

SOMS 662 Statistical Computing

Homework 1. Due Sept 14, 2004

These are some problems to try. Please let me know if you are unsure about something or find mistakes. This is the first time I've taught this so let me know if things are too difficult or easy. Please don't feel that you need to spend too much time on this homework.

1. Monahan (2.4 pg. 33)

- (a) Prove that $n!/k!(n-k)!$ is always an integer.
- (b) For your computer, for what value of n will (2.32) lead to an overflow?

2. Knuth gives some transcendental numbers in octal (base 8). Consider π ,

$$\pi = 3.11037552421026430215142306_{\text{eight}}$$

- (a) Convert these to six-digit decimal floating point numbers and check the result. If you are using matlab see if you can change formats.
- (b) Write this number as a floating point number.

3. Monahan (2.14 pg. 34) Find the values of t where cancellation will be serious in the following expressions, and rework to avoid cancellation.

- (a) $1/t - 1/\sqrt{t^2 + a}$;
- (b) $\exp(-2t^2) - \exp(-8t^2)$;
- (c) $\log(e^{(t+s)} - e^t)$;
- (d) $[(1 + e^{-t})^2 - 1]t^2e^{-t}$.

4. Monahan (2.16 pg. 34) Following (2.7.2), compute the condition number for each of the following functions $f(x)$, and give the values of x for which it is large.

- (a) $f(x) = e^x$;
- (b) $f(x) = \log(x)$;
- (c) $f(p) = \Phi^{-1}(p)$;

5. By hand, compute the LU factorization of X and the Cholesky factorization of $X'X$ where X is given by

$$\begin{pmatrix} 0 & 0 & -1 & 2 \\ -1 & -1 & 1 & 2 \\ 2 & 1 & -3 & 6 \\ 0 & 1 & -1 & 4 \end{pmatrix}$$

6. Monahan (3.4 pg. 58) Let \mathbf{A} be positive definite, and let $\mathbf{A}^{(k)}$ be the result of k steps of Gaussian elimination (without pivoting). Partition both \mathbf{A} and $\mathbf{A}^{(k)}$ at k rows and columns to form

$$\mathbf{A} = \begin{pmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{C}^T & \mathbf{D} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \mathbf{B}^* & \mathbf{C}^* \\ \mathbf{0} & \mathbf{D}^* \end{pmatrix}$$

Show that $\mathbf{D}^* = \mathbf{D} - \mathbf{C}^T \mathbf{B}^{-1} \mathbf{C}$. Where is this matrix found in statistics? If \mathbf{A} is positive definite, is pivoting necessary?

7. **(Optional)** Construct a program to determine the smallest and largest representable numbers in single precision. Determine the smallest and largest representable negative numbers in single precision. Determine the smallest and largest representable negative numbers in double precision.