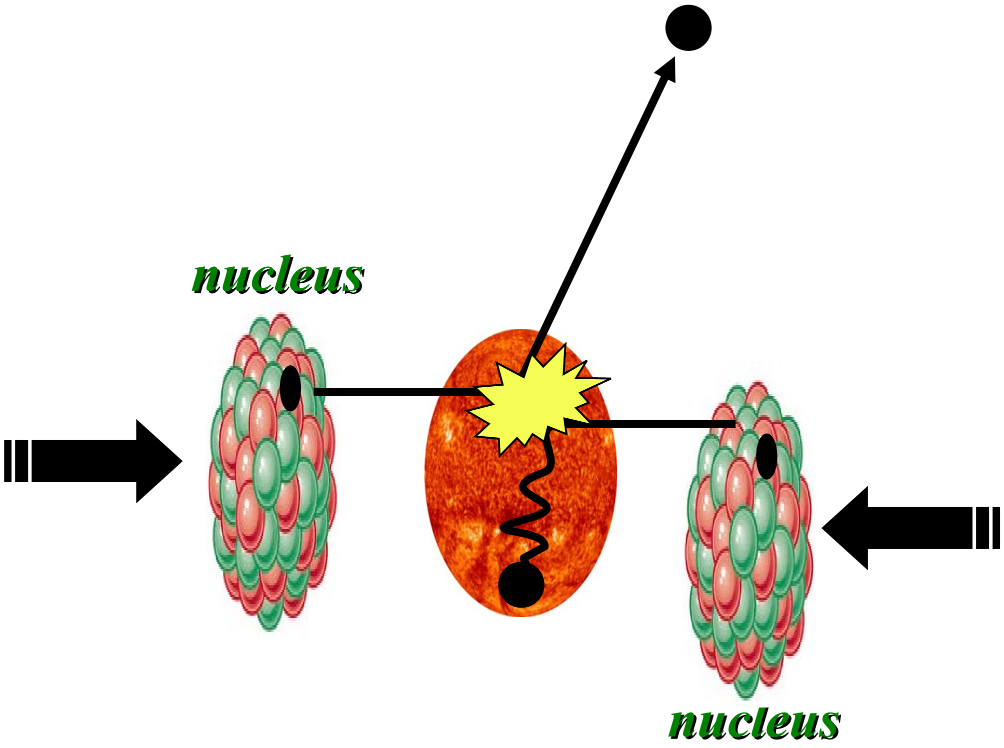


Probes of the Quark Gluon Plasma

ATLAS





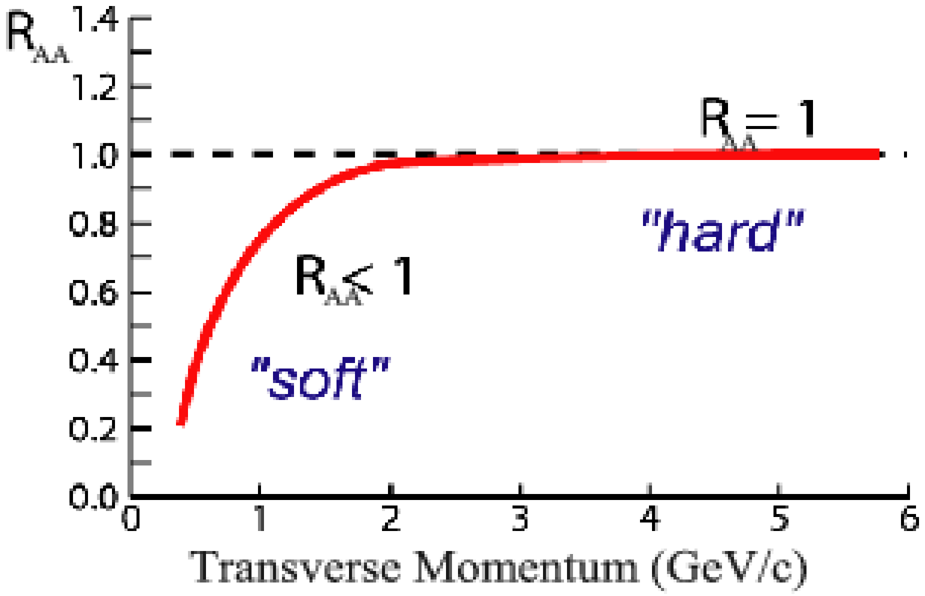


“Simple” example: Single hadrons

Nuclear modification factor

- Measure spectra of probe (jets) and compare to those in p+p collisions or peripheral A+A collisions
- If high-p_T probes (jets) are suppressed, this is evidence of jet quenching

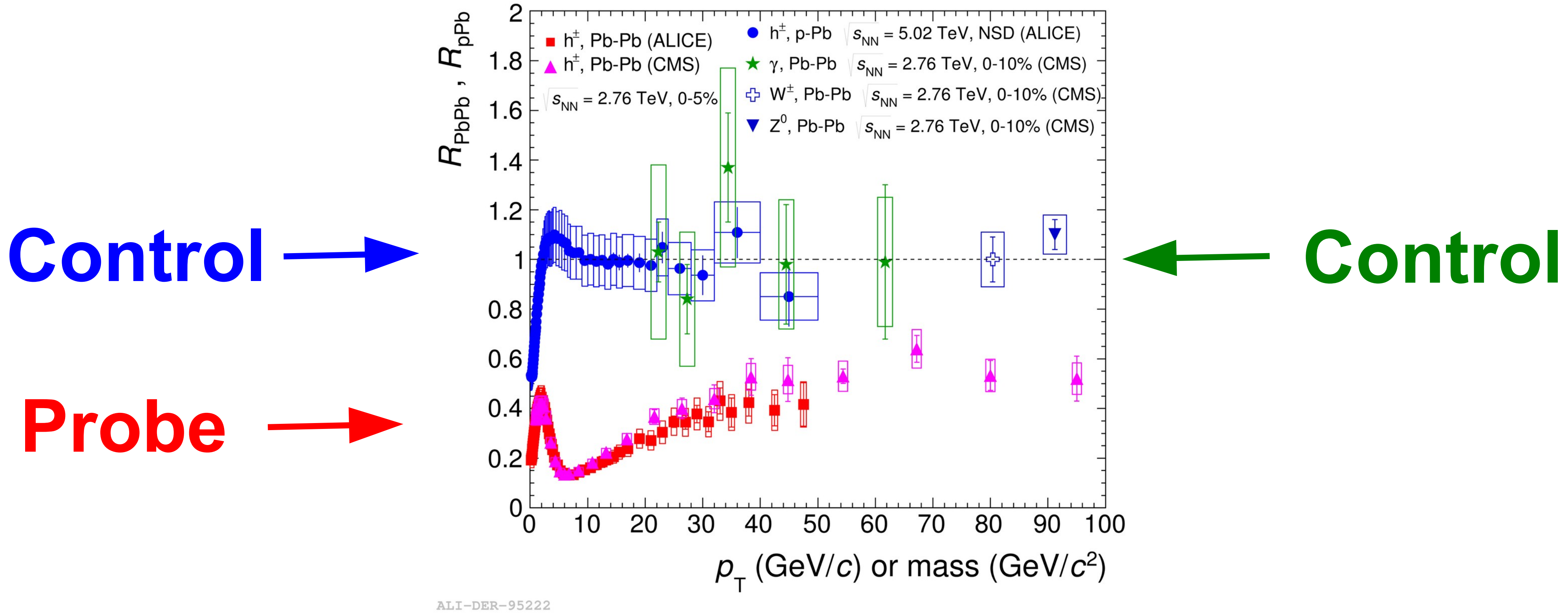
$$R_{AA} = \frac{d^2N_{AA}/dp_T d\eta}{T_{AA} d^2\sigma^{pp}/dp_T d\eta}$$



Enhancement

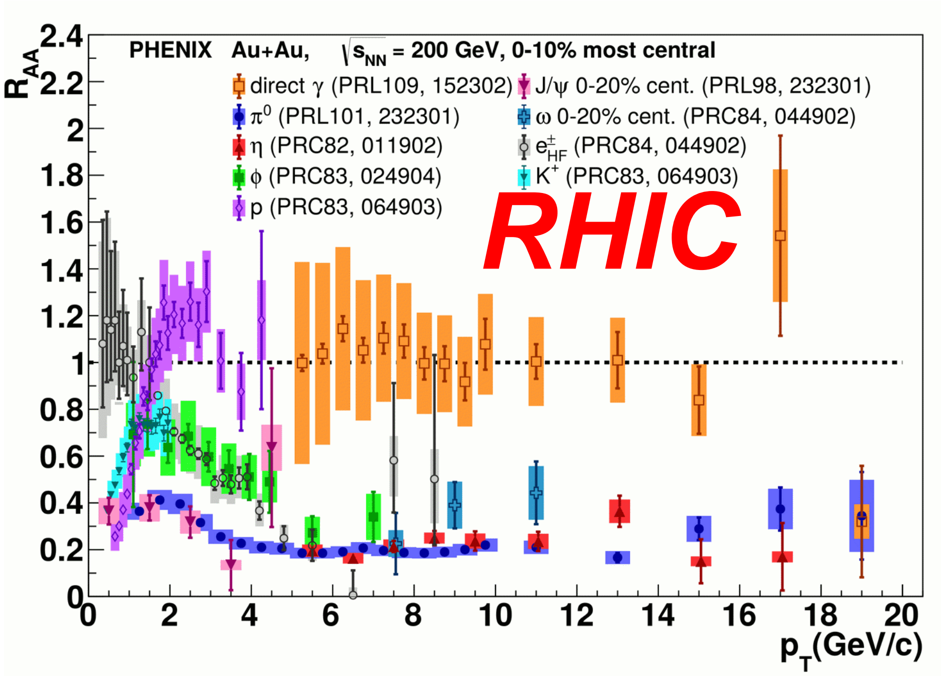
Suppression

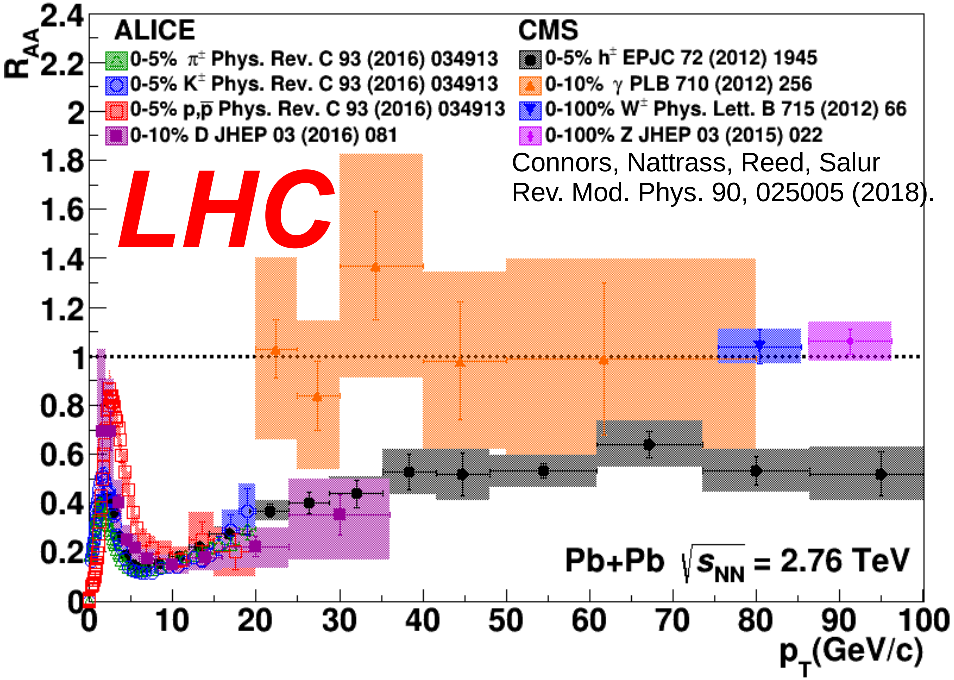
Nuclear modification factor



- Charged hadrons (colored probes) suppressed in Pb—Pb
- Charged hadrons not suppressed in p—Pb at midrapidity
- Electroweak probes not suppressed in Pb—Pb

Nuclear modification factor R_{AA}





Electromagnetic probes – consistent with no modification – medium is transparent to them

Strong probes – significant suppression – medium is opaque to them - even heavy quarks!

What am I measuring?
Definition of a jet

What is a jet?

What is a jet?

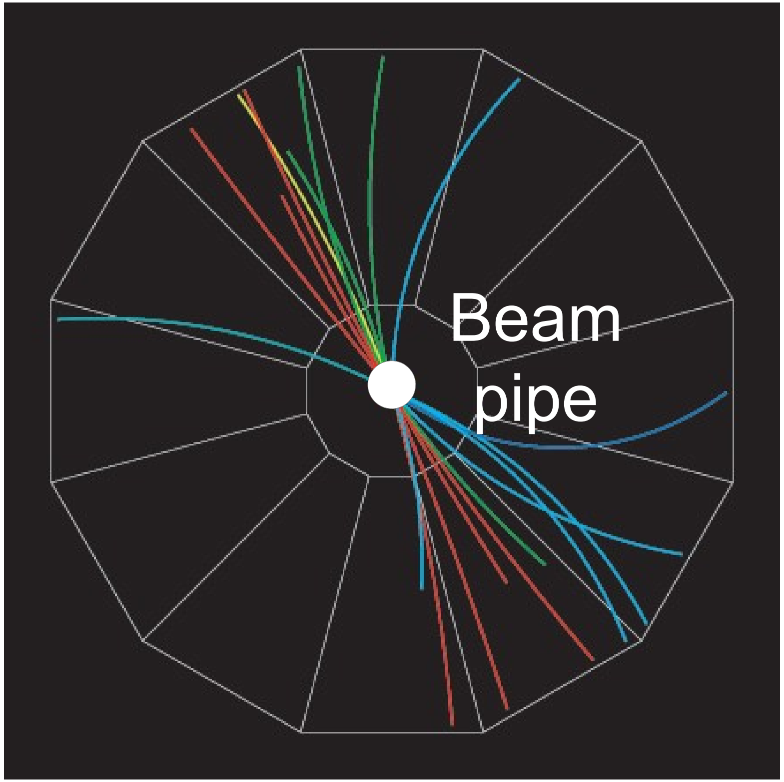
A measurement of a jet is a measurement of a parton.

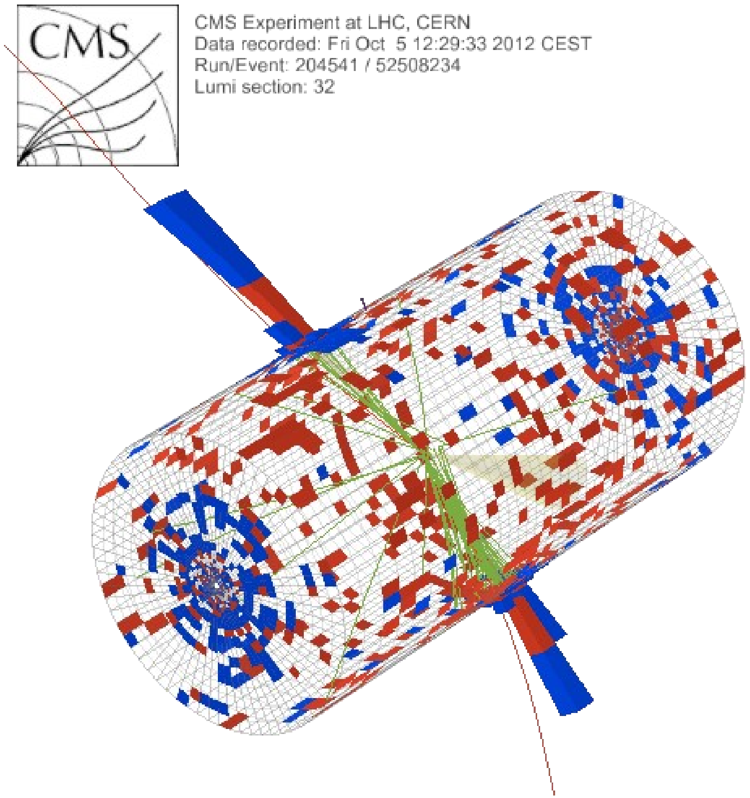
What is a jet?

~~A measurement of a jet is a measurement of a parton.~~

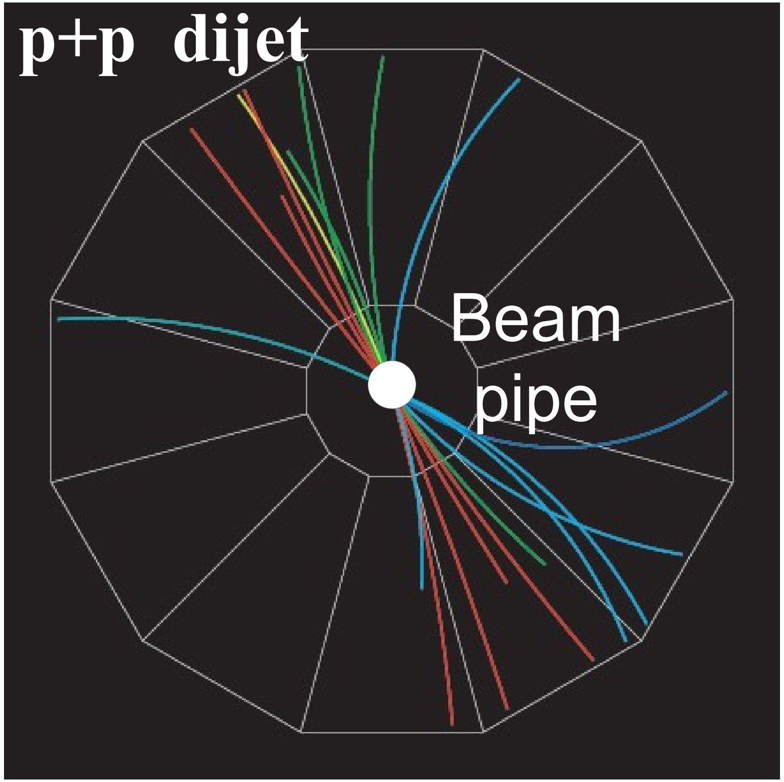
What is a jet?

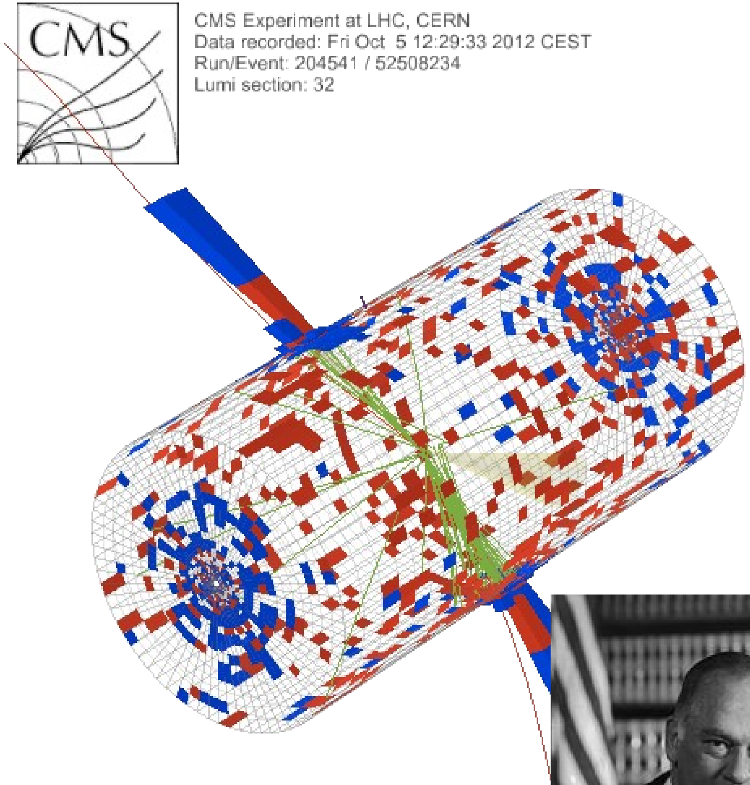
p+p dijet





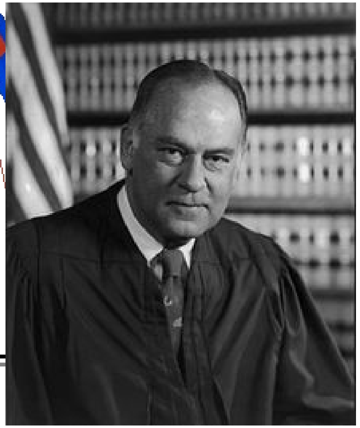
What is a jet?





"I know it when I see it"

US Supreme Court Justice Potter Stewart, Jacobellis v. Ohio



Jets in principle

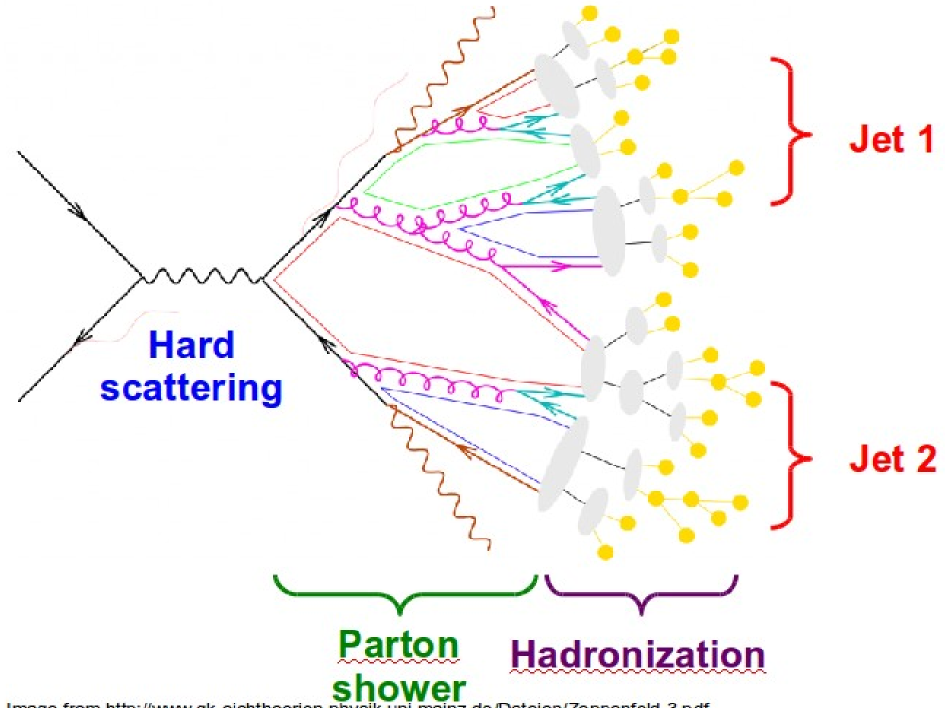
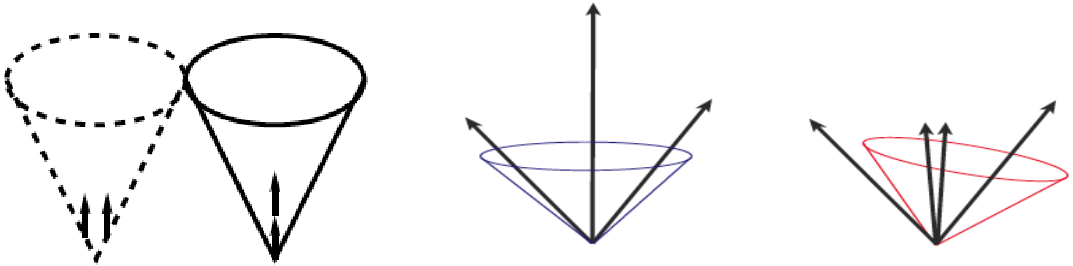
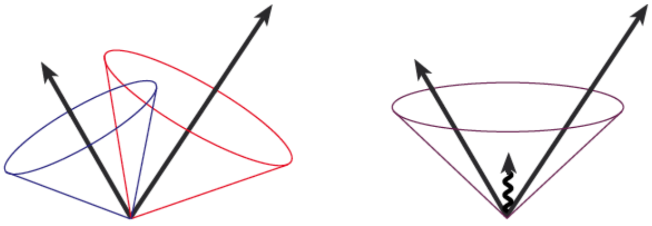


Image from http://www.gk-eichtheorien.physik.uni-mainz.de/Dateien/Zeppenfeld-3.pdf

- Jet measures **partons**
- Hadronic degrees of freedom are integrated out
- Algorithms are infrared and colinear safe





OK

BAD: 2 jets are merged in one

Jet finding *in pp collisions*

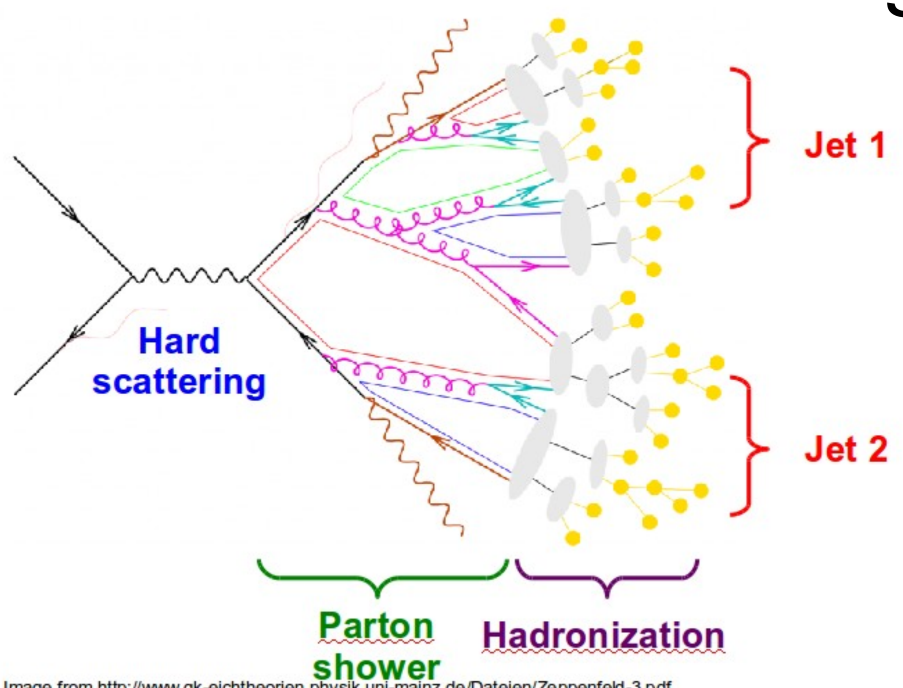
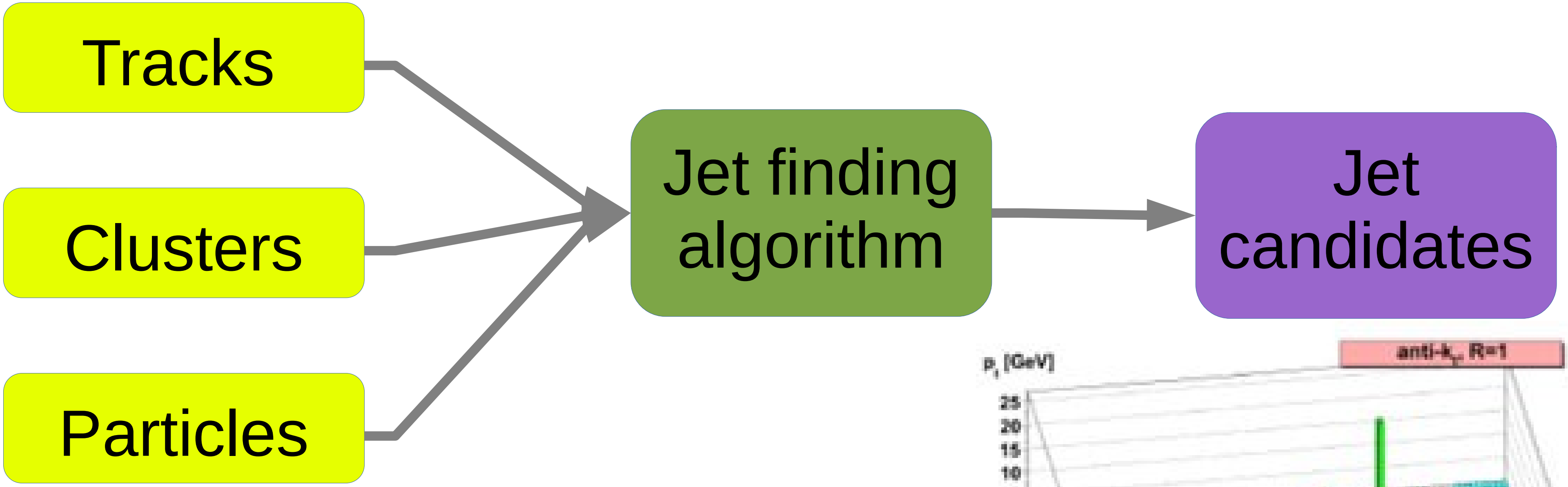


Image from http://www.gk-eichtheorien.physik.uni-mainz.de/Dateien/Zeppenfeld-3.pdf

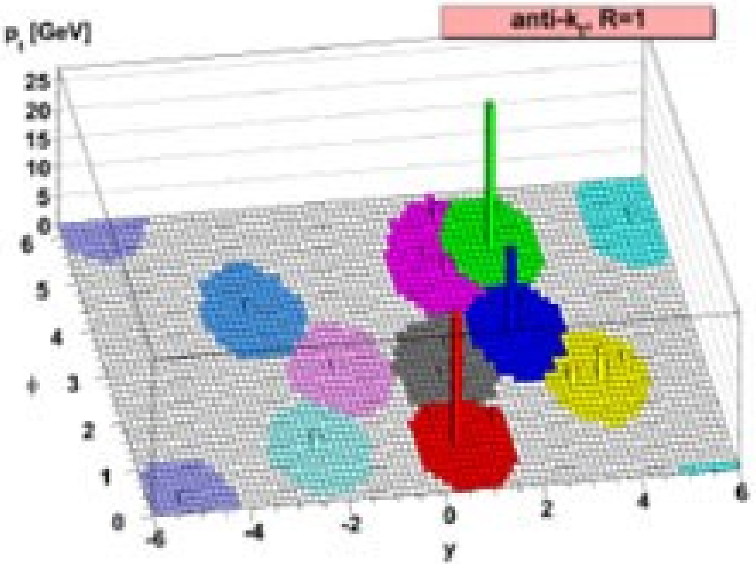
- Jet finder: groups final state particles into jet candidates
  - Anti-k_T algorithm
    JHEP 0804 (2008) 063 [arXiv:0802.1189]
- Depends on hadronization
- Ideally
  - Infrared safe
  - Colinear safe

Snowmass Accord: Theoretical calculations and experimental measurements should use the same jet finding algorithm. Otherwise they will not be comparable.

Jet finding algorithms



- Any list of objects works as input
- Use the same algorithm on theory & experiment
- Output only as good as input



k_T jet finding algorithm

Particles, clusters

↓

k_T algorithm

$$k_T = p_T, \Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$$

- For all *i,j* calculate:

$$d_{ij} = min\left(p_{T,i}^2, p_{T,j}^2\right)\frac{\Delta R_{ij}^2}{R^2}$$

$$d_{iB} = p_{T,i}^2$$

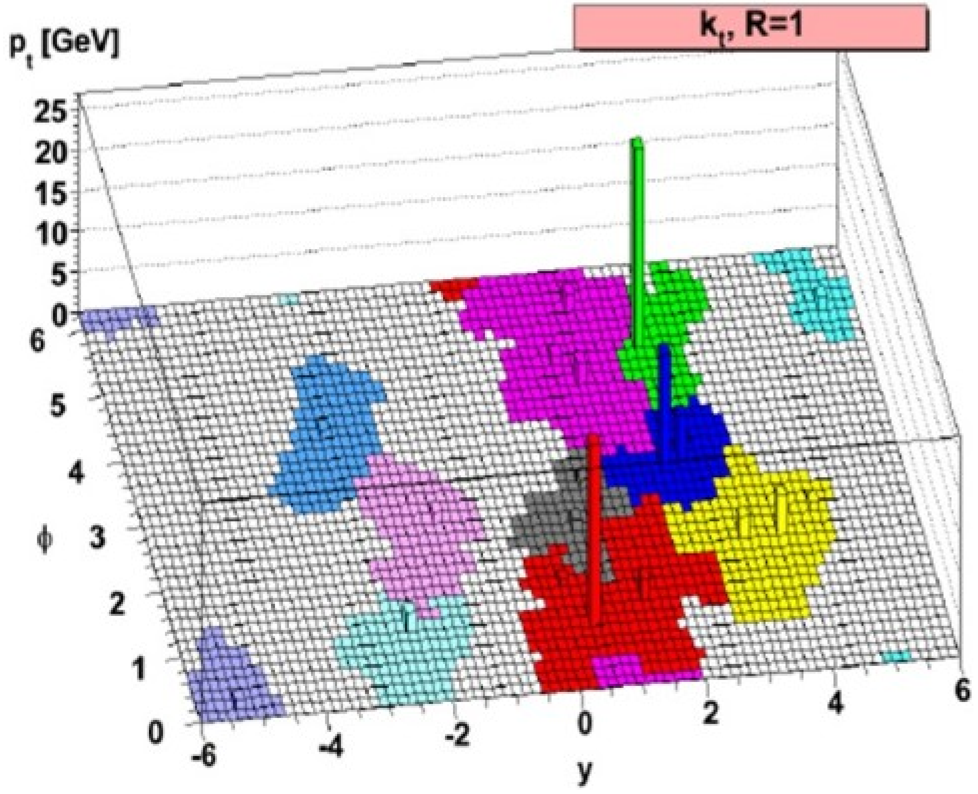
- Combine smallest d_{ij}.

  If d_{iB} smallest, $d_{iB} \rightarrow$ jet

Repeat until no particles left

↓

Jet candidates



anti-k_T jet finding algorithm

Particles, clusters

k_T algorithm

$$k_T = p_T, \Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$$

- For all *i,j* calculate:

$$d_{ij} = min\left(p_{T,i}^{-2}, p_{T,j}^{-2}\right)\frac{\Delta R_{ij}^2}{R^2}$$

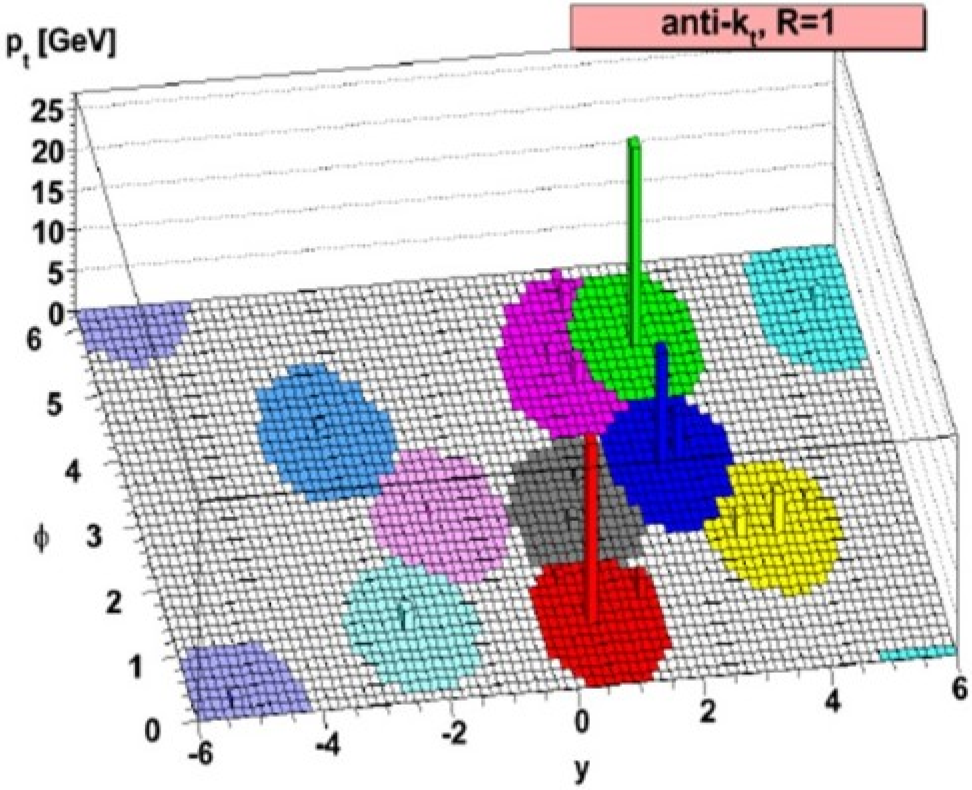
$$d_{iB} = p_{T,i}^{-2}$$

- Combine smallest d_{ij}.

  If d_{iB} smallest, $d_{iB} \rightarrow$ jet

Repeat until no particles left

Jet candidates



Cambridge/Aachen jet finding algorithm

Particles, clusters

k_T algorithm

$$k_T = p_T, \Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$$

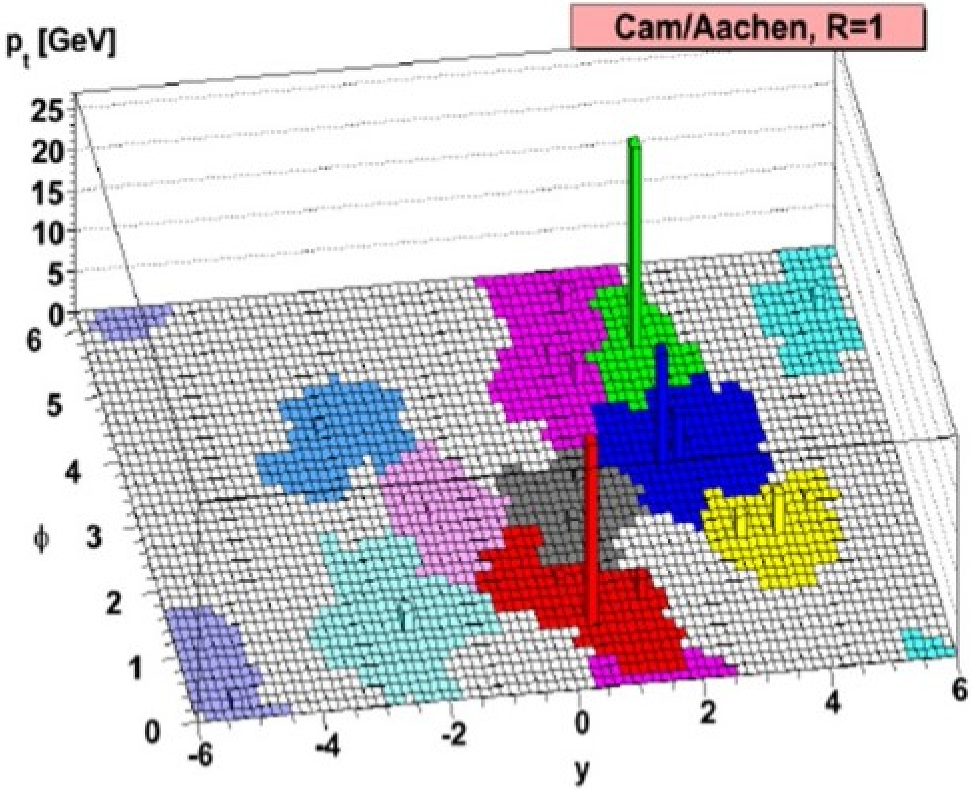
- For all *i,j* calculate:

$$d_{iB} = 1 \qquad d_{ij} = \frac{\Delta R_{ij}^2}{R^2}$$

- Combine smallest d_{ij}.
  If d_{iB} smallest, $d_{iB} \rightarrow$ jet

Repeat until no particles left

Jet candidates

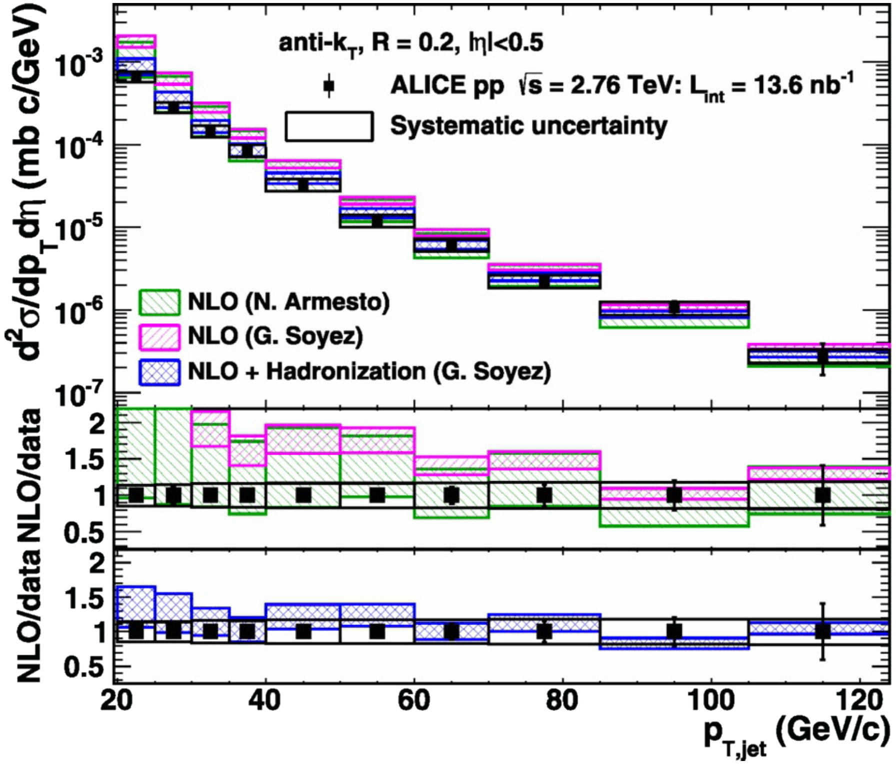


A jet is what a jet finder finds.

Jet cross-section in pp

$\sqrt{s}$ = 2.76 TeV, R = 0.2 Inclusive

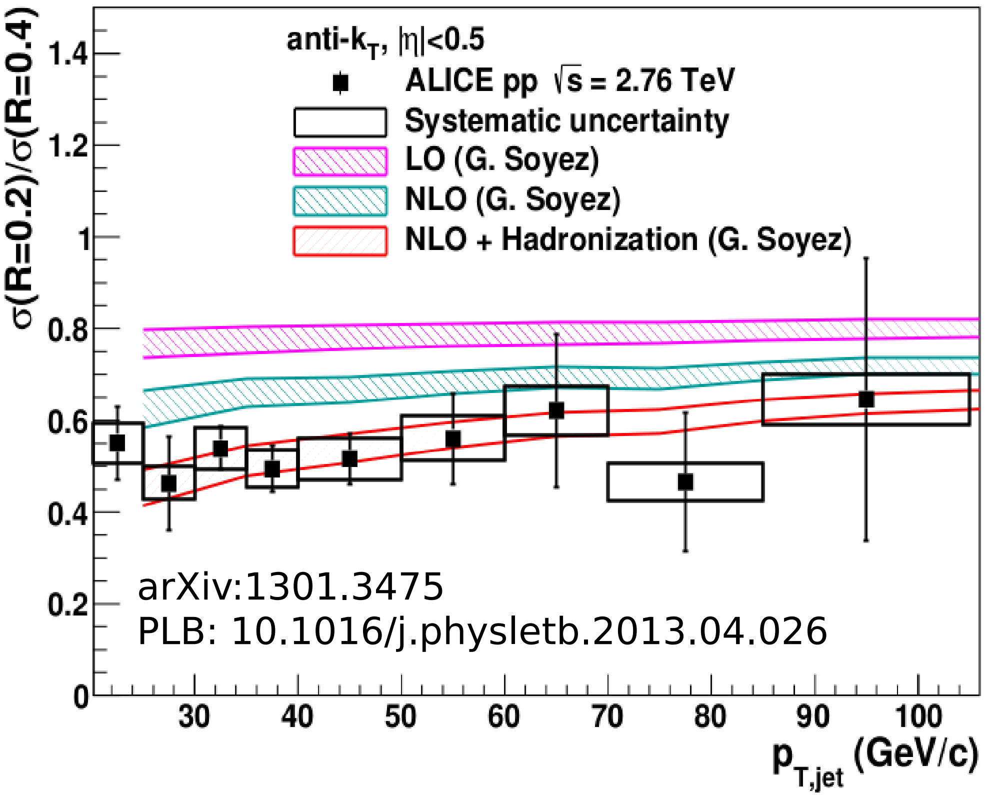
arXiv:1301.3475
PLB: 10.1016/j.physletb.2013.04.026

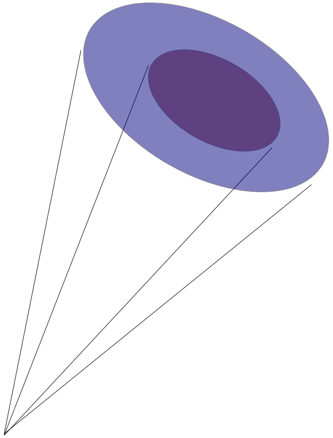


- Green and magenta bands: NLO on Parton level
- Blue band: NLO + hadronization
- Hadronization calculations necessary to describe data

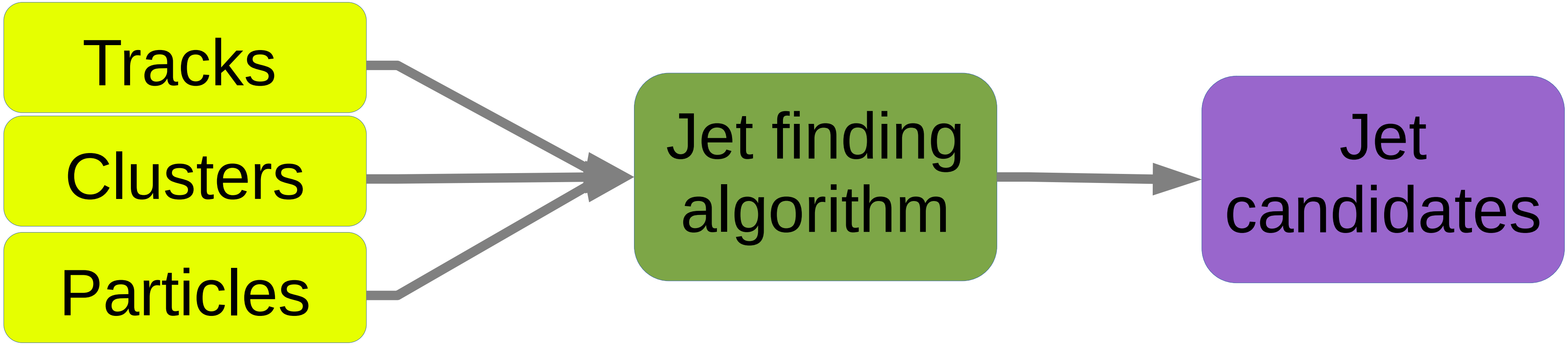
Jet ratios in pp

√s = 2.76 TeV, R = 0.2, 0.4 Inclusive





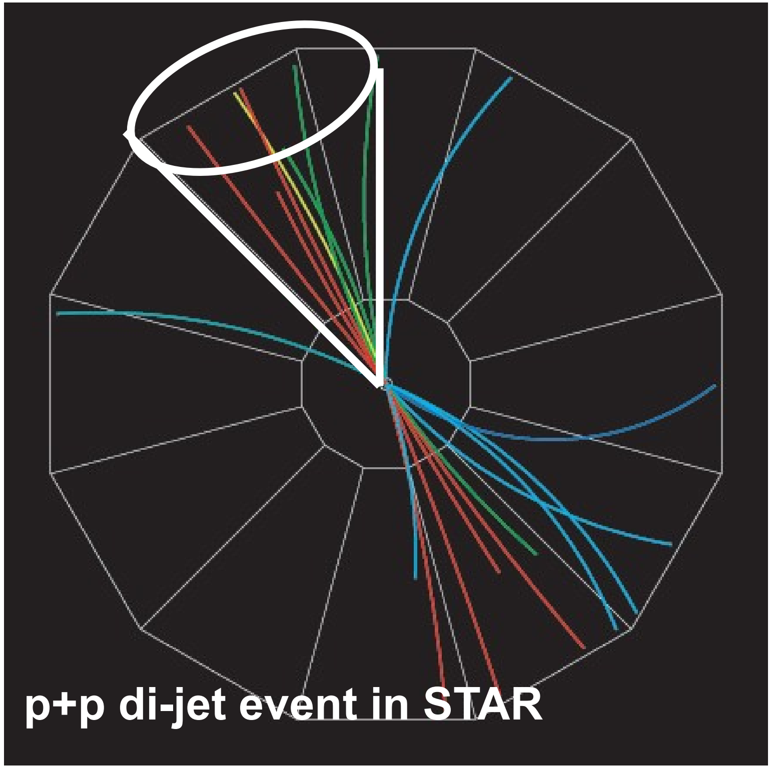
Mini-summary

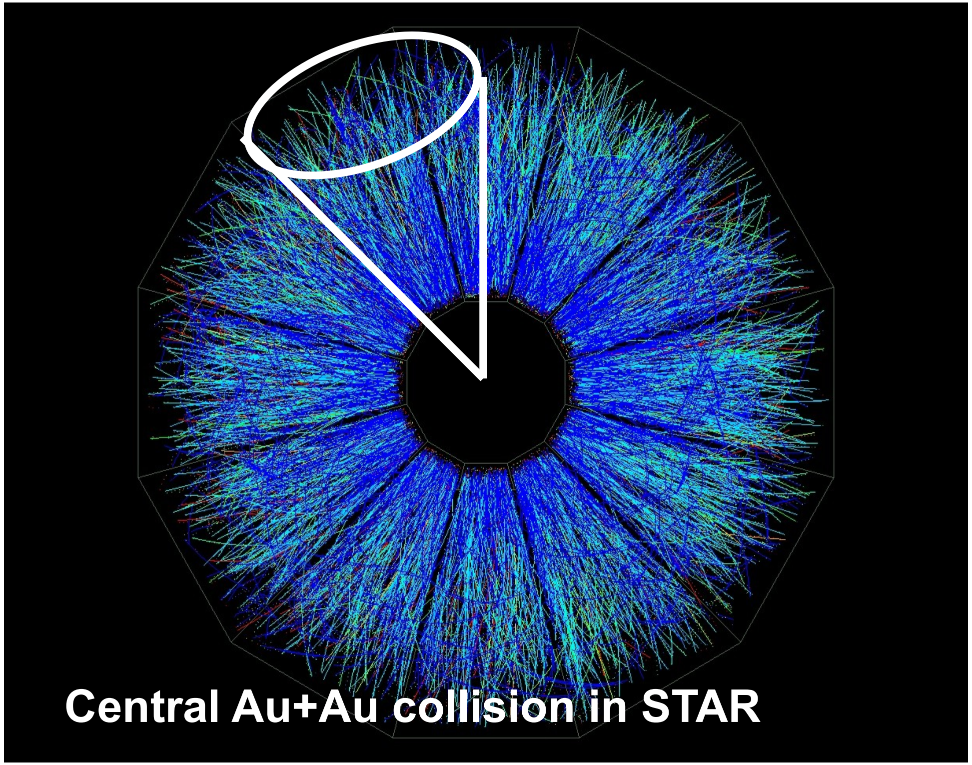


- Jets are not partons
- Good jet finders:
  - Infrared and colinear safe
  - k_T, anti-k_T, Cambridge/Aachen, SISCone
- Jet is defined by jet finder, its parameters
- PDFs, fragmentation functions non-perturbative
  → all jet measurements sensitive to somewhat non-perturbative effects
- Good agreement between theory and experiment

Jets in A+A collisions
What assumptions am I making?

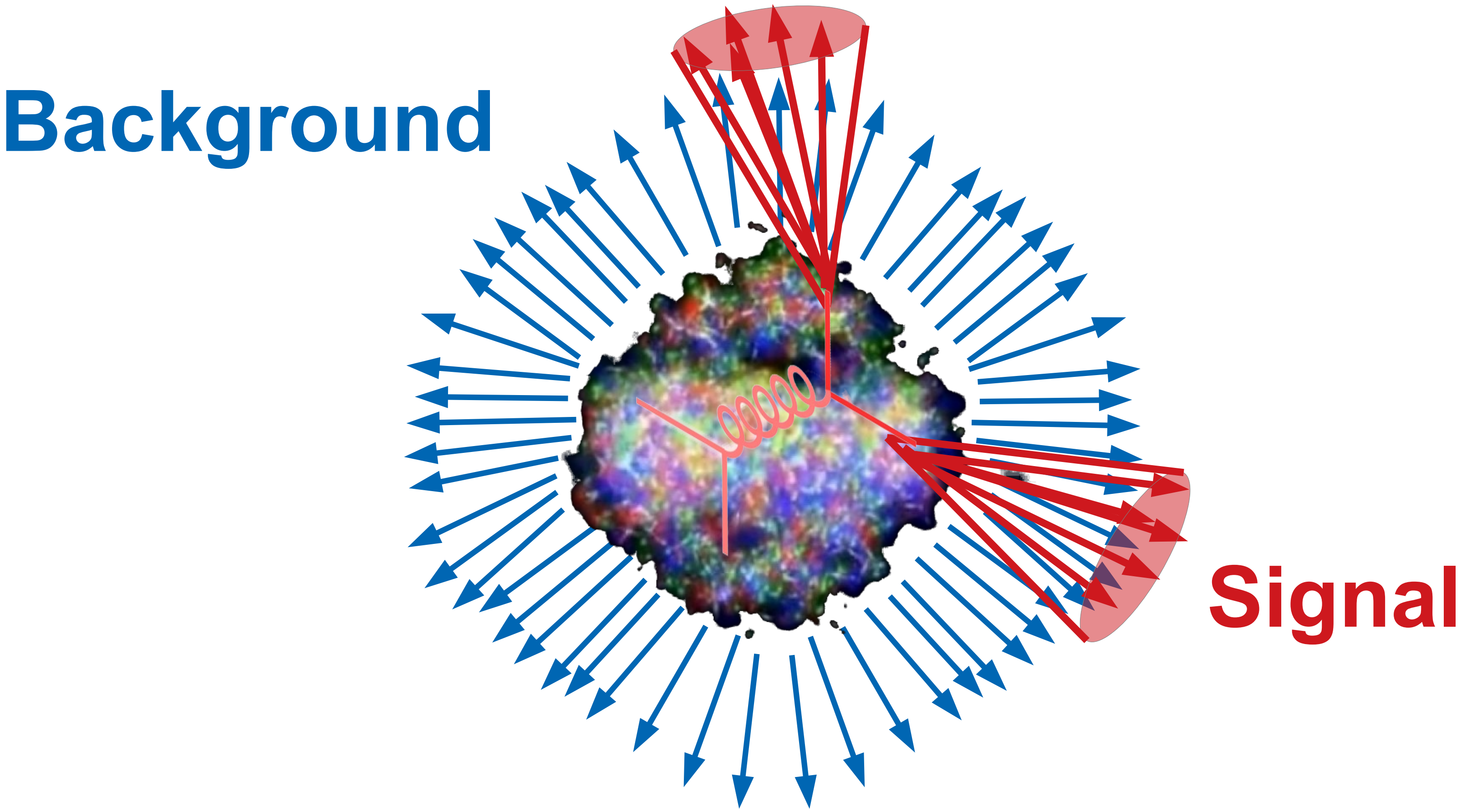
p+p vs A+A





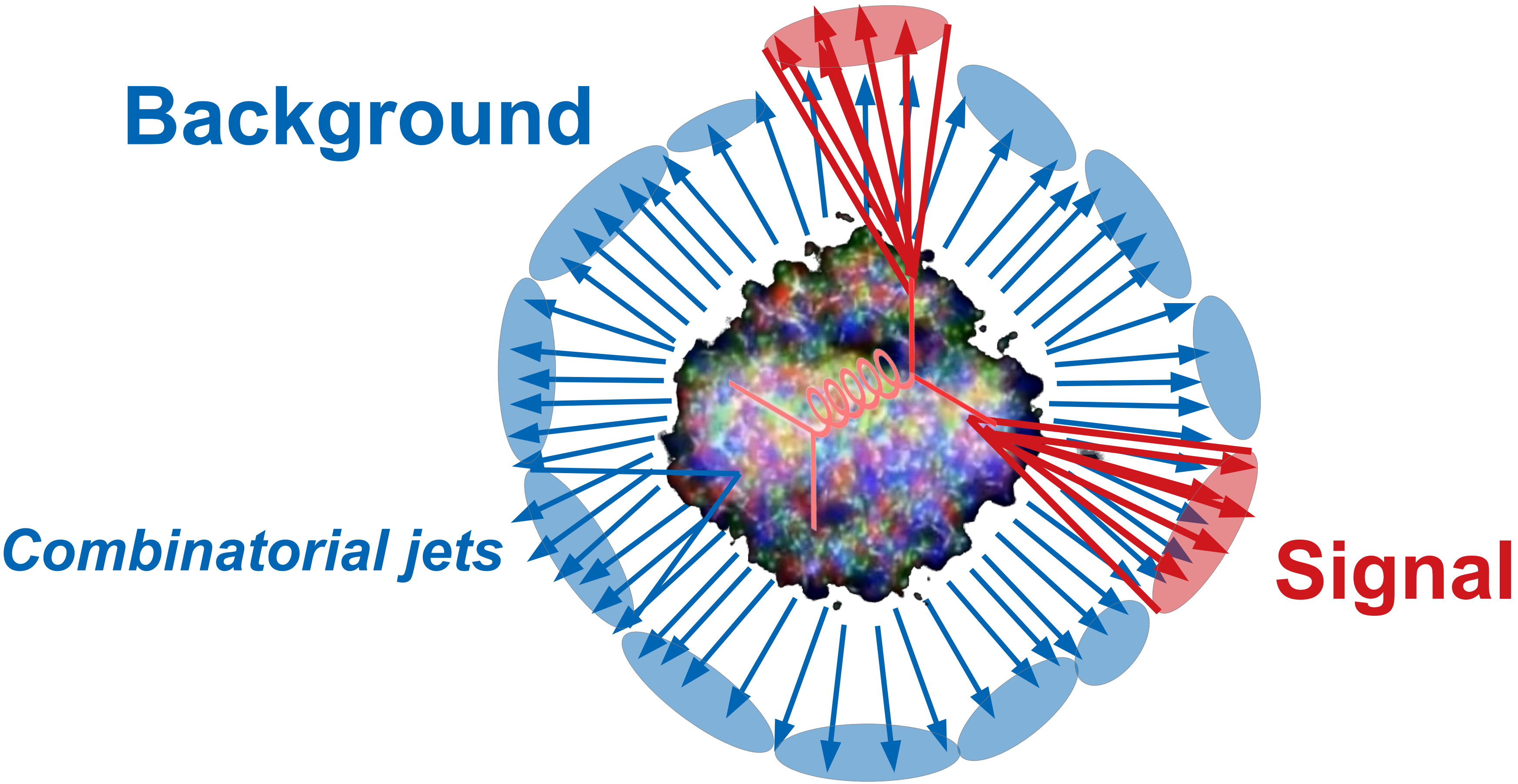
Signal vs Background:

The standard paradigm



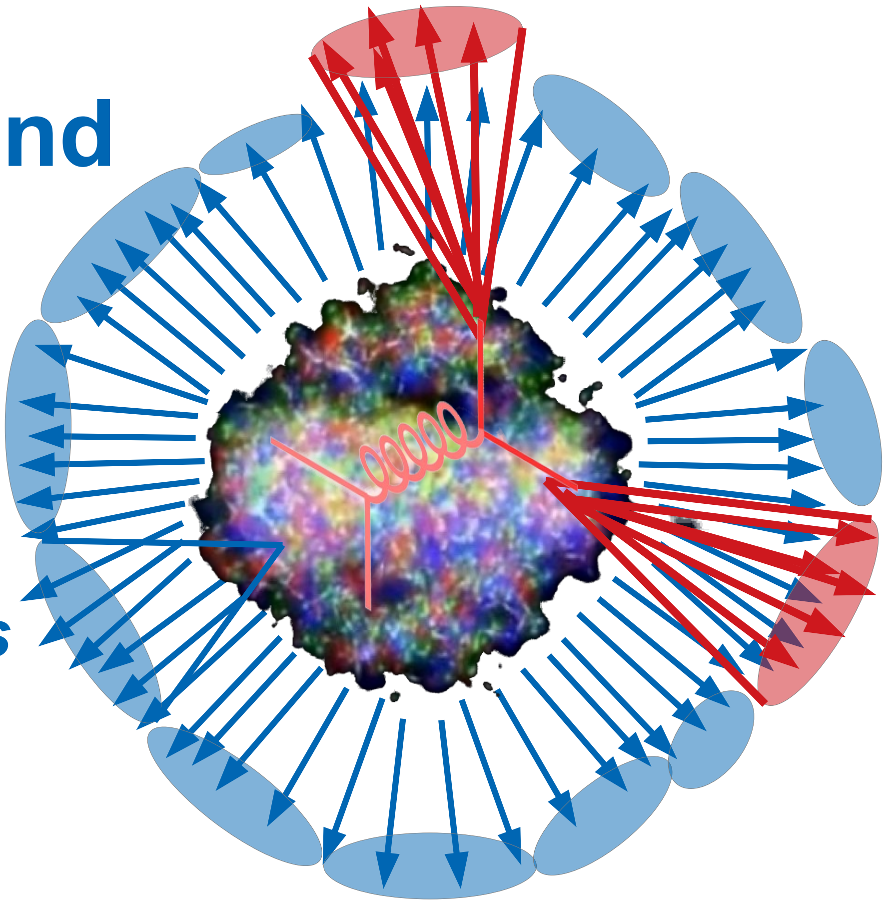
Signal vs Background:

The standard paradigm



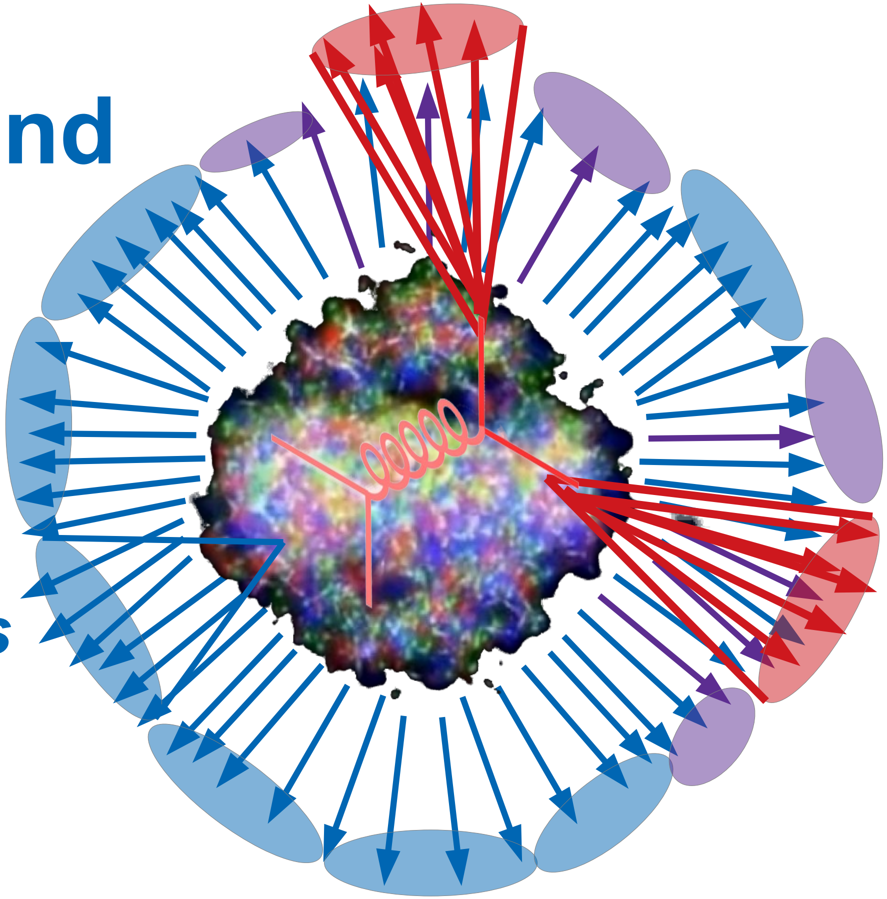
Signal vs Background:

The standard paradigm



Signal vs Background:

The standard paradigm



Background

Combinatorial jets

Signal

***Some gray areas**

Jet finding in AA collisions

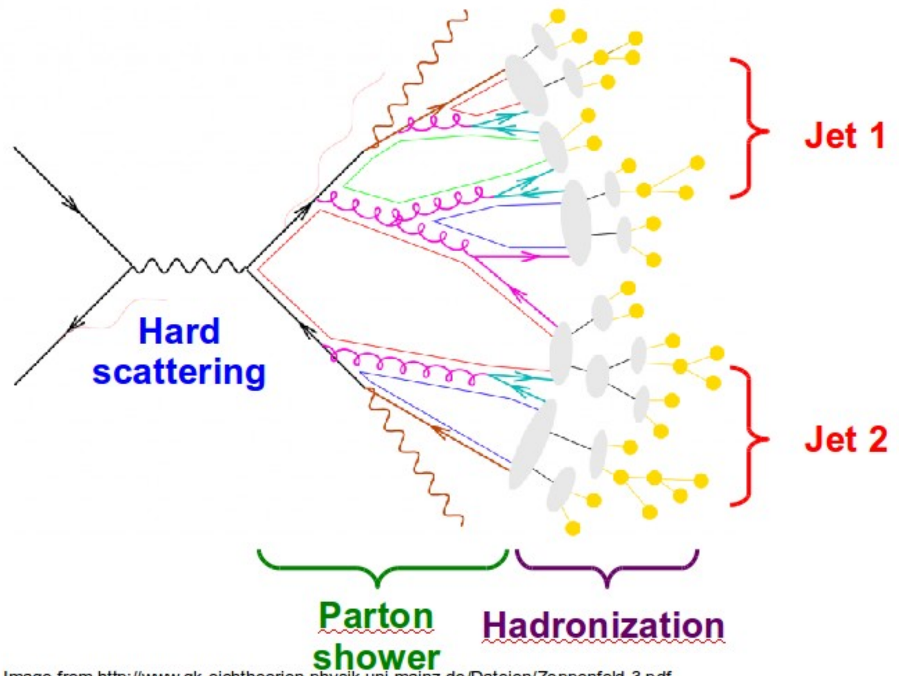
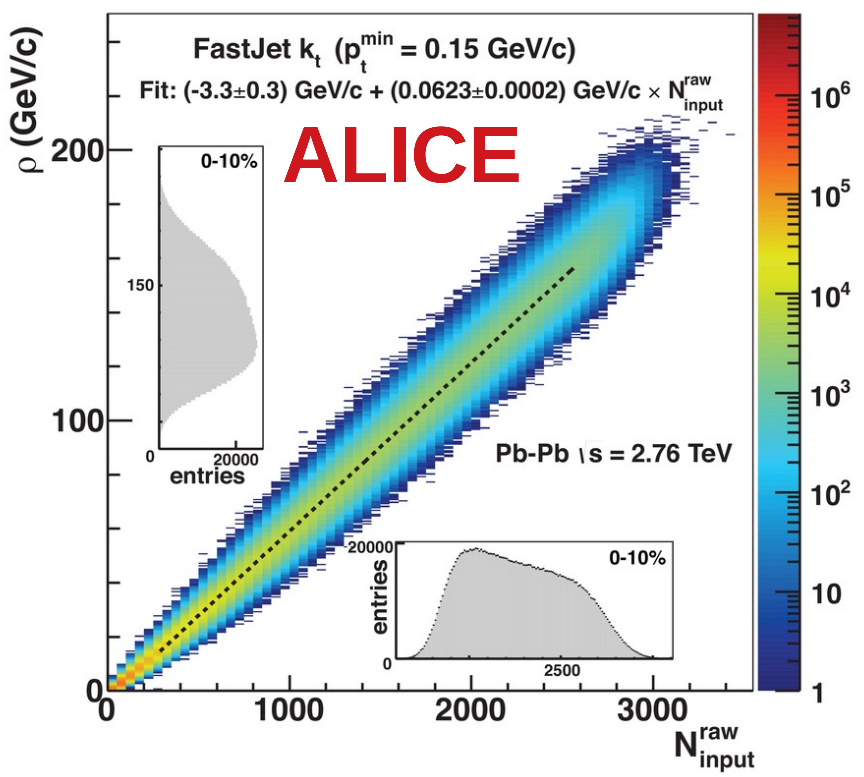
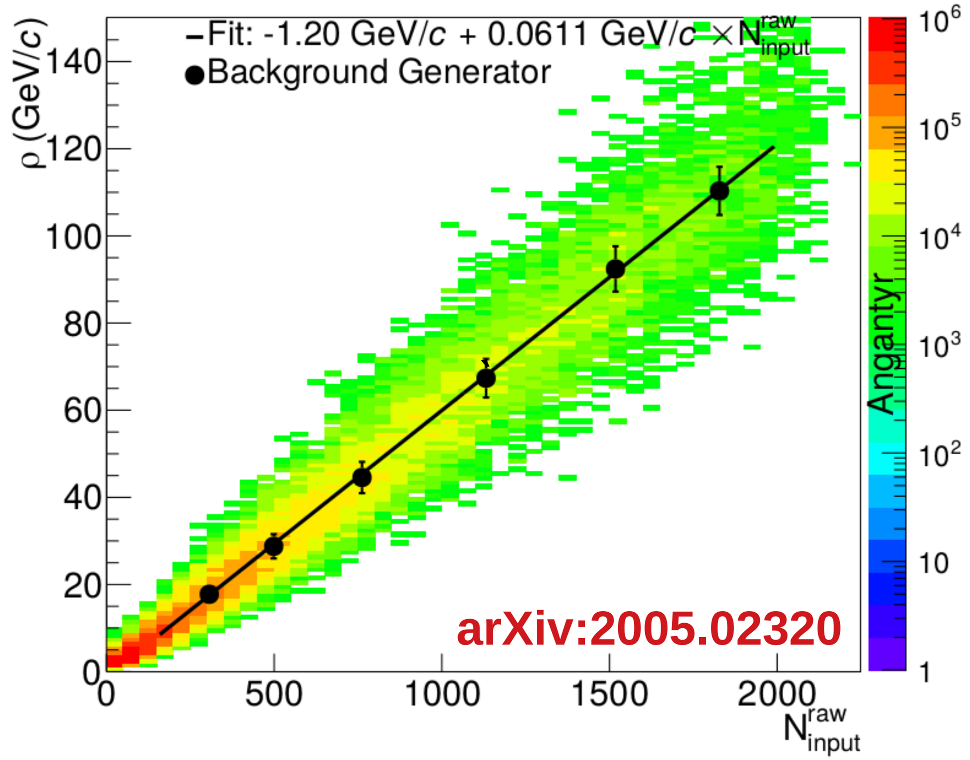


Image from http://www.gk-eichtheorien.physik.uni-mainz.de/Dateien/Zeppenfeld-3.pdf

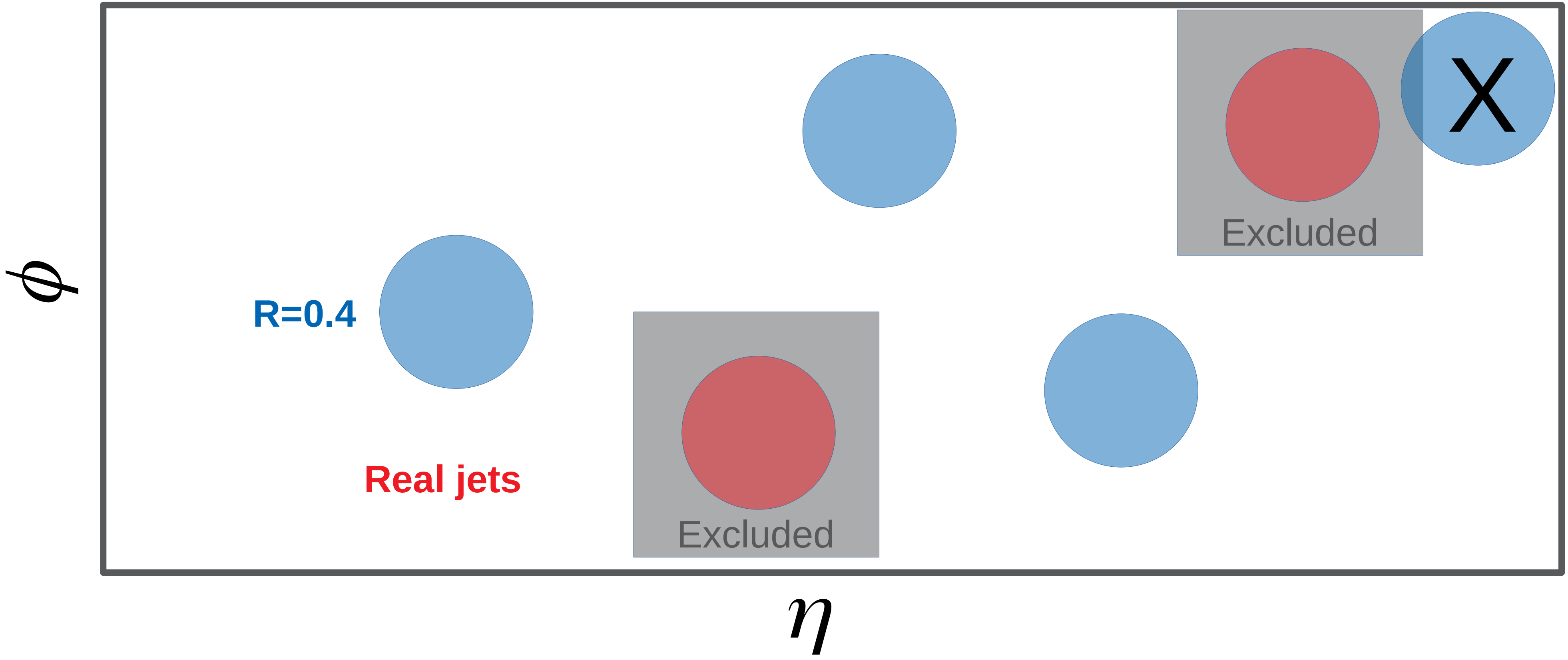
- Jet finder: groups final state particles into jet candidates
  - Anti-k_T algorithm
    JHEP 0804 (2008) 063 [arXiv:0802.1189]
- Combinatorial jet candidates
- Energy smearing from background
- Sensitive to methods to suppress combinatorial jets and correct energy
- Focus on narrow/high energy jets

Background density ρ

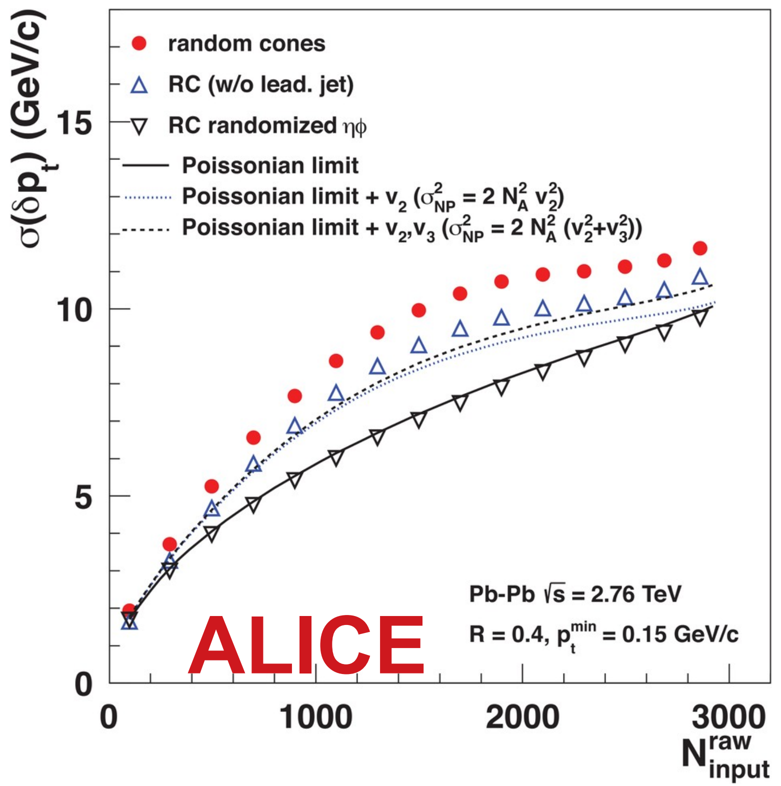


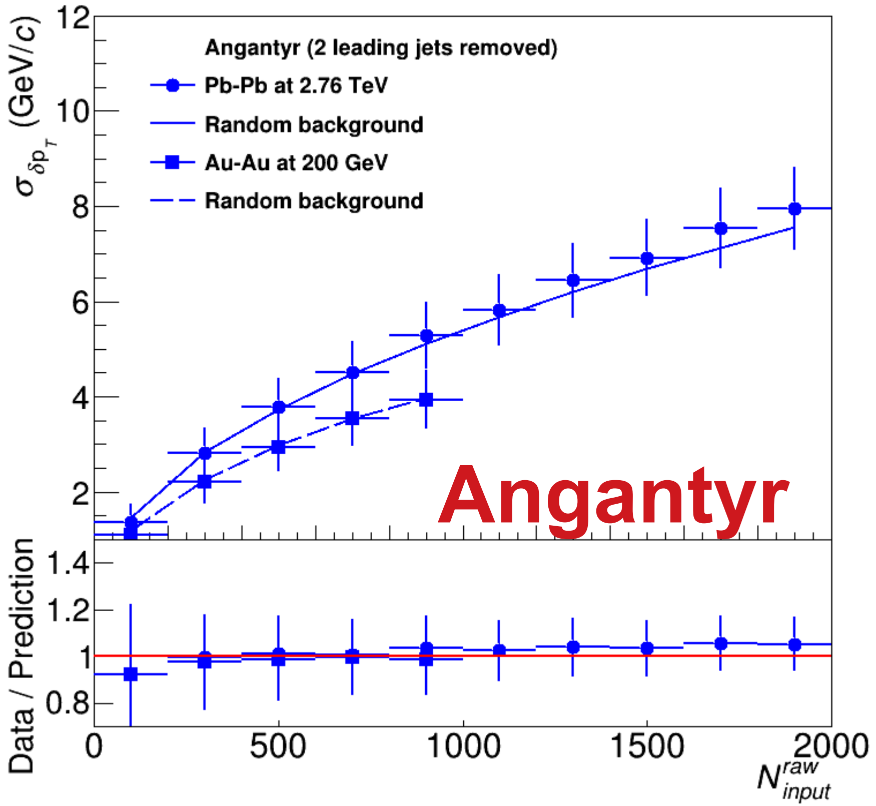


Random cones

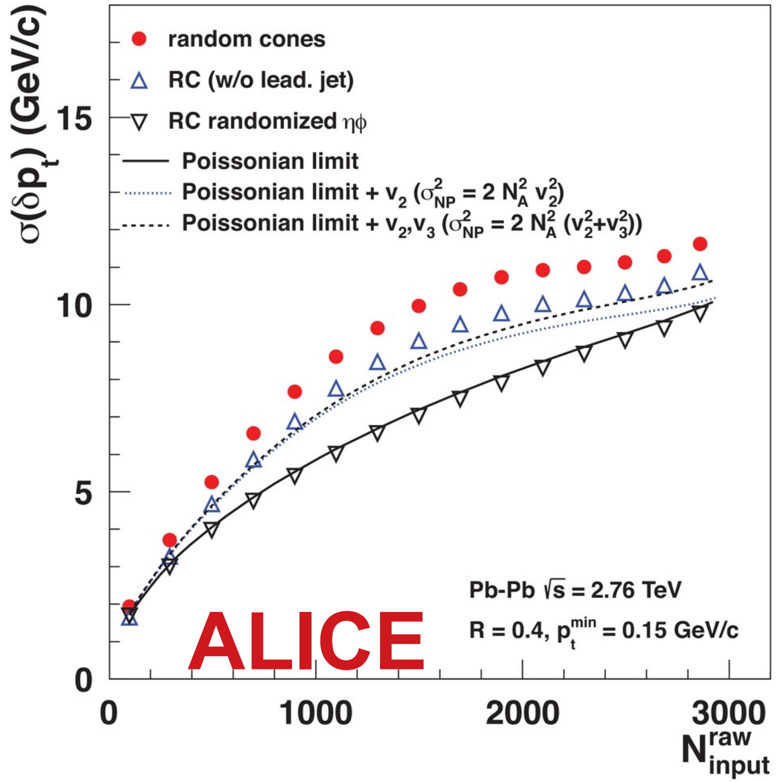


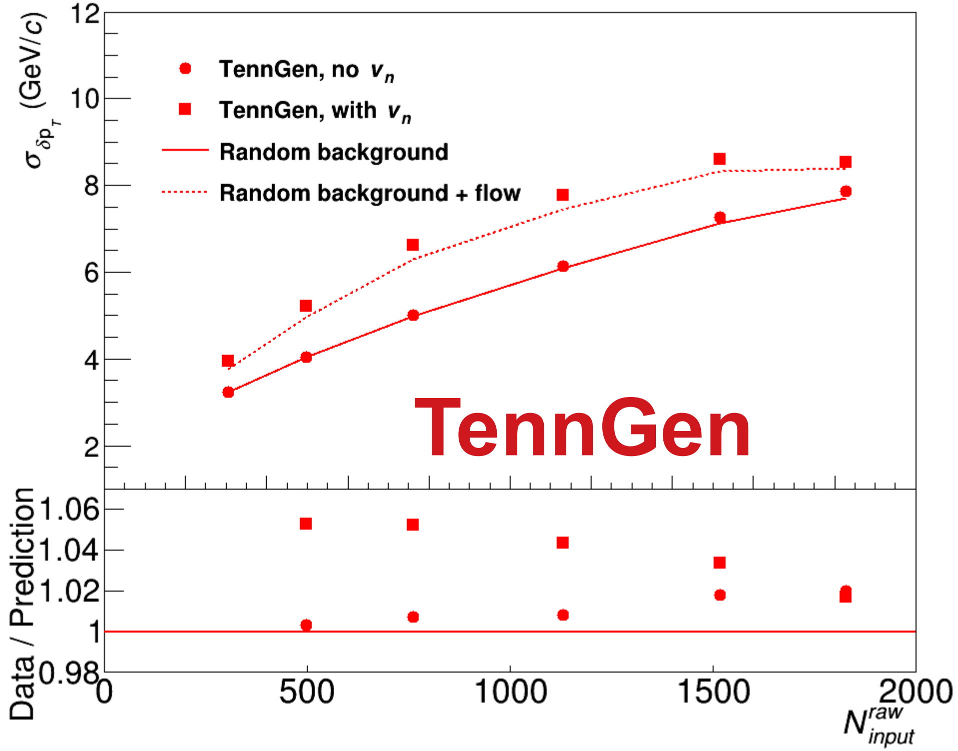
Width vs multiplicity





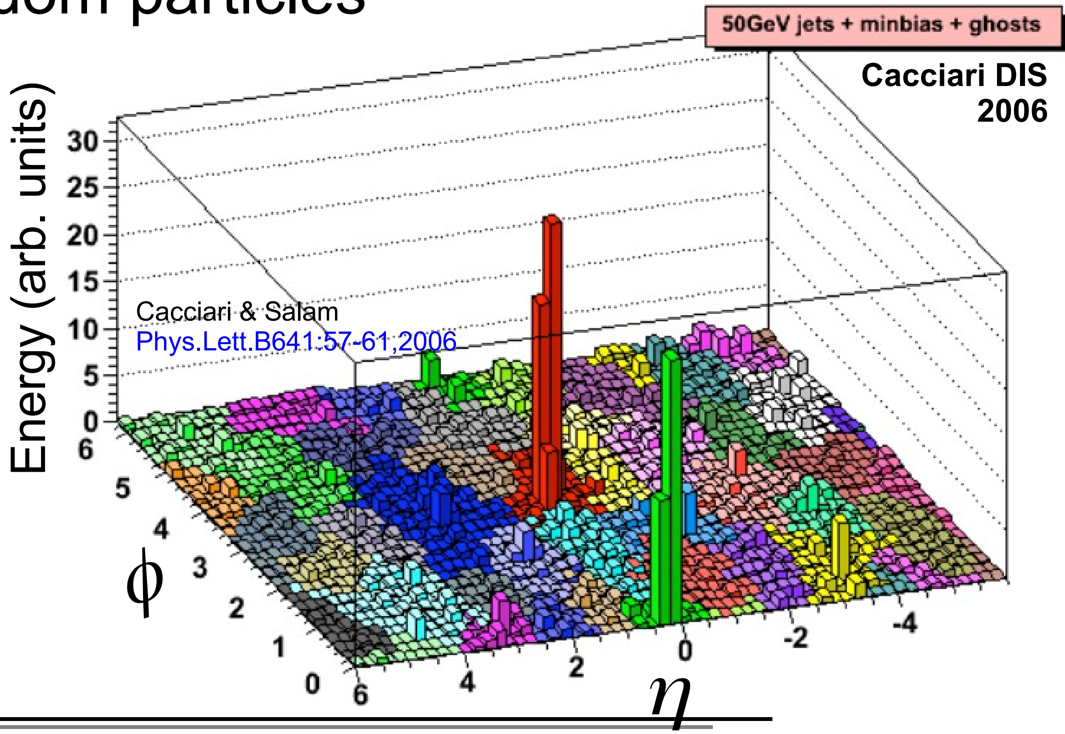
Width vs multiplicity





Mini-summary

- Jet finders put all input clusters, tracks in a jet candidate
- Background is *dominated* by random particles
  - But 5% effects from flow
- Models have background too!
  - Sensitive to multiplicity, shape of spectrum

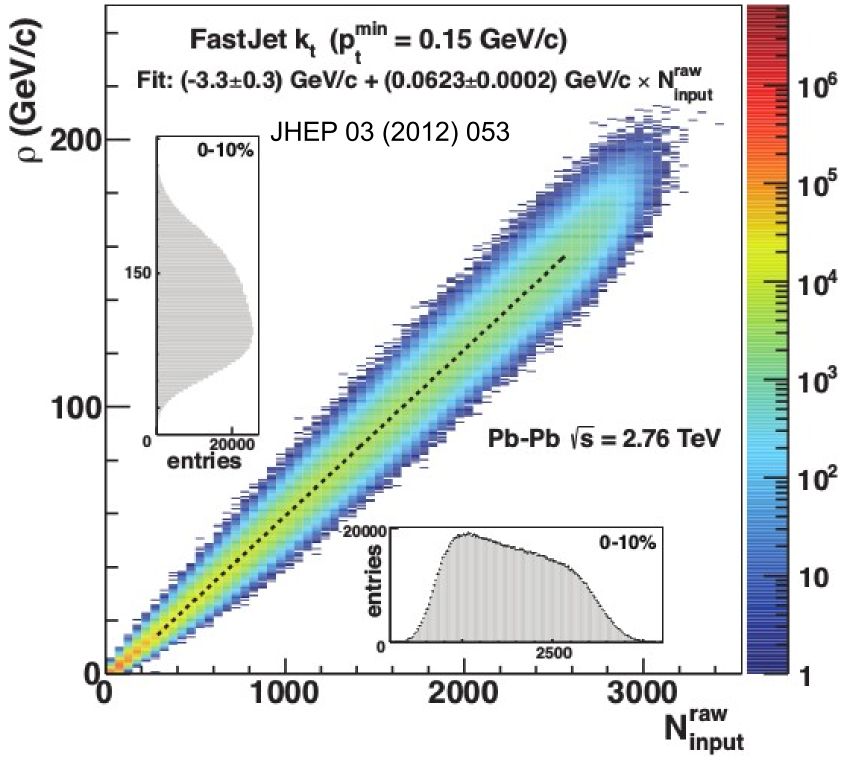


Jets in A+A collisions: Dealing with background

Focus on smaller angles

- Pros
  - Background is smaller
  - Background fluctuations smaller
- Cons:
  - Modifications expected at higher R
  - Biases sample towards quarks

Aside: "quark" and "gluon" jet only defined at leading order.

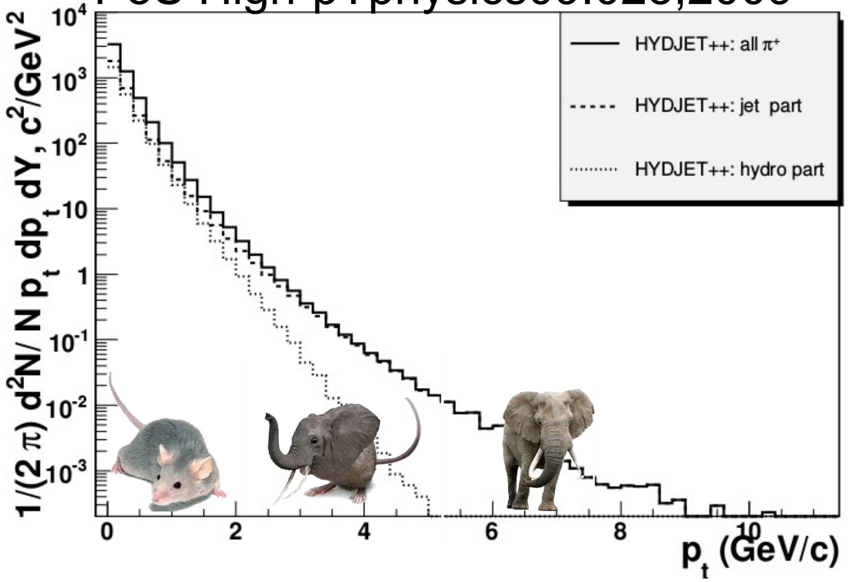


Focus on high p_T

- Pros:
  - Reduces combinatorial background
- Cons:
  - Cuts signal where we expect modifications
  - Could bias towards partons which have not interacted
  - Biases sample towards quark jets

"Quark" and "gluon" jets only defined at leading order!

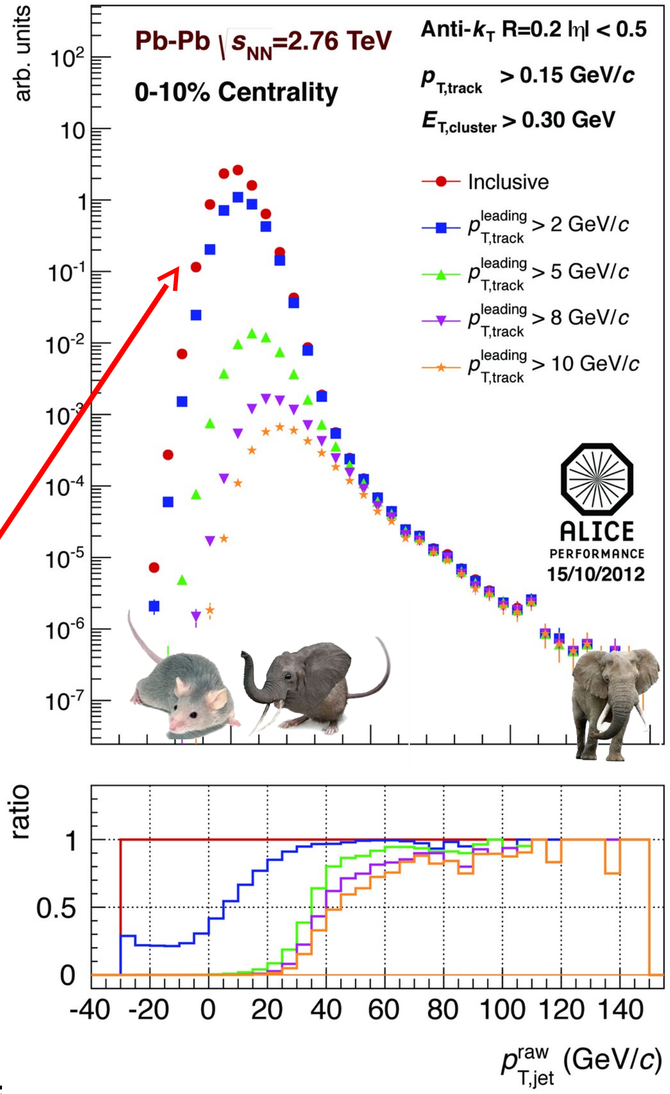
PoS High-pTphysics09:023,2009



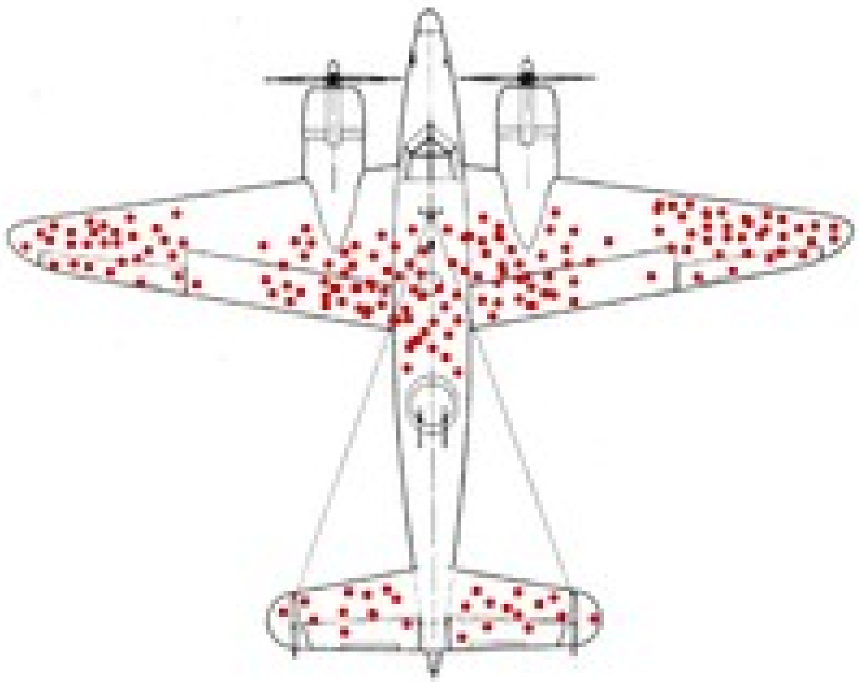
Area-based subtraction

- ALICE/STAR
- Require leading track p_T > 5 GeV/c
  - Suppresses combinatorial “jets”
  - Biases fragmentation
- No threshold on constituents
- Limited to small R

Combinatorial “jets”



Survivor bias



- WWII Example: holes planes returning indicate where it's *safer* to get hit
- We're looking at the jets which *remain*

http://walkthewilderness.net/animals-of-india-72-asiatic-elephant/

http://walkthewilderness.net/animals-of-india-72-asiatic-elephant/

What you see depends on what you're looking for

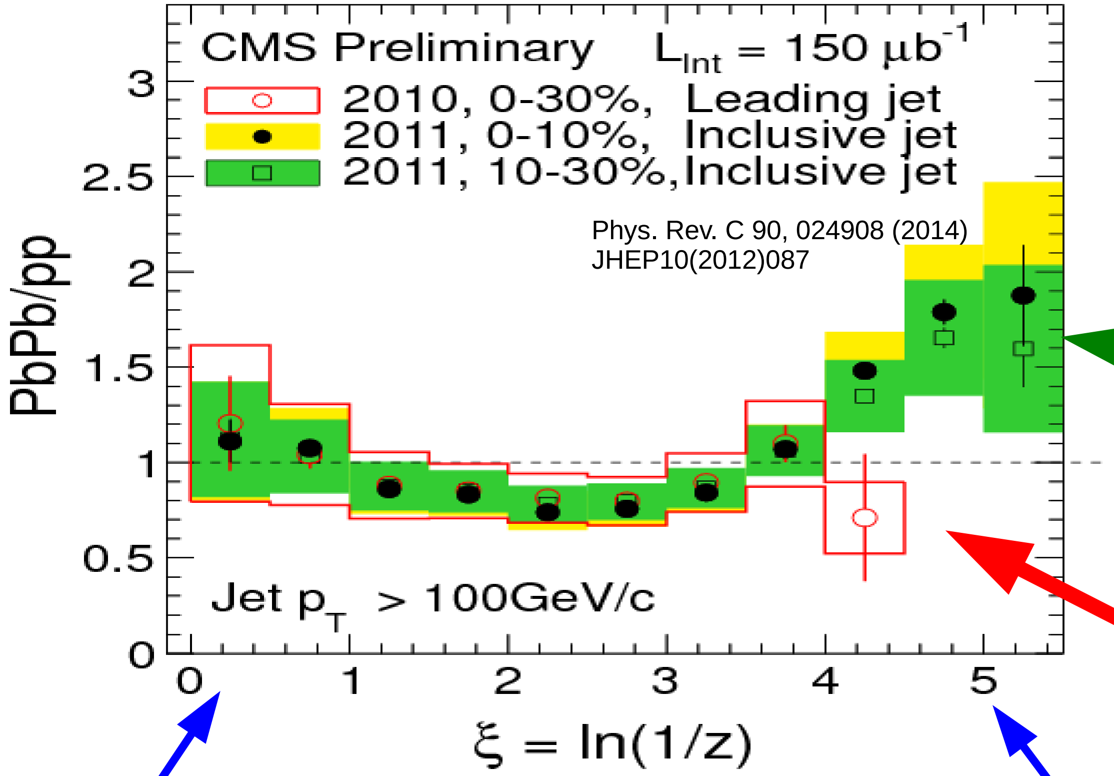
http://walkthewilderness.net/animals-of-india-72-asiatic-elephant/

Bias & background

- **Experimental background subtraction methods:** complex, make assumptions, apply biases
- **Survivor bias:** Modified jets probably look more like the medium
- **Quark/Gluon bias:**
  - Quark jets are narrower, have fewer tracks, fragment harder [Z Phys C 68, 179-201 (1995), Z Phys C 70, 179-196 (1996),]
  - Gluon jets reconstructed with k_T algorithm have more particles than jets reconstructed with anti-k_T algorithm [Phys. Rev. D 45, 1448 (1992)]
  - Gluon jets fragment into more baryons [EPJC 8, 241-254, 1998]
- **Fragmentation bias:** Experimental measurements explicitly select jets with hard fragments

http://walkthewilderness.net/animals-of-india-72-asiatic-elephant/

What you see depends on where you look



High p_T

$z = p_T/E_{jet}$

Low p_T

Mini-summary

- Most studies do one or more of the following:
  - Explicitly apply a (non-purturbative) bias
  - *Implicitly* apply a (non-purturbative) bias
  - Focus on small R
  - Focus on high pT
- May also → survivor bias
- Background subtraction should be part of definition of algorithm

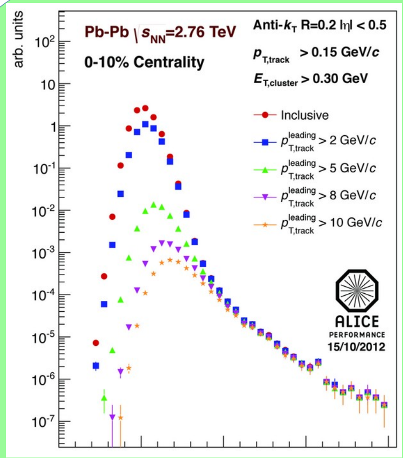
What are the dominant uncertainties?

Analysis steps

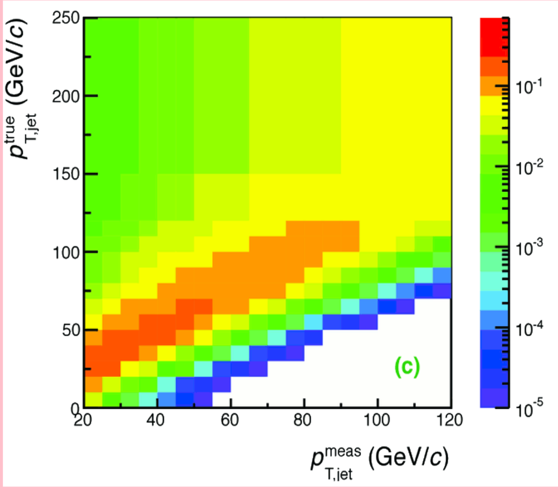
Tracks

Clusters

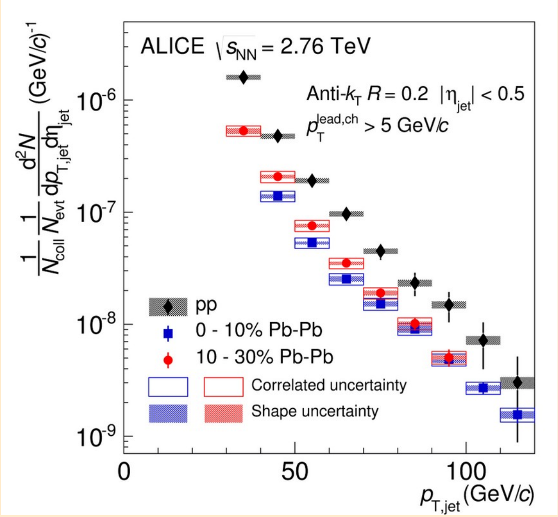
→ Jet finding algorithm → Jet candidates → Background subtraction



Jet spectrum smeared by energy resolution, background fluctuations



Unfolding – corrects for single track reco ε, E resolution, background fluctuations



Corrected spectra

Unfolding

$$\vec{v} = R\vec{\mu} + \vec{\beta}$$

$$v_i = \sum_{j=1}^{M} (R_{ij}\mu_j) + \beta_i$$

- $\vec{\mu}$: the "true" histogram
- $\vec{v}$: the actual data we measure
- $\vec{\beta}$: background
- R : the response matrix

Simple Solution (Inversion)

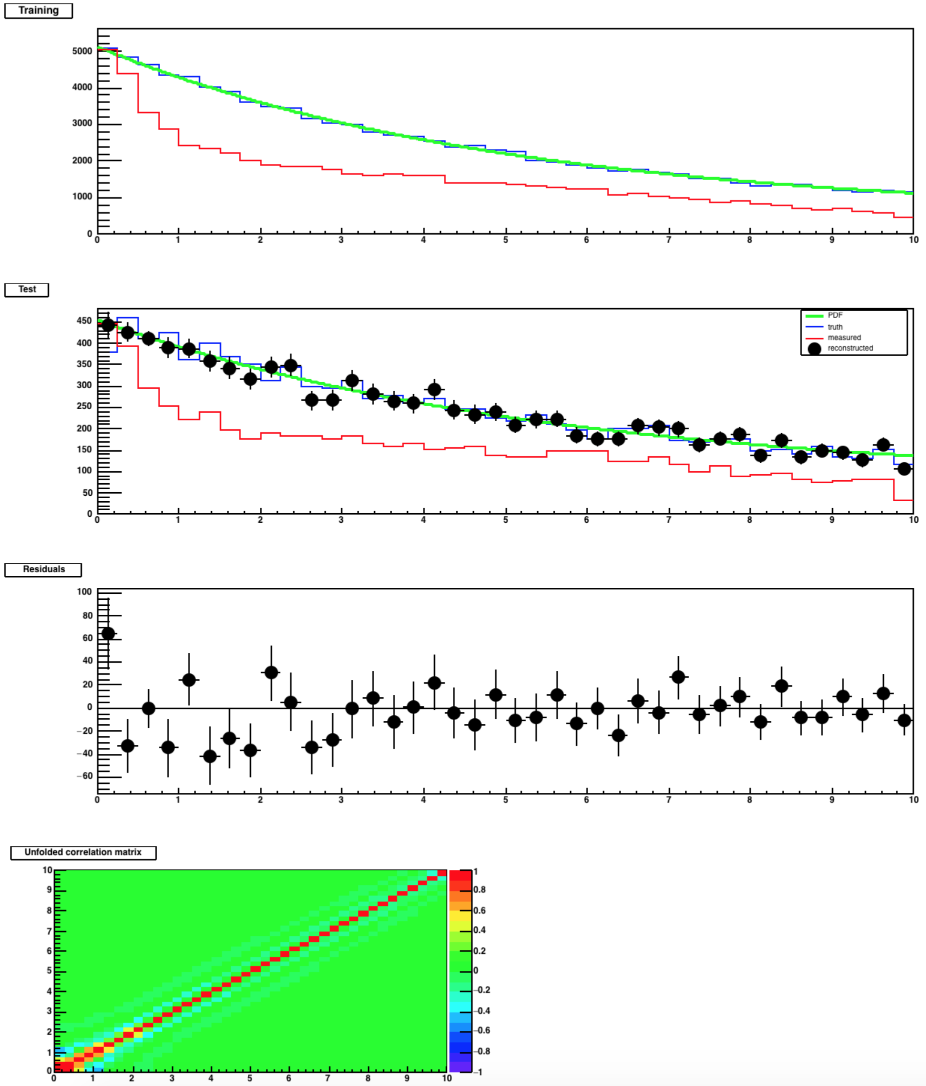
- Rearrange $\vec{v} = R\vec{\mu} + \vec{\beta}$ to get $\vec{\mu} = R^{-1}(\vec{v} - \vec{\beta})$
- Problem: we don't have $\vec{v}$, we have $\vec{n}$, the measured data, which is subject to statistical fluxuations.
- We assume n_i is the maximum likelihood estimator for v_i, then solve for the estimator $\hat{\mu} = R^{-1}(\vec{n} - \vec{\beta})$.
- R^{-1} is obtained from R through simple matrix inversion

Iterative Bayesian Method

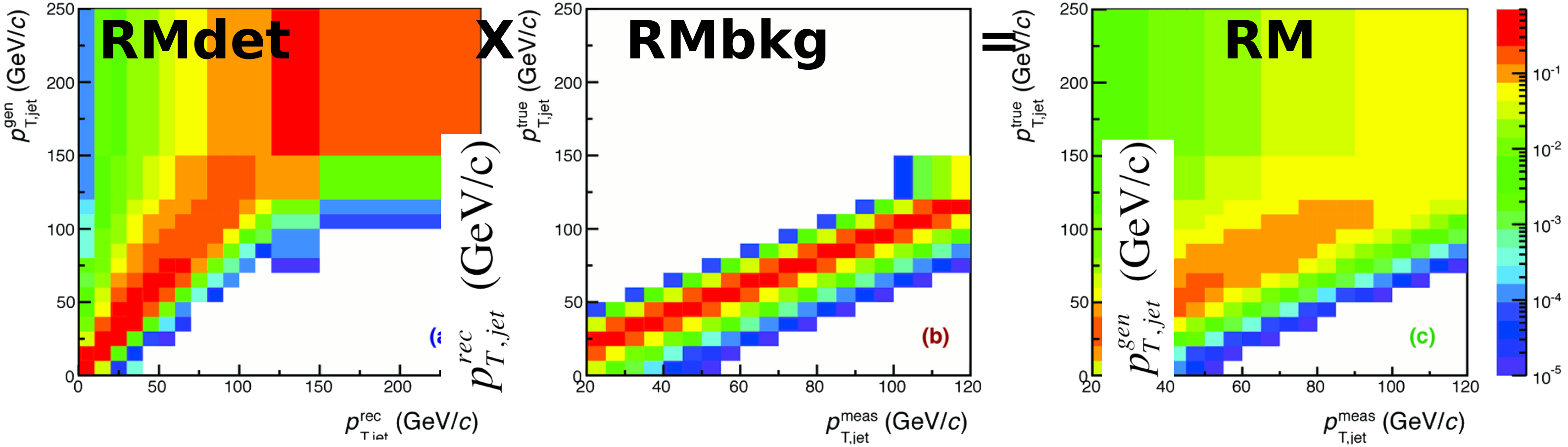
- Using prior knowledge, start with an initial guess for the distribution of true histograms $P^0\,(\hat{\mu})$
- Use Bayes' Theorem to invert the response matrix $P\left(\hat{\mu}_i|v_j^{sig}\right) = \frac{P\left(v_j^{sig}|\hat{\mu}_i\right)P^0\,(\hat{\mu}_i)}{\sum_{l=1}^{M} P\left(v_j^{sig}|\hat{\mu}_l\right)P^0\,(\hat{\mu}_l)}$
- $\hat{\mu}_i = \frac{1}{\epsilon_i}\sum_{j=1}^{N} v_j^{sig}\, P\left(\hat{\mu}_i|v_j^{sig}\right)$ where ϵ_i is the detector efficiency
- Plug in the newly obtained $P\left(\hat{\mu}_i|v_j^{sig}\right)$ and $\hat{\mu}_i$ as new priors, then repeat
- Terminate before the wildly oscillating true inverse is reached (usually ~4 iterations) to preserve some smoothness

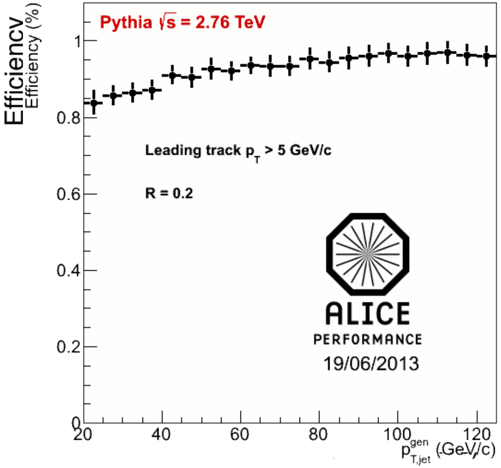
RooUnfold-Bayes

- RooUnfoldTest.cxx
- method = Bayes
- Exponential training and testing



Jets in ALICE: Response Matrix Construction





Anti-k_T R=0.2

$p_{T,track}$ > 0.15 GeV/c

$E_{T,cluster}$ > 0.30 GeV

$p_{T,track}^{leading}$ > 5 GeV/c

(a) RM_{det} Detector response matrix

(b) RM_{bkg} Background fluctuation matrix

(c) $RM_{tot} = RM_{bkg} \times RM_{det}$

Pb-Pb $\sqrt{s_{NN}}$=2.76 TeV

0-10% Centrality

ALICE

PERFORMANCE

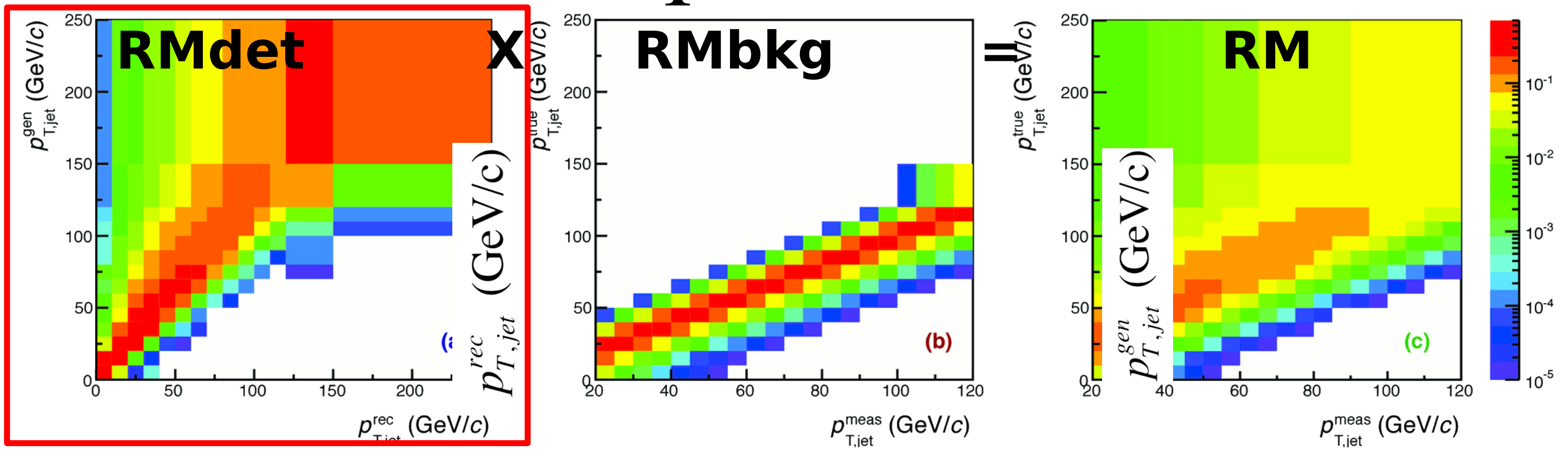
15/10/2012

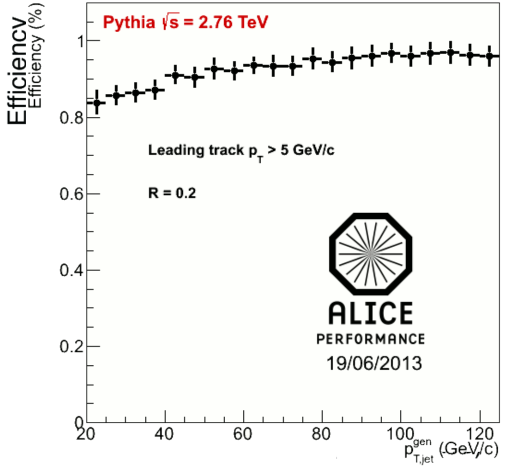
RM_{bkg} and RM_{det} are approximately factorizable

ALI−PERF−44520

Jets in ALICE: Response Matrix Construction

DETECTOR EFFECT





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ALICE PERFORMANCE

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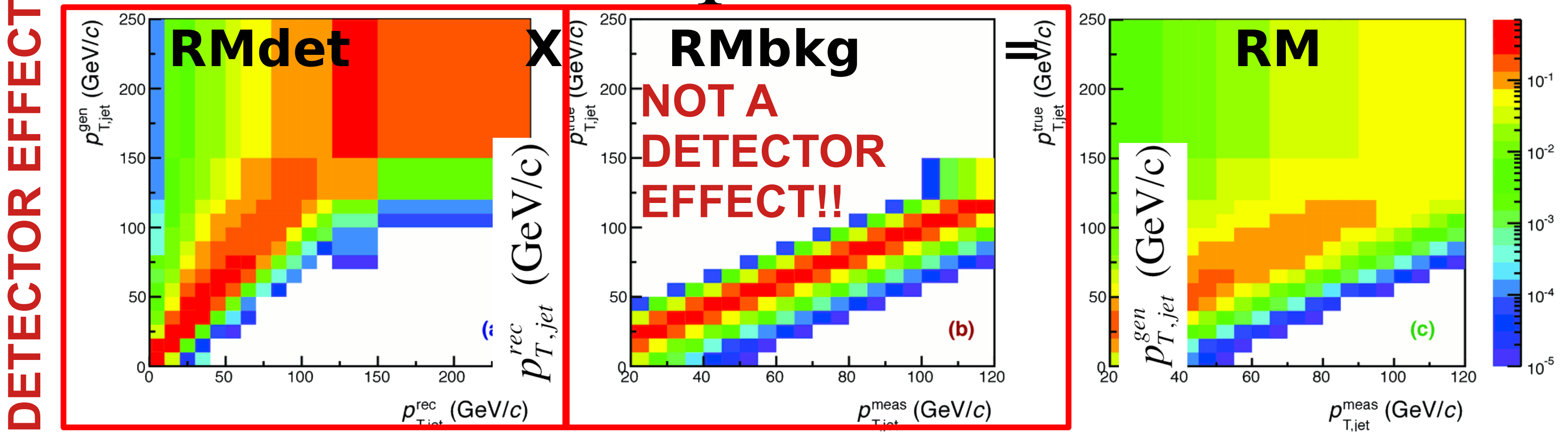
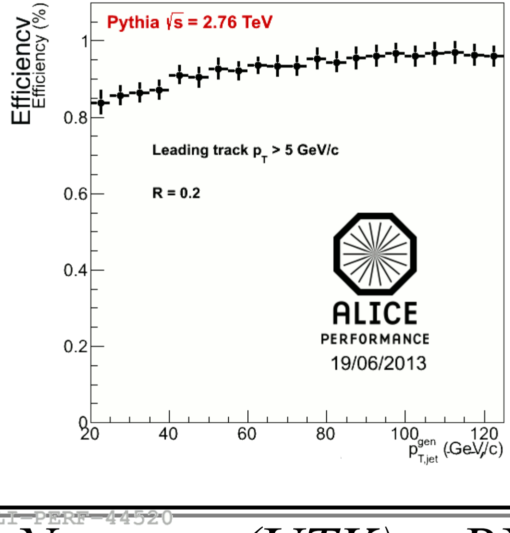
Jets in ALICE: Response Matrix Construction





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ALICE PERFORMANCE

15/10/2012

RM_{bkg} and RM_{det} are approximately factorizable

About unfolding...

- d'Agostini (author of Bayesian unfolding algorithm) says you should avoid it if you can
- Necessary when experimental resolution is poor
  - Ex: Single particle spectra $\frac{\sigma_p}{p} \ll w_{bin} \rightarrow$ unfolding unnecessary
  - Ex: Jet spectra $\frac{\sigma_p}{p} \approx w_{bin} \rightarrow$ unfolding necessary
- Algorithm assumes response matrix is correct
  - Matching reconstructed and simulated jets is non-trivial!
- Corrects for multiple experimental effects simultaneously
  - Difficult to disentangle different effects
  - Leads to non-trivial uncertainty correlations between data points due to algorithm
  - May not handle systematic correlations between effects correctly

Mini-summary

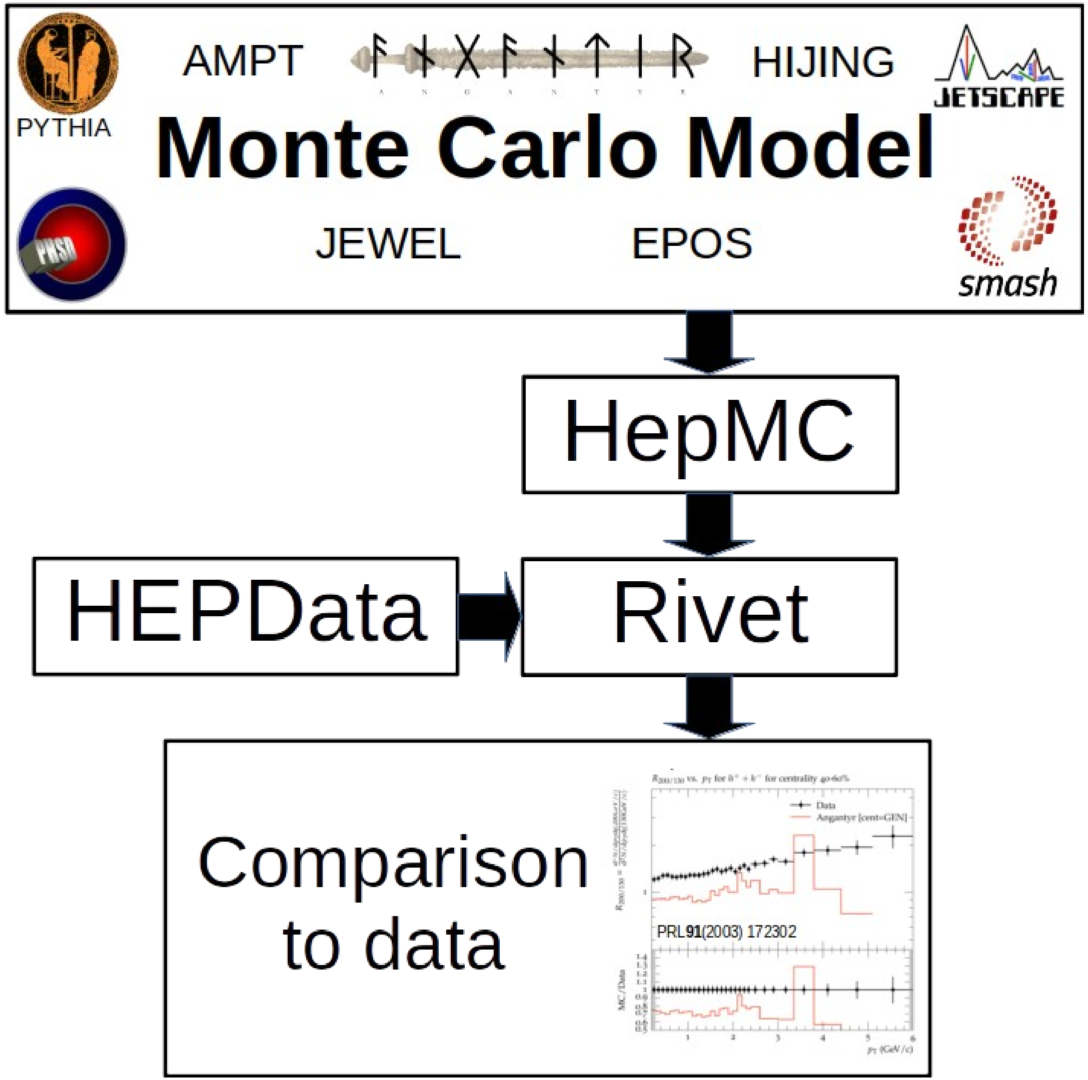
- Jet energy resolution is fundamentally large
- Unfolding is complicated, often unstable, and hard
- Construction of response matrix includes several assumptions

Jets in A+A collisions: How to compare to models

Snowmass Accord: Apply the same algorithm to data and your model. Then the measurement and the calculation are the same.

Rivet: Apply the same algorithm to data and your model. Then the measurement and the calculation are the same.

What is Rivet?

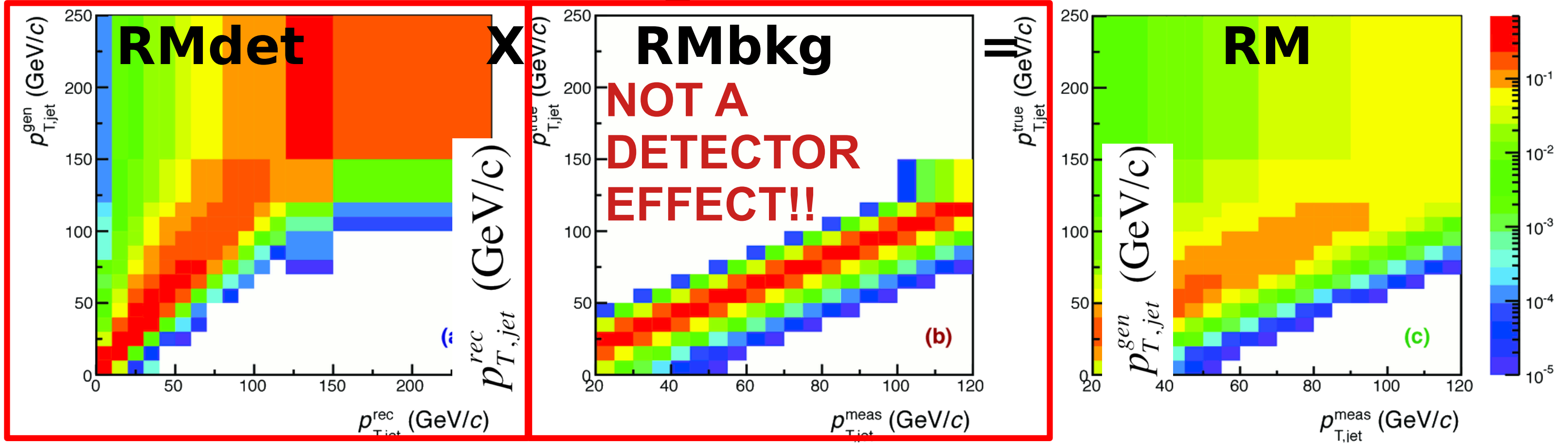

AMPT
HIJING
PYTHIA
JETSCAPE
Monte Carlo Model
JEWEL
EPOS
smash
HepMC
HEPData
Rivet
Comparison to data
Data
Angantyr [cent=GEN]
PRL 91(2003) 172302
MC / Data

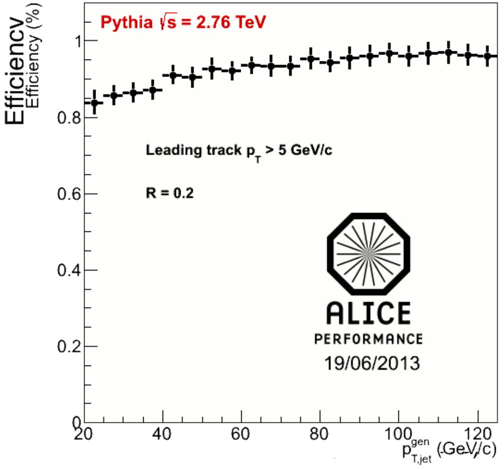
Why use Rivet?

- Facilitates comparisons between Monte Carlos and data
- It’s not that hard
- It preserves analysis details

Jets in ALICE: Response Matrix Construction

DETECTOR EFFECT





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ALICE PERFORMANCE

15/10/2012

RM_{bkg} and RM_{det} are approximately factorizable

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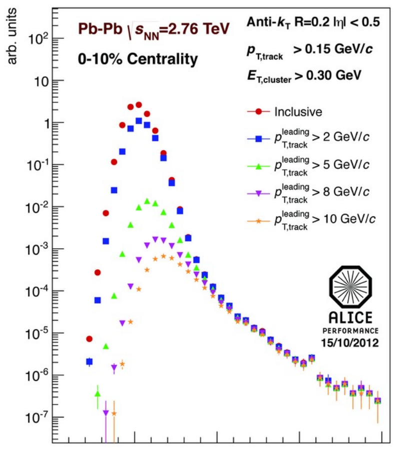
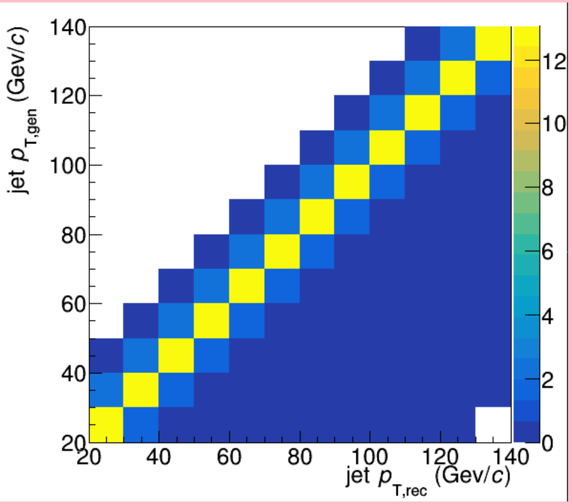
Analysis steps: Full Monte Carlo

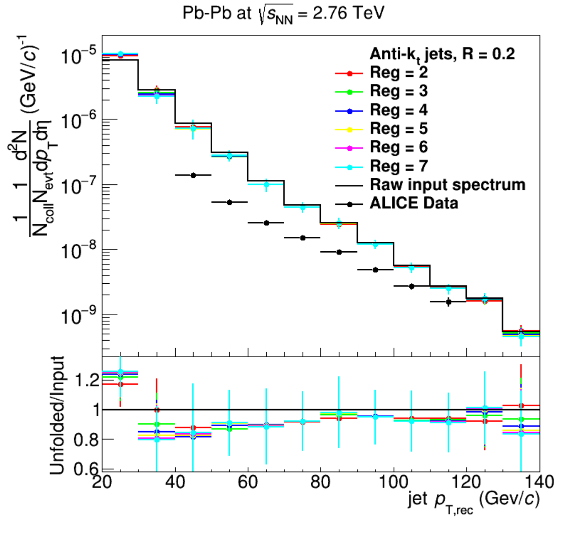
Particles → Jet finding algorithm → Jet candidates → Background subtraction



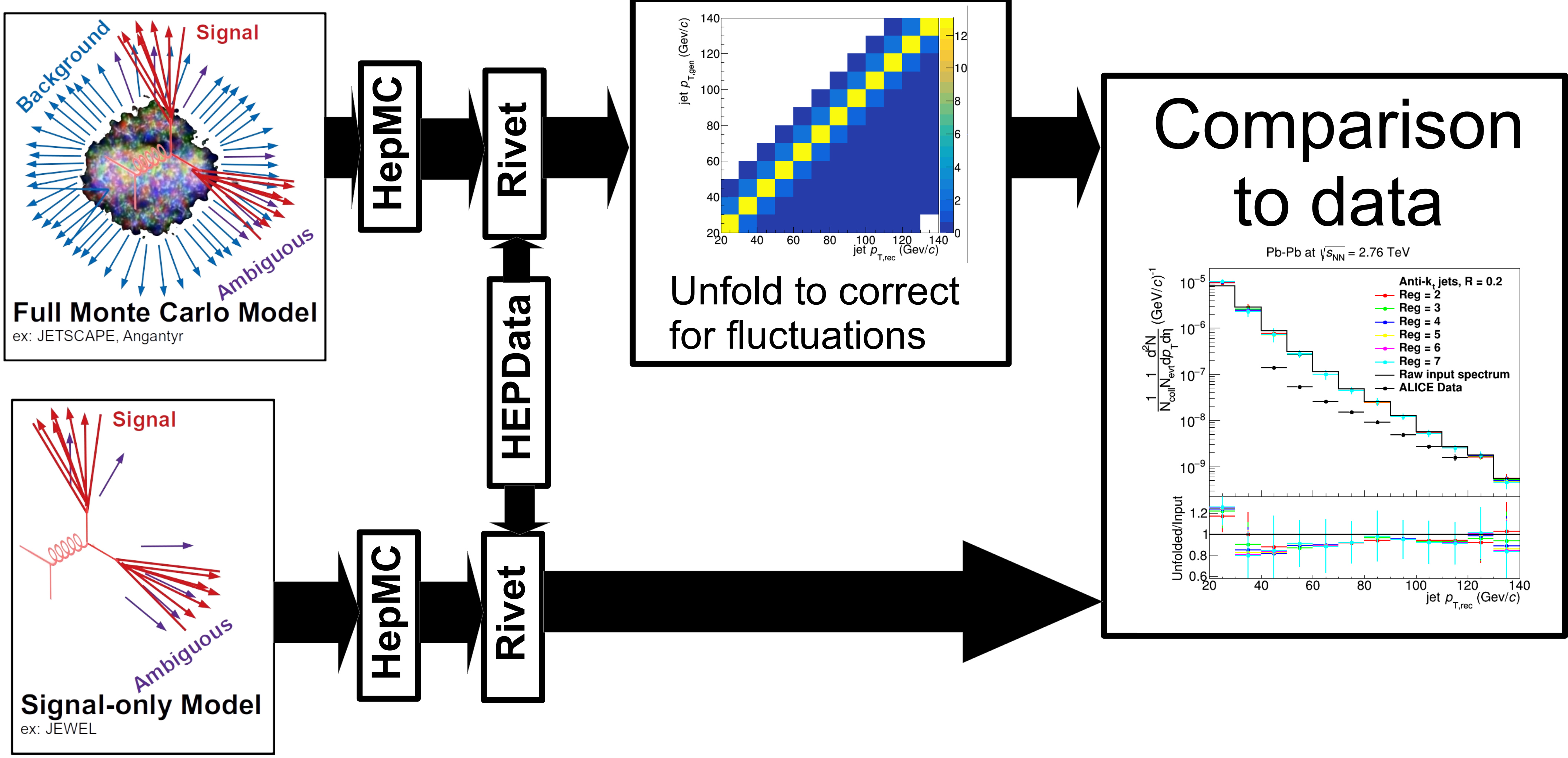
Jet spectrum smeared by energy resolution



Unfolding – ***corrects for background fluctuations***



Corrected spectra


Background
Signal
Ambiguous
Full Monte Carlo Model
ex: JETSCAPE, Angantyr
Signal
Ambiguous
Signal-only Model
ex: JEWEL
HepMC
Rivet
HEPData
HepMC
Rivet
jet $p_{T,gen}$ (Gev/c)
jet $p_{T,rec}$ (Gev/c)
Unfold to correct for fluctuations
Comparison to data
Pb-Pb at $\sqrt{s_{NN}}$ = 2.76 TeV
Anti-k_t jets, R = 0.2
Reg = 2
Reg = 3
Reg = 4
Reg = 5
Reg = 6
Reg = 7
Raw input spectrum
ALICE Data
$\frac{1}{N_{coll}}\frac{1}{N_{evt}}\frac{d^2N}{dp_T d\eta}$ (GeV/c)$^{-1}$
Unfolded/Input
jet $p_{T,rec}$ (Gev/c)

Mini-summary

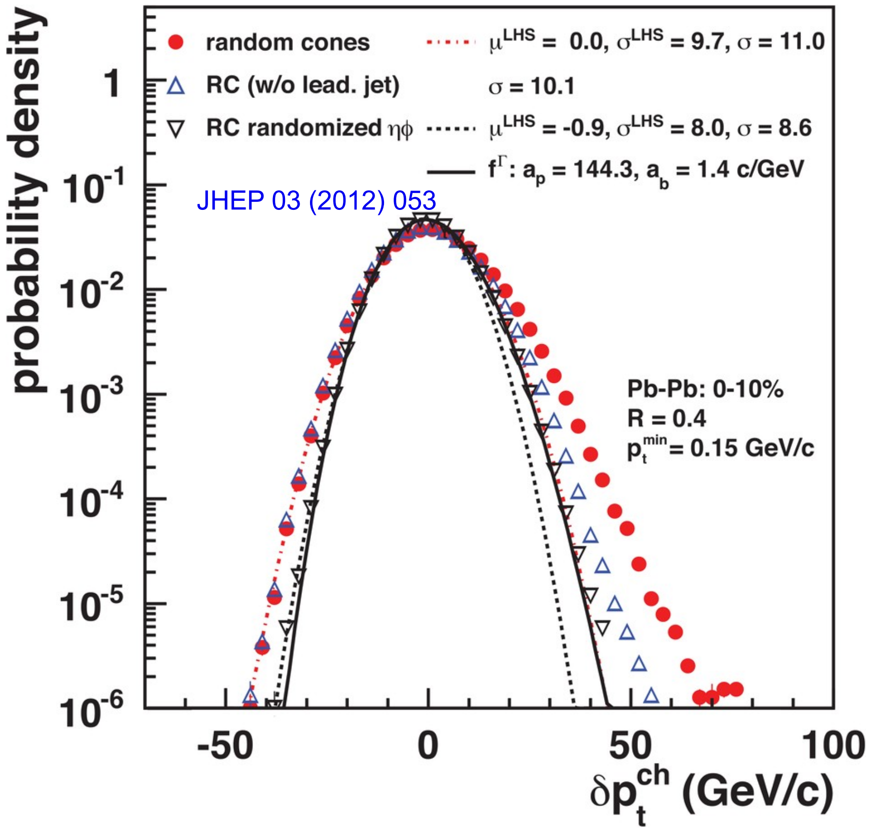
- Experimental techniques can bias measurement in subtle ways
  - Background subtraction
  - Kinematic cuts
  - Choice of jet finder, R
  - Centrality determination
  - Technique for finding reaction plane
- Unclear how these influence the measurement
- Safest to do the same analysis on data and model
  - But unfolding is necessary in a full Monte Carlo model!

Backup

Random cones in ALICE

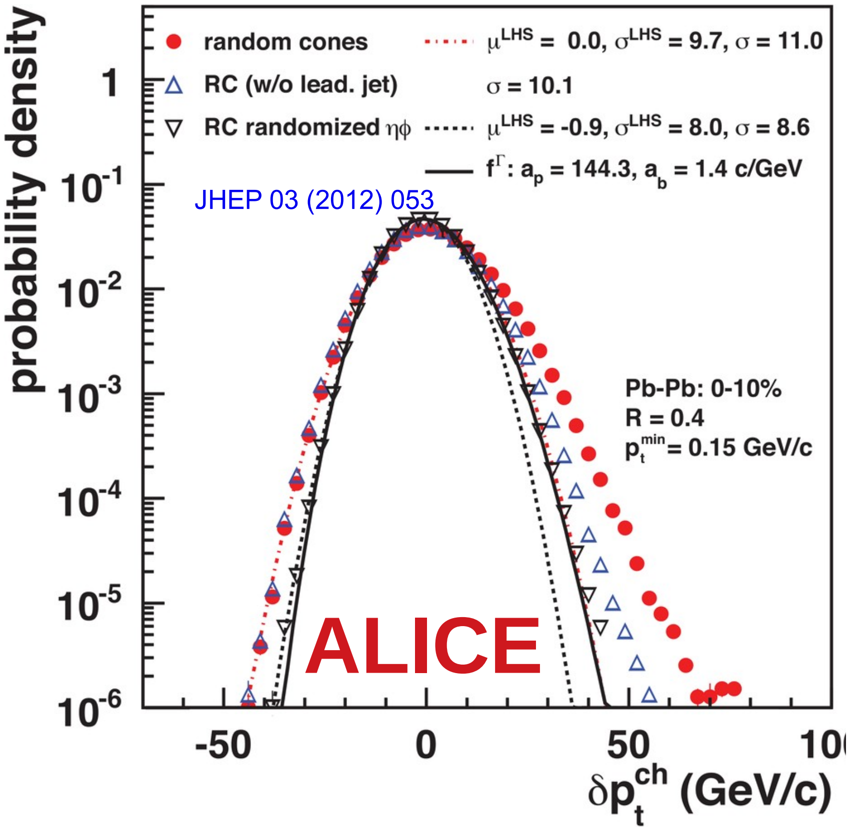
- Estimate ρ
  - k_T jet finder → jet candidates
  - ρ = Median(p_T/A)
- Draw Random cone

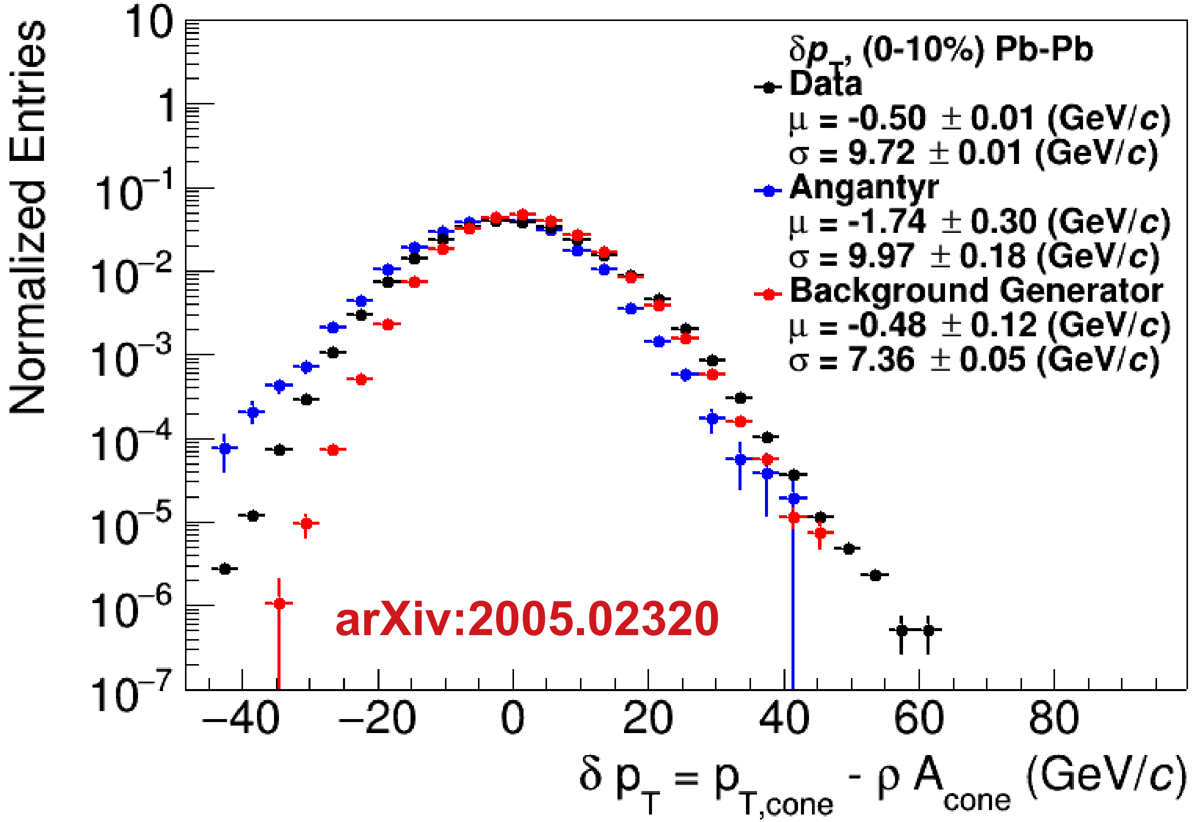
$$\delta p_T = p_T^{reco} - \rho A$$



74

Random cones





Shape of width of the distribution

Single particle spectra

$$f_\Gamma(p_T, p, b) = \frac{b}{\Gamma(p)} (b\, p_T)^{p-1} e^{-bx}$$

$$\frac{dN}{dy} \propto f_\Gamma(p_T, 2, b) = b^2 p_T e^{-k p_T}$$

$$\mu_{p_T} = \frac{p}{b}, \sigma_{p_T} = \frac{\sqrt{p}}{b}$$

Tannenbaum, PLB(498),1–2,Pg.29-34(2001)

Σp_T of N particles → N-fold convolution:

$$f_N(p_T, p, b) = f_\Gamma(p_T, Np, b) \qquad \frac{dpT^{total}}{dy} \propto f_N(p_T, Np, b)$$

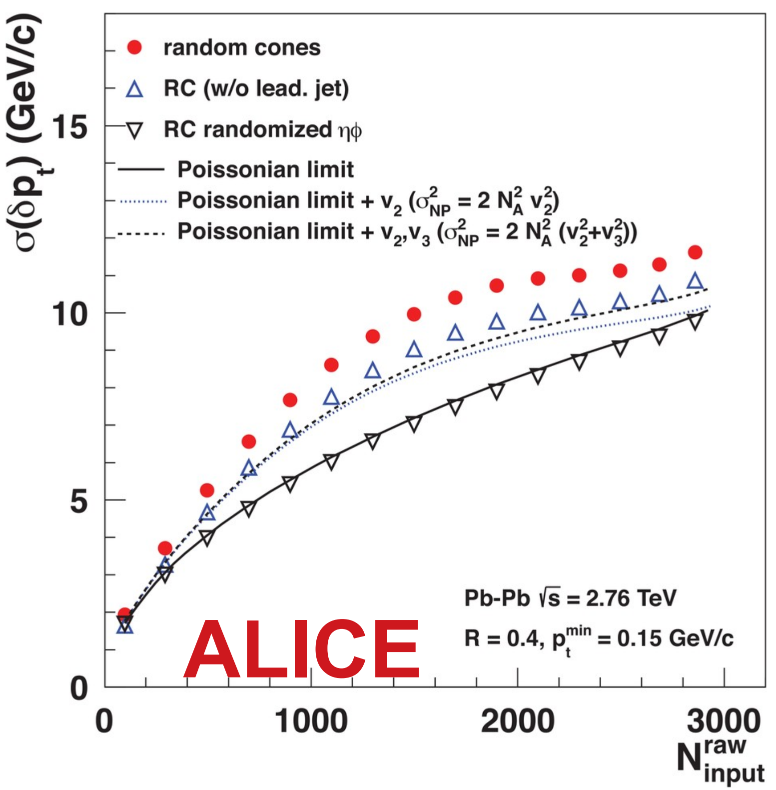
$$N = \frac{N_{total}}{A_{total}} \pi R^2 \quad \mu_{total} = \frac{Np}{b} = N \mu_{p_T}, \sigma_{total} = \frac{\sqrt{Np}}{b} = \sqrt{N} \sigma_{p_T}$$

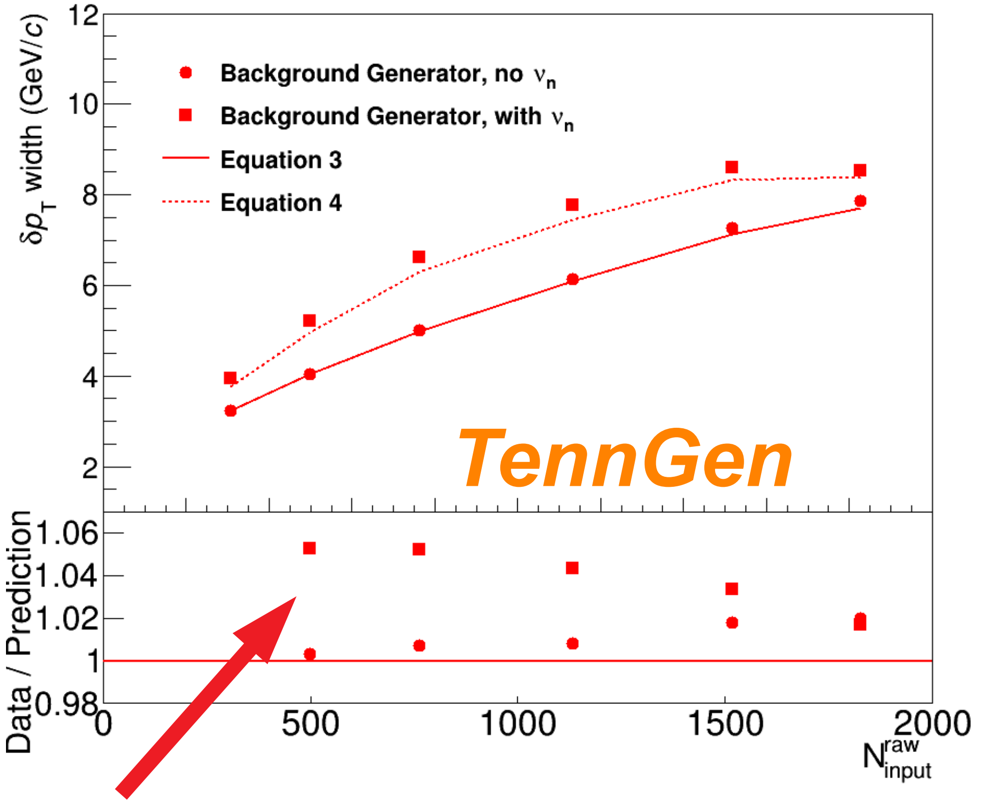
Add Poissonian fluctuations in N: $\sigma_{total} = \sqrt{N \sigma_{p_T}^2 + N \mu_{p_T}^2}$

Add non-Poissonian fluctuations in N due to flow

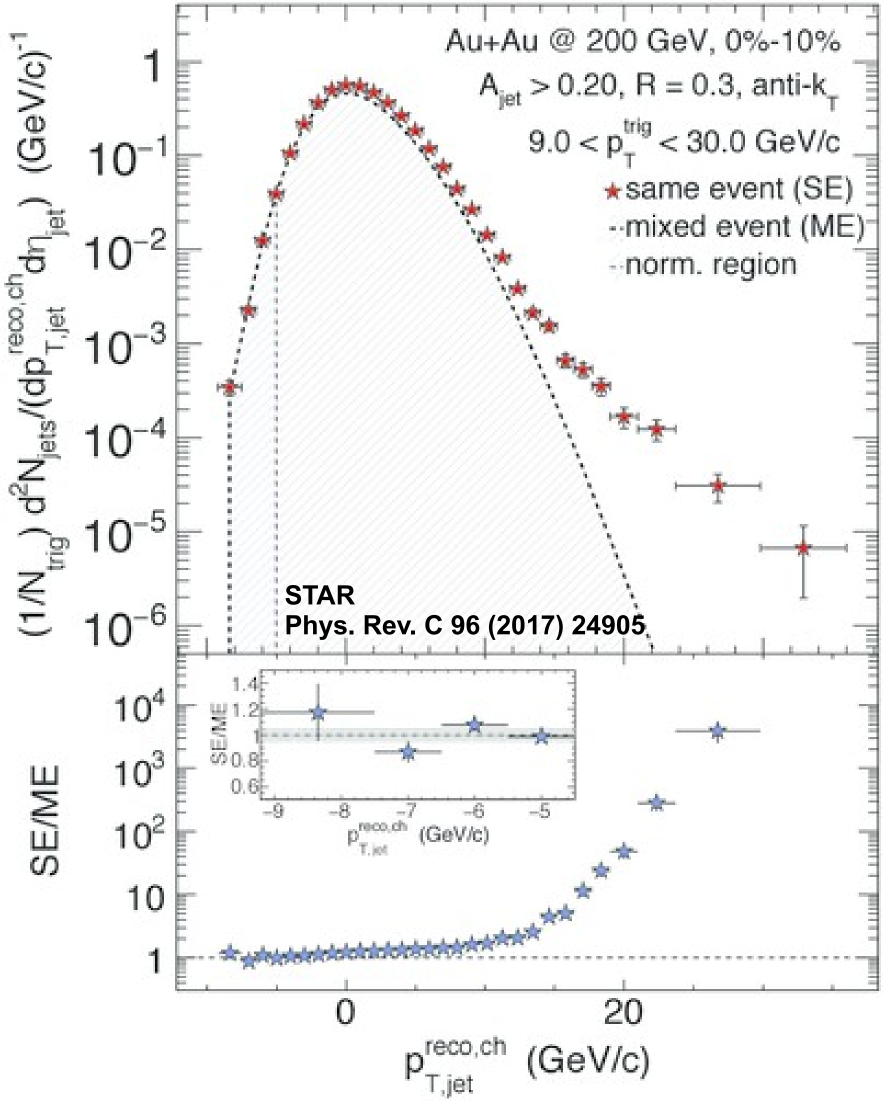
$$\sigma_{total} = \sqrt{N \sigma_{p_T}^2 + \left(N + 2 \sum_n v_n^2\right) \mu_{p_T}^2}$$

Width vs multiplicity





Small deviations



Mixed events

- Gets background up to a normalization factor
- Good agreement with the data… but 20% discrepancies still within uncertainties
- In measurement with background suppressed (h-jet correlations)
- Did not see such agreement at the LHC

Shape of width of the distribution

Single particle spectra

$$f_\Gamma(p_T, p, b) = \frac{b}{\Gamma(p)} (b\, p_T)^{p-1} e^{-bx}$$

$$\frac{dN}{dy} \propto f_\Gamma(p_T, 2, b) = b^2\, p_T\, e^{-k\, p_T}$$

$$\mu_{p_T} = \frac{p}{b}, \sigma_{p_T} = \frac{\sqrt{p}}{b}$$

Tannenbaum, PLB(498),1–2,Pg.29-34(2001)

Assumes shape

Σp_T of N particles → N-fold convolution:

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Add Poissonian fluctuations in N: $\sigma_{total} = \sqrt{N\, \sigma_{p_T}^2 + N\, \mu_{p_T}^2}$

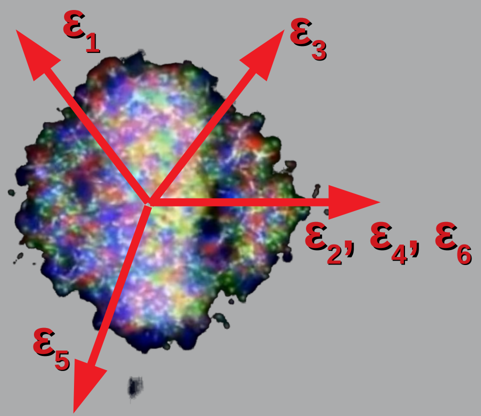
Add non-Poissonian fluctuations in N due to flow

$$\sigma_{total} = \sqrt{N\, \sigma_{p_T}^2 + \left(N + 2\sum_n v_n^2\right) \mu_{p_T}^2}$$

Assumes uncorrelated number fluctuations

TennGen background generator

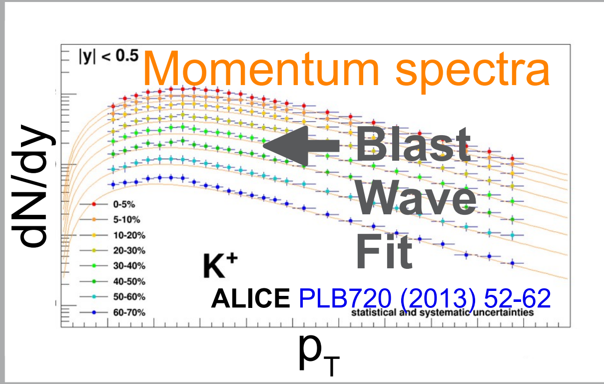
Event properties



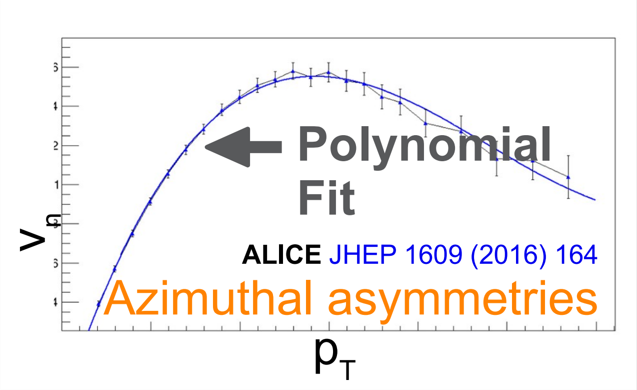
- Even event planes fixed at Ψ=0
- Odd planes at random φ
- Multiplies from ALICE PRC88 (2013) 044910

No jets! No resonances
Emulates hydro correlations

Track properties



→ Random p_T

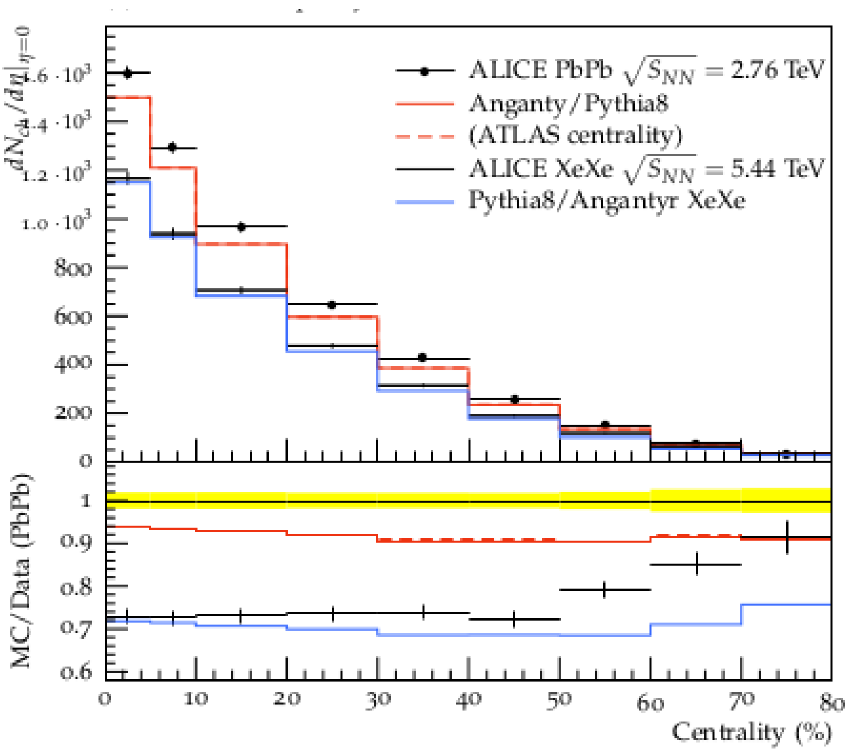


→ v_n → Random φ

PYTHIA Angantyr

JHEP (2018) 2018: 134

- Based on PYTHIA 8
  Sjöstrand, Mrenna & Skands,
  JHEP05 (2006) 026
  Comput. Phys. Comm. 178 (2008) 852.
- Based on Fritiof & wounded nucleons
- N-N collisions w/fluctuating radii → fluctuating σ



Lots of jets! And resonances!
No hydrodynamics, no jet quenching

Area-based background subtraction

Cacciari & Salam, PLB659:119–126,2008

Particles, clusters

↓

k_T algorithm

$$k_T = p_T, \Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$$

- For all *i,j* calculate:
  $$d_{ij} = min(p_{T,i}^2, p_{T,j}^2) \Delta R_{ij}^2$$
  $$d_{iB} = p_{T,i}$$
- Combine smallest d_{ij}.
  If d_{iB} smallest, $d_{iB} \rightarrow$ jet

Repeat until no particles left

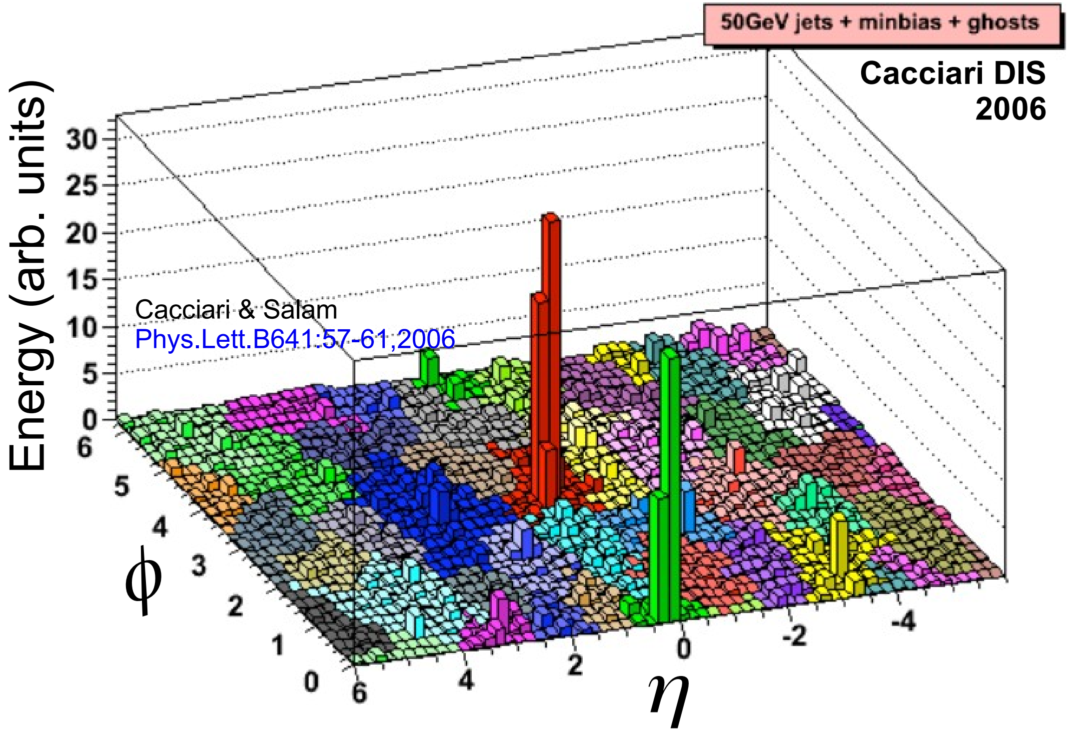
↓

Jet candidates

↓

Median $\rho = p_T/A$

$$p_T^{jet} = p_T^{reco} - \rho_{median} A^{jet}$$



Theoretical calculations

Factorization theorem

- Assumption: Parton distribution functions, perturbative cross section, fragmentation function factorize
- What people really mean by "perturbatively calculable"
  - *D and f are explicitly non-perturbative!*
  - D is for *parton c → hadron h*
    Not what is experimentally measured
- Most theories for jet quenching modify fragmentation function D

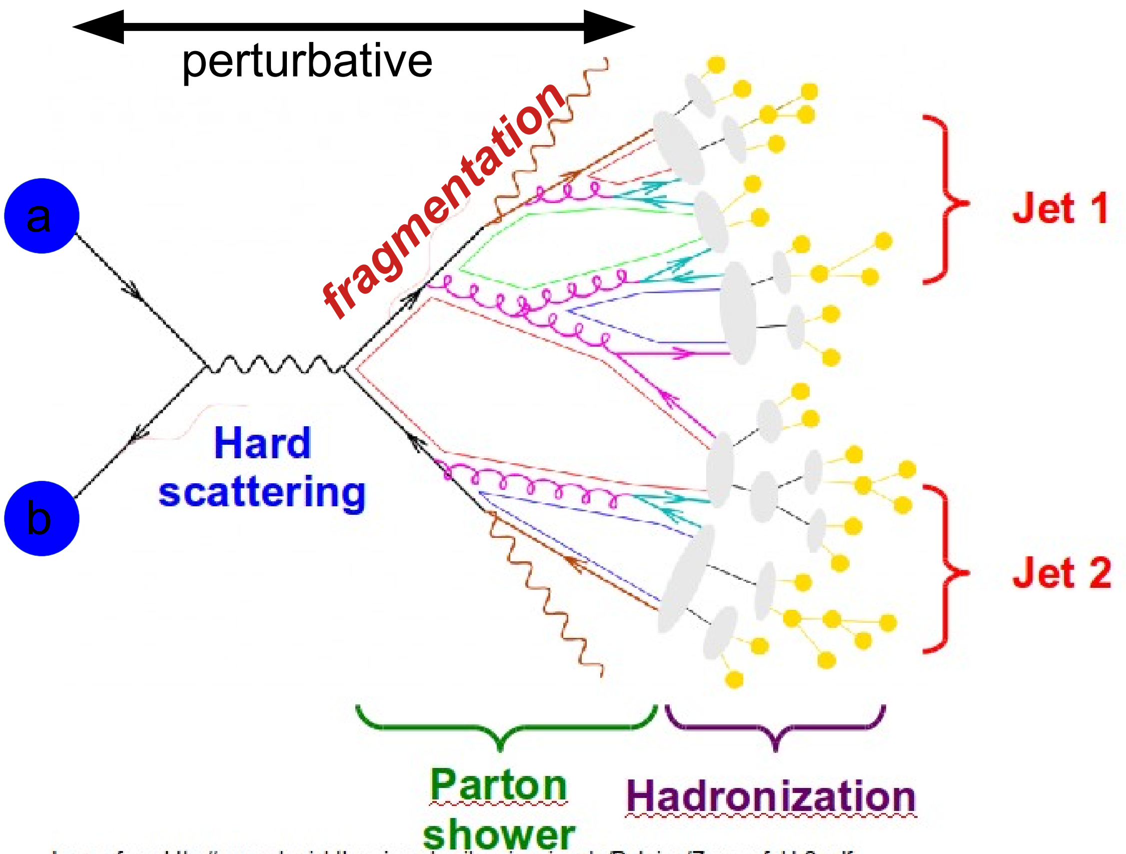


Image from http://www.gk-eichtheorien.physik.uni-mainz.de/Dateien/Zeppenfeld-3.pdf

$$\frac{d^3\sigma^h}{dy\,d^2p_T}=\frac{1}{\pi}\int d\,x_a\int d\,x_b f_a^A(x_a)f_b^B(x_b)\frac{d\,\sigma_{ab\to cX}}{d\,\hat{t}}\frac{D_c^h(z)}{z}$$

Jet finders