Regulation with Direct Benefits of Information Disclosure and Imperfect Monitoring
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Abstract
We model the optimal design of programs requiring heterogeneous firms to disclose harmful emissions when disclosure yields both direct and indirect benefits. The indirect benefit arises from the internalization of social costs and resulting reduction in emissions. The direct benefit results from the disclosure of previously private information which is valuable to potentially harmed parties. Previous theoretical and empirical analyses of such programs restrict attention to the former benefit while the stated motivation for such programs highlights the latter benefit. When disclosure yields both direct and indirect benefits, policymakers face a tradeoff between inducing truthful self-reporting and deterring emissions. Internalizing the social costs of emissions, such as through an emissions tax, will deter emissions, but may also reduce incentives for firms to truthfully report their emissions.
I. Introduction

Regulatory agencies, including the Environmental Protection Agency (EPA), commonly cite two categories of benefits associated with information disclosure programs. The first, an indirect benefit, arises from the internalization of the social costs of emissions (and consequent reductions in emissions) due to market responses to disclosures or regulatory instruments such as taxes on disclosed emissions. The second, a direct benefit, results from the disclosure of previously private information. Referring to information disclosure programs in a recent report that describes the U.S. experience with various environmental policies, the EPA states “The environmental information embodied in these approaches has economic value...even in the absence of any changes in emissions by firms” (p. 153) [23].¹ Timely information about emissions may enable potential damages to be avoided or mitigated both by affected parties and public agencies. For example, disclosure may reduce consumption of contaminated water by alerting individuals of the need for avoidance or proper treatment. Disclosure may also decrease the environmental impacts of a toxic release by accelerating clean-up efforts.

Theoretical analyses have tended to represent the social cost of emissions as a function only of emissions levels, independent of whether the presence and magnitude of emissions are publicly disclosed. The empirical work has followed a similar convention by measuring program success in terms of reductions in emissions. Neither strand of the literature has yet to explicitly account for the possibility that disclosure of harmful emissions may be directly beneficial, outside of any indirect impacts of disclosure requirements on emissions. We develop a theoretical model that attempts to reconcile this apparent inconsistency between the stated motivation for information disclosure programs and previous analyses of such programs.

¹ In fact, the report refers to the benefits of disclosure from changes in consumer or producer behavior, such as reduced emissions, as “ancillary” (p. 153).
In our model, disclosure of emissions is directly beneficial but actual emissions are imperfectly observable so policymakers face a tradeoff between inducing truthful self-reporting and deterring emissions.\(^2\) Internalizing the social costs of emissions, such as through a Pigovian tax, will deter emissions, but it may also reduce incentives for firms to truthfully disclose their emissions.

When monitoring firm behavior (such as through an audit process) is costly, a policymaker must account for three factors when designing regulatory policy: (1) the benefit of reduced emissions arising from internalizing social costs, (2) the direct social benefit of disclosure of emissions that do occur, and (3) enforcement costs. Previous analyses of environmental compliance have addressed factors (1) and (3) by considering a regulator whose objective is to minimize emissions (Garvie and Keeler [4]; Macho-Stadler and Perez-Castrillo [18]) or to minimize enforcement costs for a given level of compliance (Livernois and McKenna [17]). We model the regulator’s objective in a way that accounts for the reduction in damages arising both from disclosure of emissions and a reduction in the quantity of emissions. This framework is both more general and more representative. In this paper our principal objective is to model the optimal policy choice in this context when the instruments at the regulator’s discretion are a uniform tax on (disclosed) emissions and the frequency (or probability) of auditing a firm’s disclosure report.

In order to better understand the characteristics of the regulator’s trade-off between inducing compliance with disclosure requirements and reducing emissions, we develop a model

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\(^2\) This trade-off is present in other regulatory settings such as consumer product and food safety. Firms are required to disclose product failures and hazards, but the more costly such disclosure (either due to fines or liability exposure) the greater the incentive firms have to conceal such information. Reducing fines or limiting liability costs encourages disclosure but may dull incentives to reduce product defects. However, this tradeoff is not present in some other regulatory settings where information disclosure programs have traditionally been applied, such as income taxation.
of firm behavior in the context of an imperfect audit. An imperfect audit reveals some percentage of a firm’s actual emissions. Firms are heterogeneous in audit accuracy. That is, we allow for the possibility that some firms may be relatively more successful in hiding the extent of their misreporting either due to the nature of their emissions, evasive efforts undertaken by employees, or some other factor. Within this setting, firms optimize their choices of how much to emit and how much of their emissions to disclose in order to maximize their expected net benefits. The regulator in turn optimally chooses the policy parameters based on his expectations about how firms facing a particular regulatory environment will behave.

The model we develop adds to the literature on the role of self reporting in environmental regulation. Malik [19], Swierzbinski [22] and others have shown that incentive-compatible mechanisms for self reporting (in which firms are induced to truthfully report their emissions) can achieve enforcement cost savings and increase social welfare. The benefit of self reporting in these models arises due to the regulator having incomplete information regarding the social costs or private benefits (i.e., abatement costs) of emissions by a particular firm; disclosure then conveys important information about firm types to the regulator. In our model, disclosure allows the regulator to make inferences regarding average firm behavior but does not permit the regulator to distinguish among different types of firms. The social benefit from self reporting in our model arises very differently (and more directly) from the fact that reported emissions cause less social damage than undisclosed emissions. In our model disclosure of emissions by firms is a desirable end in itself, rather than a mechanism to achieve desirable emissions reductions in a more cost effective manner.³

³ Of course regulations requiring self reporting may serve a dual purpose, both to capture direct benefits of disclosure and to achieve enforcement cost savings from information revelation. We focus on the direct benefits of disclosure to keep our model fairly straightforward and make the implications of this regulatory motive most transparent.
This paper is organized as follows. Section II develops our main model and presents our results. We first consider the decision facing heterogeneous firms required to disclose emissions subject to a uniform tax enforced through imperfect audits. We then analyze the optimal policy choice of the regulator, who has incomplete information on firm types. Section III concludes with discussion of the implications of our model and possible extensions.

II. The Model and Results

A. The Firm’s Problem

We first analyze the decision facing firms subject to a mandatory information disclosure policy requiring them to report a level of emissions to the regulator. The compliance decision for a firm is defined by three factors: 1) the disclosure costs the firm incurs as a function of its reported emissions, 2) the penalty costs the firm incurs as a function of any emissions that are revealed in excess of the level it discloses, and 3) the nature of the auditing mechanism.4

Firms may face costs associated with emissions (whether disclosed or undisclosed) arising from a variety of sources.5 Most directly, a firm may be subject to a tax on disclosed emissions, and a subsequent penalty on unreported emissions that are later revealed. A firm may also face current or future liability costs associated with emissions, both of which may be reflected immediately in the market valuation of the firm upon the revelation of its emissions.6

Finally, the firm may face costs associated with the revelation that it failed to disclose emissions

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4 Becker’s [2] “optimal penalty” model provides the theoretical basis for the literature on environmental compliance. The main insight from his model is that potential offenders respond to the probability of detection as well as the severity of the punishment. See Polinsky and Shavell [20] (and the citations within) for a general review of the enforcement literature. Cohen [3] and Heyes [10] provide reviews of the environmental compliance and enforcement literature.
5 Firms may fail to perfectly comply in some cases simply because it is costly to collect the necessary information (e.g., a firm may bear some cost of simply measuring its own emissions). We ignore the possibility here and simply assume the firm has perfect knowledge of its emissions.
6 See Hamilton [5], Khanna et al. [15], and Konar and Cohen [16] for empirical evidence on market reactions to releases of the Toxics Release Inventory (TRI).
when required. The revelation may result from direct regulatory oversight, or through other mechanisms such as internal whistleblowers, disclosures by the media or environmental watchdog groups, or simply due to random events that bring information into the public domain.

Most previous analyses of environmental compliance assume an error-free audit process (see for example Kaplow and Shavell [14] and Innes [11]), an assumption consistent with the tax compliance literature. We define an audit to be error-free if it reveals, perhaps with some probability less than one, the exact degree of misreporting. Recently, Macho-Stadler and Perez-Castrillo [18] depart from the more common assumption in the literature of an audit that always reveals the exact degree of misreporting by allowing the probability of perfect revelation to be less than one. Notice however that the effect of this assumption is merely to decrease the probability of detection (the firm now faces a compound probability). Heyes [8] considers a similar audit structure where the probability that an audit (perfectly) detects non-compliance is endogenous. In each of these models, provided an audit occurs, it reveals either no misreporting or the exact degree of misreporting and therefore is consistent with our definition of an error-free audit. The assumption of error-free audits seems best suited to situations where firms make dichotomous choices to comply with a regulation or not. However, in the case of environmental information disclosure requirements, where incurred penalties are likely to vary with the degree of noncompliance, the firm’s decision may be more accurately modeled as choosing the optimal degree of compliance. Therefore, we model compliance as a continuous choice and assume each firm faces an imperfect audit, one that reveals a percentage of the firm’s actual emissions.

Consider the problem facing a $k$-type firm (firms vary in type with regard to the distribution of possible audit outcomes in a manner described below). Let $e_k$ represent the $k$-

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7 Malik [19] is an exception. He models a binary compliance decision allowing for errors in auditing the firm’s compliance status. In contrast, we model compliance with the information disclosure requirement as a continuous choice in order to focus our analysis on behavioral changes at the intensive, rather than extensive, margin.
type firm’s emissions and denote its benefit of emitting as \( B(e_k) \) where \( B'(e_k) > 0 \) and \( B''(e_k) < 0 \). Let \( z_k \) denote the share of actual emissions reported by the \( k \)-type firm, so the reported quantity of emissions is \( e_k z_k \). For clarity and tractability, we assume that for each unit of reported emissions, the firm incurs a constant per unit cost, denoted \( \alpha \), which we characterize as the “tax” on emissions. Similarly, if the audit reveals a level of emissions that exceeds reported emissions, the firm incurs a constant per unit cost, denoted \( \beta \), on the revealed but unreported emissions. We refer to \( \beta \) as the “penalty.”

Each firm is audited with (independent) probability \( p \) and has private information, represented by the parameter \( k \), regarding the distribution of audit outcomes if it is audited. That is, if a firm of type \( k \) is audited, the audit reveals a quantity of emissions equal to \( e_k (u + k) \) where \( u \) is a random variable with probability density function \( f(u) \) and cumulative distribution function \( F(u) \) on the interval \([1 - d, 1 + d]\). Each audited firm receives an independent draw of \( u \) but all draws are from the common distribution \( F \) (i.e., the distribution of audit outcomes differs among firms only with respect to their \( k \) type). We assume \( f(u) \) is unimodal, symmetric around one, and independent of the firm’s actual emissions. That is, the scale of the firm or its

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8 Both disclosure and penalty costs could of course be non-linear. For example, the penalty cost function might increase at an increasing rate with the magnitude of the violation if regulators take the view that large infractions should be punished severely while minor infractions receive a much milder treatment. The linearity assumption renders the model much more tractable and avoids issues associated with the optimal size of a firm as a function of the regulatory environment, which is beyond the scope of our analysis.

9 There are several other ways in which we might incorporate firm heterogeneity with regard to enforcement. For example, we could assume that firms differ in their perceived penalties for non-reporting or in their probabilities of being found noncompliant as in Innes [12]. We thank an anonymous referee for suggesting these possibilities to us.

10 Because the audit process has two-sided errors yielding the possibility that emissions are “revealed” in excess of the actual level (as in Harford [6]), it is possible that a firm would find it optimal to over comply, reporting emissions in excess of its actual level. In the regulatory context we model in the next section the regulator will never find it optimal to induce overcompliance from firms on average (across \( k \) types), and for simplicity we will assume none over-comply. Arora and Gangopadhyay [1], Shimshack and Ward [21], among others explicitly focus on overcompliance with environmental regulations.
emissions level does not impact the effectiveness of audits, so the audit is equally likely to reveal any given percentage of actual emissions regardless of the firm’s true emissions level.

The value of $k$ varies across firms and the regulator knows only the distribution of $k$, denoted $G(k)$ with support $[-\varepsilon, \varepsilon]$. Assuming firms are heterogeneous in this manner ensures that the regulator cannot infer a particular firm’s true emissions from its report. The expected value of $k$ is assumed to be zero so that on average across firms audits are unbiased. An additional assumption, that $d + \varepsilon < 1$, is required to ensure an interior solution on $z$ for firms of all types.

Given our assumptions and the values of $\alpha$, $p$, and $\beta$, the $k$-type firm faces a constant per unit cost of emitting, denoted $\mu_k$, where:

$$\mu_k \equiv \mu(z_k) = \alpha z_k + p\beta \int_{z_k-k}^{d} (u+k-z_k) f(u) du .$$

The firm’s objective is to choose the report, $z_k$, and emissions, $e_k$, to maximize the expected net benefits of emitting:

$$\max_{e_k, z_k} \{ B(e_k) - e_k \mu_k \} .$$

It is clear from equation (2) that with a constant tax and penalty and independence between the audit effectiveness and actual emissions levels, firm $k$’s optimal choice of $z_k$ is independent of $e_k$. The first order condition for an interior solution on $z_k$ is given by:

$$\alpha = p\beta \int_{z_k-k}^{d} dF(u) = p\beta \left[ 1 - F(z_k^* - k) \right]$$

\[ \text{11 If the regulator knew a firm’s type then he could infer the optimal report, consequent cost of emitting, and optimal emissions quantity. In such a circumstance it is less clear that disclosure of emissions by the firm would yield direct benefits.} \]
where \( z^*_k \) denotes the optimal reported share of emissions that minimizes \( \mu_k \). The first order condition indicates that the firm’s optimal report, \( z^*_k \), equates the marginal cost of reported emissions, \( \alpha \), and the expected marginal benefit of reported emissions. The expected marginal benefit reflects the expected avoided per unit penalty on revealed but unreported emissions.

Rearranging (3) we obtain a simple expression for \( z^*_k \):

\[
z^*_k = F^{-1}\left(1 - \frac{\alpha}{p\beta}\right) + k.
\]

(4)

The form of firm heterogeneity we have introduced enters the model fairly simply; the firm-specific audit parameter simply shifts the optimal report, \( z^*_k \).

**Lemma 1.** Given \( \alpha \), \( \beta \), and \( p \), an interior solution on \( z^*_k \) exists for \( \frac{\alpha}{p\beta} \) sufficiently less than one with \( z^*_k \) defined by expression (4) above. For an interior solution, the firm’s optimal report, \( z^*_k \), is decreasing in the tax on reported emissions, \( \alpha \); increasing in the probability of audit, \( p \); increasing in the penalty on revealed but unreported emissions, \( \beta \); and increasing in the firm-specific audit parameter, \( k \).

Assuming an interior solution exists for firms of all \( k \)-types at \( z^*_k \), the first order condition on \( e_k \) can be stated as follows:

\[
\mu^*_k = B'(e^*_k)
\]

(5)

where \( \mu^*_k \equiv \mu(z^*_k) \) denotes the marginal cost of emitting given the optimal report. The unit-cost of emissions, \( \mu^*_k \), for a particular firm depends both directly on \( k \) and on the resulting \( z^*_k \) (with \( \mu^*_k \) of course increasing in \( k \)). Equation (5) implicitly defines the firm’s demand for emissions, as
a function of the marginal cost of emitting (given $z_k^*$), which we denote $e_k^* = e(\mu_k^*) = B'^{-1}(\mu_k^*)$, where $e'(\mu_k^*) < 0, e''(\mu_k^*) \geq 0$. Lemma 2 states the comparative static results for the optimal level of emissions, $e_k^*$.

**Lemma 2.** The $k$-type firm’s optimal level of emissions, $e_k^*$, decreases with the tax on reported emissions, $\alpha$; the penalty on revealed but unreported emissions, $\beta$; the probability of audit, $p$; and the firm-specific audit parameter, $k$.

**B. The Regulator’s Problem**

We model the situation facing the regulator as a minimization problem and assume his objective function, denoted $V$, is comprised of three terms: (1) the total damages from emissions by firms net of expected taxes and fines paid by the firms; (2) enforcement costs; (3) the firms’ net benefits from emitting. The first component of $V$ accounts for both the direct and indirect benefits of emissions disclosure. Because the regulator knows only the distribution of firm types, his objective function depends on expected firm behavior. Thus, to develop $V$, we first consider these three components for a $k$-type firm.

The (total) damages associated with the emissions of a $k$-type firm are given by $D(e_k, z_k) = h(e_k)(m - sz_k)$ with $h' > 0, h'' \geq 0$. The parameters $m$ and $s$ with $s < m$ allow us to isolate the effects of the direct benefit of disclosure. $m$ parameterizes the damage from undisclosed emissions; with no disclosure the total and marginal damages from emissions are $m \cdot h(e_k)$ and $m \cdot h'(e_k)$ respectively. $s$ parameterizes the difference between the damage of undisclosed and disclosed emissions; with full disclosure the total and marginal damages from emissions are $(m - s) \cdot h(e_k)$ and $(m - s) \cdot h'(e_k)$ respectively. For an interior solution on $z_k$, the
marginal damage of emissions is \( \frac{\partial D(e_k, z_k)}{\partial e_k} = h'(e_k)(m - sz_k) > 0 \). Thus, changes in \( m - sz_k \)

pivot the marginal (and total) damage of emissions. \(^{12}\) For the special case of a linear damage function \( h''(z_k) = 0 \), \( m - sz_k \) represents the constant damage per unit of emissions given the firm’s degree of disclosure, \( z_k \). We assume it is never optimal to induce over compliance \( (z_k > 1) \) from a firm of any type, which requires that the maximum audit bias \( \varepsilon \) is not too large. \(^{13}\) The \( k \)-type firm pays expected total taxes and fines equal to \( e_k \cdot \mu_k \). Thus, component (1) for the \( k \)-type firm is

\[ h(\theta_k)(m - sz_k) - e_k \cdot \mu_k. \]

The \( k \)-type firm’s net benefit from emissions (component (2)) is represented by

\[ B(e_k) - e_k \cdot \mu_k. \]

Lastly, enforcement cost per firm is constant and equal to \( pw \). Thus, combining terms, the \( k \)-type firm’s contribution to \( V \), denoted \( V_k \), given its optimal emissions and report is:

\[ V_k = h(\theta_k)(m - sz_k) - B(e_k') + pw. \]

Taking expectations across firms yields the expression for \( V \):

\[ V = \int_{-\varepsilon}^{\varepsilon} \{ h(\theta_k'(m - sz_k') - B(\theta_k') + pw \} dG(k). \]

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\(^{12}\) One might consider an alternative damage function of the form \( D(e_k, z_k) = h(e_k)(z_k + \gamma(1 - z_k)) \), where \( \gamma > 1 \) parameterizes the additional marginal damage caused by undisclosed emissions relative to disclosed emissions. However, the key comparative static results on the regulator’s choice variables are ambiguous with respect to a change in \( \gamma \). The reason is that an increase in \( \gamma \) increases the reduction in damage achieved by disclosure, but also increases the total harm from emissions for any given level of disclosure.

\(^{13}\) Presumably mitigation of social damages from emissions is maximized with full disclosure \( (z_k = 1) \), and disclosure in excess of this could in fact have a social cost (e.g. by leading to excessive avoidance or clean-up efforts regarding the polluted resource). Therefore it would never be optimal for the regulator to induce over-compliance in expectation across firm types. Our assumption that no type of firm will overstate actual emissions amounts to assuming that the cost of enforcement is sufficiently great and the maximum audit bias \( (\varepsilon) \) small enough that at optimum even firms with a very high \( k \) type report \( z_k < 1 \).
We assume the regulator minimizes $V$ with respect to his choice of $\alpha$, the tax on reported emissions, and $p$, the audit probability.\footnote{In modeling the policy choices available to the regulator we have not allowed the regulator to choose a deposit-refund instrument in lieu of a tax. Swierzbinski [22] finds a deposit-refund system to be optimal in a model of regulation with self reporting. However, as discussed earlier, the role of self reporting in Swierzbinski’s model is quite different than in ours because it arises as a result of the regulator’s uncertainty about a firm’s pollution abatement costs (absent any direct benefits of disclosure). A deposit-refund scheme would not be optimal in general in our context because it raises the enforcement cost of internalizing social damages. Although a deposit-refund scheme could be optimal in our context under certain conditions, we’ve chosen to constrain the regulator to using a Pigovian tax both for simplicity and because deposit-refund mechanisms are not broadly utilized in environmental regulation (particularly in the U.S., see EPA [23]).} Therefore, we assume $\beta$, the marginal penalty on revealed but unreported emissions, is exogenous. In the context of our model the regulator would always do best to set this penalty as high as possible because doing so achieves the highest compliance given any tax with the least enforcement costs. This fairly standard result leads us to simply assume that the regulator faces some constraint on the magnitude of the penalty that can be imposed.$^{15}$

The first order conditions for an interior solution to the regulator’s problem follow:

\[
\begin{align*}
\int_{-\infty}^{\infty} & \left\{ h'(e_k) e'\mu_k \frac{\partial h_k}{\partial \alpha} (m - s \mu_k^*) - B'(e_k) e'\mu_k \frac{\partial h_k}{\partial \alpha} - h(e_k, \mu_k^*) k \frac{\partial z_k^*}{\partial \alpha} \right\} dG(k) = 0 \quad (6) \\
\int_{-\infty}^{\infty} & \left\{ h'(e_k) e'\mu_k \frac{\partial h_k}{\partial p} (m - s \mu_k^*) - B'(e_k) e'\mu_k \frac{\partial h_k}{\partial p} - h(e_k, \mu_k^*) k \frac{\partial z_k^*}{\partial p} + w \right\} dG(k) = 0 \quad (7)
\end{align*}
\]

Equation (6) indicates that the regulator chooses $\alpha^*$ to equate the marginal benefit of a higher tax (due to lower emissions) with the marginal cost of a higher tax (due to less truthful reporting). Similarly equation (7) illustrates that $p^*$ equates the marginal benefit of increased audit frequency (greater disclosure and reduced emissions) and the marginal cost (additional audit resources).

$^{14}$ See, for example, Becker [2] and Harrington [7]. This assumption can also be grounded in the argument that the marginal penalty may include factors which are outside the regulator’s control such as the market’s reaction to news that a firm underreported its actual emissions or explicit fines and increased liability resulting from an independent judiciary process (Garvie and Keeler [4]).
Both a higher tax and higher audit probability achieve greater internalization of social costs (and thus a reduction in emissions), but each is costly in a different way. A higher tax reduces disclosure, which is costly when disclosure has direct benefits. A higher audit probability is directly costly as more resources are devoted to enforcement. To understand the interplay between these choices, consider the two extreme cases regarding the value of disclosure. First, suppose disclosure has no direct benefit so $s = 0$. In this case there is no interior solution on $\alpha$; it is optimal to set $\alpha^* \geq p\beta$ (in which case all firms disclose nothing). This achieves the greatest internalization of social costs (arising entirely through fines rather than taxes) with the least expenditure on enforcement. The optimal audit probability, $p^*$, will reflect the marginal benefit of reduced emissions resulting from internalization relative to the marginal cost of auditing, and an interior solution will exist for $w$ sufficiently large. At the other extreme, suppose $s = m$ so that with full disclosure, emissions are no longer damaging. In such a case the optimal policy involves zero tax on reported emissions. Full compliance with the disclosure requirement can then be achieved with a negligible audit probability. Although this extreme case may seem unrealistic, it conveys important intuition: as $s$ approaches $m$ the optimal policy may be minimal taxation and infrequent auditing. Auditing is costly for the regulator and high compliance rates can still be achieved with a low probability of audit when the tax on reported emissions is also low.

An interior solution in both dimensions of the regulator’s choice will exist if $s$ is sufficiently large but strictly less than $m$ (i.e., the damage of emissions are sufficiently reduced but not completely eliminated by disclosure) and if the cost of auditing, $w$, is sufficiently large. We assume this is the case and focus our analysis on the comparative statics at an interior solution. To sign the comparative statics on $\alpha^*$ also once again requires that the maximum audit
bias $\varepsilon$ is not too large (in a manner that is defined in the proof in the appendix). With these assumptions, we state the following proposition:

**Proposition 1.** The regulator’s optimal tax, $\alpha^*$, is increasing in $m$ and decreasing in $s$. The optimal audit probability $p^*$ is decreasing in the cost of auditing, $w$.

**Proof.** See appendix.

The comparative static results regarding the optimal tax are broadly intuitive. The regulator trades-off internalizing social costs with a higher tax against the consequent reduction in disclosure; the more valuable is disclosure (due to higher $s$), the lower the optimal tax. Conversely, the more socially costly all emissions are (as represented by $m$), the higher the optimal tax in order to achieve greater internalization of these costs and lower resulting emissions. The effect of the cost of auditing, $w$, on $\alpha^*$ is ambiguous. A higher cost of auditing, $w$, does not directly affect the optimal tax but will of course reduce the optimal audit probability, $p^*$. Whether the optimal tax increases or decreases with an increase in $w$ depends on how the decrease in the audit probability affects the marginal benefit and cost of the tax. The expression for $\frac{\partial \alpha^*}{\partial w}$ is provided in the appendix.

Unlike the comparative statics for the optimal tax, the directions of the effects of $m$ and $s$ on the optimal audit frequency are in general ambiguous (see appendix for both expressions). Consider first the effect of $m$. As the damage of emissions rises (holding constant the reduction that occurs due to disclosure, $s$) the marginal benefit of internalization rises. For this reason it seems intuitive that the optimal audit probability would rise as well, since raising $p$ increases firms’ costs of emitting. However, an increase in $m$ increases the optimal tax $\alpha^*$ as stated in Proposition 1. This in turn increases $\mu^*$ for firms of all types and reduces emissions *ceteris*
A reduction in emissions reduces the marginal benefit of increased disclosure and therefore reduces the value of auditing with regards to achieving higher rates of disclosure. If the firm’s elasticity of demand for emissions is very high, then the optimal response to an increase in $m$ may be to raise the tax to reduce emissions but reduce the audit probability. Thus, we cannot exclude the possibility that $\frac{\partial p^*}{\partial m} < 0$. However, were the regulator restricted to choosing only $p$, with $\alpha$ fixed, then we find unambiguously $\frac{\partial p^*}{\partial m} > 0$.

The ambiguity of the effect of an increase in $s$ on the optimal audit probability is more easily understood. An increase in $s$ has opposing effects on the value of auditing. A higher $s$ increases the value of disclosure, which increases the marginal benefit of auditing. However, the higher $s$ decreases the value of internalizing the damages of emissions because the higher $s$ reduces the damages of emissions at any given level of disclosure. This decreases the marginal benefit of auditing. Either effect may dominate.

IV. Conclusion

When information disclosure has direct social benefits but is costly for a firm and enforcement is costly and imperfect a regulator must confront the competing objectives of inducing disclosure and internalizing social costs. This tension is clearly present in many environmental regulatory contexts where the harm from emissions can be mitigated if potentially impacted parties have better information about the nature and quantity of emissions. It also exists in other regulatory settings such as product safety regulation. Disclosure of product defects and hazards has direct social benefits, but it is desirable that firms face a cost (either liability or fines) when their products cause harm in order to induce care.
There are certainly many avenues for future work in this area. One could imagine two policymakers, one of whom chooses a tax and the other the audit probability (e.g., legislature and executive or regulatory agency) but who have different objective functions and interact strategically. A regulator may have other policy instruments at his discretion, including choosing the audit probability for a firm in a dynamic setting based on past behavior. One also might consider an endogenous audit process in which the probability of audit is a decreasing function of disclosed emissions. We have not modeled the choice between putting enforcement resources into more frequent audits or more effective audits. Clearly a regulator must achieve an optimal balance, and the model we’ve developed could provide a framework for exploring this issue. We have assumed that disclosure costs (tax) and penalties are constant per unit, and that audit effectiveness is independent of firm size or total emissions. Relaxing these assumptions significantly complicates the analysis, but could inform important issues regarding how regulation affects industry structure.
Appendix—Proof of Proposition 1:
Rewrite the regulator’s objective function as follows:

\[ V = \int_{-\epsilon}^{\epsilon} \left\{ h(e_k(\mu_k^*)(m-sz_k^*)) - e_k^* \lambda_k^* - \int_{\mu_k}^{\epsilon} e(\rho) d\rho + pw \right\} dG(k) \]

since \(-B(e_k^*) = -e_k^* \lambda_k^* - \int_{\mu_k}^{\epsilon} e(\rho) d\rho\) where \(\lambda_k^*\) represents the \(k\)-type firm’s choke price for emissions. This substitution employs the fact that the \(k\)-type firm’s maximized benefit of emitting (given \(z_k^*\)) can be represented by the area under its emissions demand curve up to \(e_k^*\).

Then, the first order conditions are given by:

\[ \int_{-\epsilon}^{\epsilon} \left\{ h'(e_k) e'(\mu_k^*) \frac{\partial \mu_k^*}{\partial \alpha} (m-sz_k^*) - \mu_k^* e'(\mu_k^*) \frac{\partial \mu_k^*}{\partial \alpha} - h(e_k(\mu_k^*)) \frac{\partial z_k^*}{\partial \alpha} \right\} dG(k) = 0 \] (A1)

\[ \int_{-\epsilon}^{\epsilon} \left\{ h'(e_k) e'(\mu_k^*) \frac{\partial \mu_k^*}{\partial p} (m-sz_k^*) - \mu_k^* e'(\mu_k^*) \frac{\partial \mu_k^*}{\partial p} - h(e_k(\mu_k^*)) \frac{\partial z_k^*}{\partial p} + w \right\} dG(k) = 0 \] (A2)

Define the \(k\)-type firm’s contribution to \(V\) as follows:

\[ V_k = h(e_k(\mu_k^*)) (m-sz_k^*) - e_k^* \lambda_k^* - \int_{\mu_k}^{\epsilon} e(\rho) d\rho + pw, \] so \( V = \int_{-\epsilon}^{\epsilon} [V_k] dG(k) \).

For a given \(k\), the elements of the Hessian matrix of \(V_k\) are given below. Assume initially that \(h'(e_k^*)(m-sz_k^*) - \mu_k^* \geq 0\) for all \(k\). This condition states that at the regulator’s optimum no firms bear a higher cost if emitting than the marginal social damage of their emissions. Since (A1) implies that \( \int_{-\epsilon}^{\epsilon} h'(e_k) e'(\mu_k^*) dG(k) > 0\), this condition is not very restrictive. Below we address the case when this does not hold.

\[ \frac{\partial^2 V_k}{\partial \alpha^2} = \hat{f}_{11} = e'(\mu_k^*) \left\{ h'(e_k) (m-3sz_k^*) - \mu_k^* \frac{\partial z_k^*}{\partial \alpha} - (z_k^*)^2 \right\} - sh(e_k) \frac{\partial^2 z_k^*}{\partial \alpha^2} \]

\[ + e''(\mu_k^*) \left\{ h'(e_k) (m-3sz_k^*) - \mu_k^* \frac{\partial z_k^*}{\partial \alpha} + h''(e_k) e'(\mu_k^*) (m-sz_k^*) (z_k^*)^2 \right\} > 0 \forall k \]

\[ \frac{\partial^2 V_k}{\partial p^2} = \hat{f}_{22} = e'(\mu_k^*) \left\{ h'(e_k) (m-3sz_k^*) - \mu_k^* \frac{\partial z_k^*}{\partial p} - 2sh(e_k) \frac{\partial^2 z_k^*}{\partial p^2} - h''(e_k) e'(\mu_k^*) (m-sz_k^*) \left( \frac{\partial \mu_k^*}{\partial p} \right)^2 \right\} - sh(e_k) \frac{\partial^2 z_k^*}{\partial p^2} \]

\[ + e''(\mu_k^*) \left\{ h'(e_k) (m-3sz_k^*) - \mu_k^* \left( \frac{\partial \mu_k^*}{\partial p} \right)^2 + h''(e_k) e'(\mu_k^*) (m-sz_k^*) \left( \frac{\partial \mu_k^*}{\partial p} \right)^2 \right\} > 0 \forall k \]

\[ \frac{\partial^2 V_k}{\partial \alpha \partial p} = \hat{f}_{12} = e'(\mu_k^*) \left\{ h'(e_k) (m-3sz_k^*) - \mu_k^* \frac{\partial z_k^*}{\partial \alpha} - sh(e_k) \frac{\partial^2 z_k^*}{\partial \alpha \partial p} - h''(e_k) e'(\mu_k^*) (m-sz_k^*) \left( \frac{\partial \mu_k^*}{\partial \alpha} \right)^2 \right\} \]

\[ - sh(e_k) \frac{\partial^2 z_k^*}{\partial \alpha \partial p} + e''(\mu_k^*) \left\{ h'(e_k) (m-3sz_k^*) - \mu_k^* \left( \frac{\partial \mu_k^*}{\partial \alpha} \right)^2 + h''(e_k) e'(\mu_k^*) (m-sz_k^*) \left( \frac{\partial \mu_k^*}{\partial \alpha} \right)^2 \right\} \]

Note that for an interior solution, \(\hat{H} = \hat{f}_{11} \hat{f}_{22} - \left( \hat{f}_{12} \right)^2\) must be positive. Therefore, \(\hat{f}_{11} > 0 \forall k\) since \(\hat{f}_{22} > 0 \forall k\).
Additional second order effects follow:

\[
\frac{\partial^2 V_k}{\partial \alpha \partial m} = \hat{f}_{1m} = h'(e_k)e'(\mu_k^*)z_k^* < 0 \forall k \quad (A3)
\]

\[
\frac{\partial^2 V_k}{\partial p \partial m} = \hat{f}_{2m} = h'(e_k)e'(\mu_k^*)\frac{\partial \mu_k^*}{\partial p} < 0 \forall k \quad (A4)
\]

\[
\frac{\partial^2 V_k}{\partial \alpha \partial s} = \hat{f}_{1s} = -h(e_k)\frac{\partial z_k^*}{\partial \alpha} - h'(e_k)e'(\mu_k^*)z_k^* > 0 \forall k \quad (A5)
\]

\[
\frac{\partial^2 V_k}{\partial p \partial s} = \hat{f}_{2s} = -h(e_k)\frac{\partial z_k^*}{\partial p} - h'(e_k)e'(\mu_k^*)\frac{\partial \mu_k^*}{\partial p} \forall k \quad (A)
\]

\[
\frac{\partial^2 V_k}{\partial \alpha \partial w} = \hat{f}_{1w} = 0 \forall k
\]

\[
\frac{\partial^2 V_k}{\partial p \partial w} = \hat{f}_{2w} = 1 \forall k 
\]

Having signed these expression for any specific value of the random variable \( k \) we can sign the comparative statics.

\[
\frac{\partial \alpha^*}{\partial m} = \int_{-\alpha}^{\alpha} \left[ \frac{1}{H} - \hat{f}_{1m} \hat{f}_{2m} \hat{f}_{12} \right] dG(k) = \int_{-\alpha}^{\alpha} \left[ \frac{1}{H} - \hat{f}_{1m} \hat{f}_{2m} \hat{f}_{12} \right] dG(k) > 0 \quad \text{since} \quad |\hat{H}| = \hat{f}_{11} \hat{f}_{22} - \hat{f}_{12}^2 > 0 \forall k \quad \text{and}
\]

\[
- f_{1m} \hat{f}_{22} + \hat{f}_{2m} \hat{f}_{12} = h'(e_k)e'(\mu_k^*)\left[ h'(e_k)(m - sz_k^*) - \mu_k^* \left( \frac{\partial \mu_k^*}{\partial p} - z_k^* \frac{\partial^2 \mu_k^*}{\partial p^2} \right) \right] + h'(e_k)e'(\mu_k^*) \left( \frac{\partial \mu_k^*}{\partial \alpha} - z_k^* \frac{\partial z_k^*}{\partial \alpha} \right) > 0 \forall k
\]

\[
\frac{\partial \alpha^*}{\partial s} = \int_{-\alpha}^{\alpha} \left[ \frac{1}{H} - \hat{f}_{1s} \hat{f}_{2s} \hat{f}_{12} \right] dG(k) = \int_{-\alpha}^{\alpha} \left[ \frac{1}{H} - \hat{f}_{1s} \hat{f}_{2s} \hat{f}_{12} \right] dG(k) > 0 \quad \text{since} \quad |\hat{H}| > 0 \forall k \quad \text{and}
\]

\[
- \hat{f}_{1s} \hat{f}_{22} + \hat{f}_{2s} \hat{f}_{12} = h(e_k)h'(e_k)e'(\mu_k^*)\left[ (m - sz_k^*)\frac{\partial \mu_k^*}{\partial p} \right] \left( \frac{\partial \mu_k^*}{\partial \alpha} - z_k^* \frac{\partial z_k^*}{\partial \alpha} \right) + h(e_k)h'(e_k)e'(\mu_k^*)z_k^* \left( \frac{\partial \mu_k^*}{\partial p} - z_k^* \frac{\partial z_k^*}{\partial p} \right) + h(e_k)h'(e_k)e'(\mu_k^*) \left( \frac{\partial \mu_k^*}{\partial \alpha} - z_k^* \frac{\partial z_k^*}{\partial \alpha} \right) < 0 \forall k
\]
The results above show that if $h'(e^*_k)(m-s^*_k) - \mu^*_k \geq 0$ for all $k$ then the comparative statics have the indicated signs. Although this condition must hold for firms in expectation at any regulator’s optimum, it may not hold for firms of high $k$ type if $c$ is large and $w$ and $s$ sufficiently small. However, $- \hat{f}_{1m}\hat{f}_{22} + \hat{f}_{1m}\hat{f}_{22} > 0$ and $- \hat{f}_{1s}\hat{f}_{2s} + \hat{f}_{1s}\hat{f}_{2s} > 0$ will still hold for all $k$ unless $h'(e^*_k)(m-s^*_k) - \mu^*_k$ is large in absolute value for some $k$. Furthermore, even if this is the case for a small mass of firms at the upper end of the range of $k$, it will not change the sign of $\frac{\partial \alpha^*}{\partial m}$ or $\frac{\partial \alpha^*}{\partial s}$ unless there is sufficient mass in the $G$ distribution on this upper range of $k$. We assume $\varepsilon$ is sufficiently small that this outcome does not occur.

We now derive the comparative static results for the optimal audit probability for any specific value of the random variable $k$. The comparative static result for $w$ on $\alpha^*$ is generally ambiguous:

$$\frac{\partial \alpha^*}{\partial w} = \frac{1}{|\hat{H}|} \left[ -\hat{f}_{1w}\hat{f}_{12} + \hat{f}_{1w}\hat{f}_{22} \right]$$

Since $|\hat{H}| > 0 \forall k$, the sign of $\frac{\partial \alpha^*}{\partial w}$ equals the sign of $\hat{f}_{12}$, which is given above and is generally ambiguous.

The signs of $\frac{\partial p^*}{\partial m}$ and $\frac{\partial p^*}{\partial s}$ are generally ambiguous. The respective expressions of the results for any value of $k$ follow:

$$\frac{\partial p^*}{\partial m} = \frac{1}{|\hat{H}|} \left[ -\hat{f}_{11}\hat{f}_{1m} + \hat{f}_{12}\hat{f}_{2m} \right]$$

$$\frac{\partial p^*}{\partial s} = \frac{1}{|\hat{H}|} \left[ -\hat{f}_{11}\hat{f}_{1s} + \hat{f}_{12}\hat{f}_{2s} \right]$$

where

$$- \hat{f}_{11}\hat{f}_{2m} + \hat{f}_{12}\hat{f}_{1m} = h'(e^*_k)[\nu(\mu^*_k)]h'(e^*_k)(m-2sz^*_k) - \mu^*_k\left( z^*_k \frac{\partial z^*_k}{\partial p} - \frac{\partial z^*_k}{\partial \alpha} \frac{\partial \alpha}{\partial p} \right)$$

$$+ h(e^*_k)h'(e^*_k)\nu(\mu^*_k)\left( \frac{\partial \mu^*_k}{\partial p} \frac{\partial z^*_k}{\partial \alpha} + z^*_k \frac{\partial \mu^*_k}{\partial \alpha} \frac{\partial \alpha}{\partial p} \right)$$

and

$$- \hat{f}_{11}\hat{f}_{2s} + \hat{f}_{12}\hat{f}_{1s} = h'(e^*_k)[\nu(\mu^*_k)]h'(e^*_k)(m-2sz^*_k) - \mu^*_k\left( z^*_k \frac{\partial z^*_k}{\partial p} - \frac{\partial z^*_k}{\partial \alpha} \frac{\partial \alpha}{\partial p} \right)$$

$$+ h(e^*_k)h'(e^*_k)\nu(\mu^*_k)\left( \frac{\partial \mu^*_k}{\partial p} \frac{\partial z^*_k}{\partial \alpha} + z^*_k \frac{\partial \mu^*_k}{\partial \alpha} \frac{\partial \alpha}{\partial p} \right)$$

for all $k$. We assume $\varepsilon$ is sufficiently small that this outcome does not occur.
\[- \dot{f}_{11} \dot{f}_{24} + \dot{f}_{14} \dot{f}_{11} = h(e_k) h''(e_k) \left[ e'(\mu_k^*) \right]^2 \left( m - sz_k^* \right)^2 \frac{z_k^*}{z_k^*} \left( \begin{array}{c}
\frac{\partial z_k^*}{\partial p} - \frac{\partial ^2 z_k^*}{\partial p \partial \alpha} \\
\frac{\partial z_k^*}{\partial \alpha} - \frac{\partial ^2 z_k^*}{\partial p \partial \alpha} 
\end{array} \right) \]

+ h'(e_k) \left[ e'(\mu_k^*) \right] z_k^* \left[ h'(e_k) \left( m - sz_k^* \right) - \mu_k^* \left( \frac{\partial z_k^*}{\partial p} - z_k^* \frac{\partial z_k^*}{\partial \alpha} \right) \right] + h(e_k) h''(e_k) e'(\mu_k^*) z_k^* \left( \begin{array}{c}
\frac{\partial ^2 z_k^*}{\partial p^2} - \frac{\partial ^2 z_k^*}{\partial p \partial \alpha} \\
\frac{\partial ^2 z_k^*}{\partial \alpha^2} - \frac{\partial ^2 z_k^*}{\partial p \partial \alpha}\end{array} \right)

+ h(e_k) \left[ e'(\mu_k^*) \right] \frac{\partial z_k^*}{\partial \alpha} \left( \frac{\partial z_k^*}{\partial p} - z_k^* \frac{\partial z_k^*}{\partial \alpha} \right) + \left[ h(e_k) - h'(e_k) \right] e'(\mu_k^*) z_k^* \left( \begin{array}{c}
\frac{\partial z_k^*}{\partial p} - z_k^* \frac{\partial z_k^*}{\partial \alpha} \\
\frac{\partial z_k^*}{\partial \alpha} - z_k^* \frac{\partial z_k^*}{\partial \alpha}\end{array} \right)

+ \left[ h(e_k) \right] e''(\mu_k^*) \left[ h'(e_k) \left( m - sz_k^* \right) - \mu_k^* \right] z_k^* \left( \begin{array}{c}
\frac{\partial z_k^*}{\partial p} - \frac{\partial ^2 z_k^*}{\partial p \partial \alpha} \\
\frac{\partial z_k^*}{\partial \alpha} - \frac{\partial ^2 z_k^*}{\partial p \partial \alpha}\end{array} \right)
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